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R

ROGERI COTESII
HARMONIA MENSURARUM.

HARMONIA MENSURARUM,

SIVE

ANALYSIS & SYNTHESIS

Per RATIONUM & ANGULORUM MENSURAS

PROMOTÆ:

ACCEDUNT ALIA

OPUSCULA MATHEMATICA:

PER

ROGERUM COTESIUM.

——————— *Si propius stes,*
Te capient magis.———————

EDIDIT ET AUXIT
ROBERTUS SMITH
Collegii S. *Trinitatis* apud *Cantabrigienses* Socius;
Astronomiæ & Experimentalis Philosophiæ
Post COTESIUM Professor.

CANTABRIGIÆ, MDCCXXII.

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VIRO

RICHARDO MEAD, M.D.

COLLEGII REGALIS

MEDICORUM LONDINENSIUM

ET

SOCIETATIS REGIÆ

SOCIO

S. P. D.

ROBERTUS SMITH.

Cum promiſſam hujus Operis editionem qua potui fide ac diligentia abſolviſſem; non parum mihi gratulatus ſum, tum quod amicorum expectationi ſatis (ut ſpero) feciſſem, tum quod juſtam ad Te, Vir præſtantiſſime, ſcribendi occaſionem (quam diu mihi

mihi optaveram) nactus eſſem. Nec dubito, quin voluptatis hujus pars aliqua ad Te quoque redundet; cum probe intelligam nihil Tibi antiquius eſſe magiſque in votis, quam ut optimæ artes incrementa ſumant, noviſque indies inventis illuſtrentur. Non eſt itaque, cur hoc ſtudii genus apud Te efferam & verbis ornem: qui non ſolum in aliorum laude illud ſemper poſuiſti publice, ſed in uſu atque oblectatione Tecum habuiſti. Neque de Auctore hujus operis mihi opus eſt dicere, cujus virtutem ex oculis nuper eſſe ſublatam præſens ætas cum maxime dolet; & ingenio eum raro & inſigni præditum fuiſſe vel hoc ipſum Opus poſteris teſtabitur, Scriptoriſque memoriam vetabit interire. Diſcrucior itaque ſubinde; cum tali me amico orbatum ſentio, quicum diu in eodem cubiculo amantiſſime & conjunctiſſime vixi. Feram autem moderatius dolorem quem ex Cotesii deſiderio accipio, ſi Te mihi aliquo obſervantiæ & amoris pignore

pignore arctius adjunxero. Is ego ſum profecto, qui doctorum & illuſtrium hominum familiaritatem imprimis expeto; qua vel ad otium nihil dulcius, vel ad negotium nihil utilius. Itaque cur ego non ad Te præcipue accurram? ad quem egregii quique viri atque omni literarum genere ornati maximo affectu ducuntur: qui omnia illa, quæ a ſummo atque optimo quovis expectari debent, de Tua amicitia non tam ſibi pollicentur olim, quam nunc repræſentata poſſident. Quam meritiſſimam laudem Tuam ita boni omnes ubique prædicant; ut ſæpe mihi mirari ſubeat tantum Tibi animi corporiſque robur eſſe, ut & in maxima celebritate atque oculis civium verſari poſſis, & in otio ac ſeceſſu non Tibi ſolum ſed amicis vacare: ut ſimul in gravi Medicinæ munere principatum exerceas, ſimul omnium fere literarum cognitioni ſtudioque incumbas. Neque aliter hæc fieri poſſe exiſtimem, quam peculiari quodam munere naturæ; quæ certe parum

duxit

duxit naſcenti Tibi unum quid magnum & ſingulare tribuiſſe, niſi cum omnibus ſuis dotibus foret conjunctum. Quibus ego invitatus ac fretus, hoc & abs Te ſpero & mihi polliceor, munuſculum meum quanto cum ſtudio a me profectum ſit, tanta Te humanitate fore accepturum.

CANTABRIGIÆ
e Collegio S. Trinitatis
Apr. 16 1722.

EDITORIS

EDITORIS

PRÆFATIO.

IN manus tibi trado, Lector benevole, præstantissimi Viri & excellentis ingenii magnum monumentum: postumum quidem; sed adeo cultum & perpolitum, ut non tam dolendum sit quod ultima Auctoris manus non accesserit, quam quod majora indies inquirenti immatura mors supervenerit. Problemata maximi momenti, de Logarithmorum constructione ac insigni quodam usu; de curvarum Longitudine, Areis, Solidis inde genitis & solidorum Superficiebus; de corporum Viribus Attractivis; de fluidorum Densitate ac Resistentia; de planetarum Trajectoriis & id genus multa, non solum nova ratione hic composita sed ex apertis fontibus per analysin derivata reperies: utramque sane tam facilem & elegantem qualem alibi frustra quæras: ut in hoc potissimum incertus sis, utrum plus admireris Auctoris nostri inventa, an ipsam inveniendi rationem.

Is enim aliorum methodis non contentus, ad Problematum Resolutiones novam sibi & compendiariam patefecit viam. Audiamus ipsum in hac Logometria *narrantem.* " *In eum* " *potissimum finem praecedentia conscripsi, ut allatis aliquot Ex-* " *emplis ostenderem, qua commodissima ratione Logometrici* " *(& Trigonometrici) Canonis usus in Geometriam recipi, & ad* " *Resolutionem Problematum difficiliorum adhiberi possit. Problematum nempe solutiones, quas ad Hyperbolae & Ellipseos Quadraturam ante ipsum alii perduxerunt, primus ille ad Rationum & Angulorum Mensuras perduxit. Hae mensurae sunt quantitates cujuscunque generis; quarum magnitudines magnitudinibus rationum & angulorum sunt analogae. Termini autem rationis dimetiendae in problematis difficilioribus sumuntur a lateribus trianguli rectanguli; in quo reperitur etiam angulus dimetiendus. Haec sunt Principia & pars theoretica novae hujus Geometriae: quam cum ad Praxin perducere volumus, primo quidem per Tabulas memoratas quaerendus est vel rationis dimetiendae Logarithmus, vel Numerus Graduum in angulo dimetiendo; deinde datis tribus terminis inveniendus est quartus proportionalis: sic enim semper habebitur utriusvis generis mensura, quae problemati proposito satisfaciat.* " *His Principiis (inquit Auctor) haud* " *facile crediderim generaliora dari posse; cum tota Mathesis* " *vix quicquam in universo suo ambitu complectatur praeter* " *angulorum & rationum Theoriam. Neque sane commodiora sperabit, qui animadverterit Effectionis hujus facilitatem; quae in omnibus problematis nihil quicquam desciscit ab ea praxi simplicissima quam modo descripsi. Nimius essem fortasse si hujus methodi commoditates reliquas sigillatim explicarem; quippe quas exem-*

exemplorum inductio clarius illustraret, quam ulla verborum descriptio. Generatim itaque hoc solum animadvertendum censeo; nempe, cum rationum magnitudines (pariter ac ipsi termini) per quantitates cujuscunque generis exponi possunt; Geometris integrum esse quodlibetcunque earum genus pro re nata seligere: ut neque necesse sit Logometricæ cujuspiam Figuræ subsidium comparare, neque alias fere ullas quantitates adhibere, quam quas ipsæ suppeditent propositi problematis conditiones. Id quod intelligi potest de angulorum mensuris.

Istiusmodi Problematum Constructiones perquam elegantes traduntur in Logometria; *quæ est Operis hujus Pars prima & in Actis* Regiæ Societatis *nostræ non ita pridem edita. Huic jam Pars secunda & tertia hoc consilio accedunt, ut Problemata resolvendi facultatem consequi possis. Scilicet in Parte secunda continentur* Theoremata tum Logometrica tum Trigonometrica; *quæ datarum Fluxionum Fluentes exhibent, non per areas conicas pro more consueto, sed per mensuras rationum & angulorum. Atque his Theorematis alia insuper adjicere constituerat Auctor, si ulteriorem vitæ usuram Deus dedisset. Auctoris itaque institutum prosecutus sum; novasque composui Theorematum Tabulas superioribus illis longe ampliores.*

In Parte tertia, ubi agitur de Problematum Analysi & Constructione, *Theorematum usus uberrimis exemplis comprobatur & illustratur. Mirabilis autem* Mensurarum *utriusque generis* Harmonia, *quæ in hac problematum resolutione reperitur, etiam in operis fronte inscribi meruit. Ad calcem Libri addidi*

Notas aliquas; quibus ea potissimum explicavi quæ ad Operis scopum maxime pertineant: Et, cum a Mathematico perfecto hæc tria præcipue requirantur, Resolutio, Compositio & Demonstrandi Ratio, adjeci Methodum componendi Demonstrationes Logometricas & Trigonometricas, *variis Exemplis patefactam & explanatam. Quo in genere non dubito quin ipse Auctor hujusmodi quippiam adjungere voluisset, si inventa sua perficere & in lucem edere ipsi concessum fuisset.*

Sic tibi novæ hujus Geometriæ imaginem quandam vel potius umbram descripsi & repræsentavi; quod solum licuit in re nova & non vulgari. Sed si ipsos tanti luminis fontes intimosque recessus penitus explorare & aperire velis, in quibus præcipua artis arcana, obscuritate nimium involuta, hactenus latuerunt; spondeo tibi polliceorque incredibilem animi voluptatem summa cum utilitate conjunctam. Nec vereor equidem ne vanis te promissis lactasse videar, si Auctoris hujus inventa singula & limatum scribendi genus ad cœlum usque laudibus efferam. Sed ne forte existimes; vel maximarum rerum difficultatem novitatemque mihi supra meritum placuisse, vel Auctoris intimam amicitiam & cognationem ipsam judicio meo lenocinatam fuisse; ut minuam hanc opinionem & spem tibi erigam magis, unum instar omnium appellabo judicem NICOLAUM SANDERSONUM; *mihi mutuo amore suoque merito conjunctissimum; in Academia nostra* Matheseos Professorem & Societatis Regiæ Socium: *Qui, cum ab ipsa infantia captus oculis fuerit, neque se unquam vidisse nunc meminerit, magno tamen naturæ miraculo non modo recondita quæque Matheseos arcana Mentis oculis*

perlu-

perlustravit, sed & ejus pomæria protulit; quin & omnem de Lumine, Umbra, Coloribus totaque Optica Doctrinam prodigioso ingenio comprehendit. Is talis tantusque Vir Auctoris nostri inventa & scribendi rationem cum maxime admiratur, & in deliciis habet ultroque laudat. Atque hæc quidem quanquam ita sint; non tamen tanta mea est fiducia, quin fieri posse putem ut promissis meis eventus parum respondeat. Quod si acciderit; hoc unum modo te admonere liceat, ut te ipse diligenter excutias; ne aut prava quædam superstitio ita tibi jam furtim obrepserit, ut nihil quicquam novari velis, aut in his penitus expendendis negligentius quam par est versatus fueris.

Quæ potissimum ad Astronomiam instruendam perficiendamque Auctor noster excogitaverit, in Operibus Miscellaneis *hic edita invenies. Tractatum scilicet admodum utilem in Mixta Mathesi ad* Errorum Limites definiendos computandosque; *quos per sensuum nostrorum finitam vim, variasque Instrumentorum imperfectiones penitus devitare non possumus. Huc accedit de* Methodo Differentiali NEWTONIANA *tractatus; multo ante ab Auctore nostro conscriptus, quam* NEWTONI *de eodem argumento tractatus in lucem ederetur. His etiam subnectitur tertius tractatus* de Constructione Tabularum per Differentias. *Nimirum ut satis subsidii sit ad tabulæ cujusvis constructionem, præcipuos aliquot & rariores numeros vel per Theoriam vel per Observationes utcunque invenisse. Horum namque intervalla, per propositiones hic expositas, numeris quotlibet intermediis ita compleri possunt, ut universorum lex & progressio æque justa sit & æquabilis, ac si singuli per theoriam primitus inventi fuissent.*

Horum

Horum operum agmen claudunt Schedulæ tres; de Deſcenſu Gravium, de Motu Pendulorum in Cycloide *&* de Motu Projectilium. *Quorum omnium theoriam tanta felicitate in compendium adduxit mireque contraxit, (quanquam juvenis admodum erat cum hæc ſcriberet,) ut vel ipſa brevitas perſpicuitati plurimum proſit. Id quod pariter dicendum eſt de aliis nonnullis quæ nondum edidi. Scilicet* de Compendio Arithmetices, de Reſolutione Æquationum *&* de Problemate Dioptrico univerſali*: in quo elegantiſſima ratione geometrica definivit Viſibilis Magnitudinem apparentem, Situm (erectum vel everſum,) Locum apparentem gradumque Diſtinctionis; dum viſibile conſpicitur per quotlibet Lentes, cujuſcunque generis & ad quæcunque intervalla tum inter ſe tum inter oculum & viſibile diſpoſitas. Quo uno problemate Dioptricam tantum non univerſam exhauſit. Scripſit etiam Tractatum* de Natura Curvarum; *quarum genera & ſpecies diſtinguit figuraſque deſcribit & primarias proprietates eruit ex æquationibus algebraicis. Et quanquam agat præcipue de curvis primi Generis (ſcilicet de Conicis Sectionibus, quæ originem trahunt ex æquationibus quadraticis); inſigniores tamen earum proprietates ad ſuperiorum generum curvas pertinere ſimilibus argumentis oſtendit: quibus alia ſubnectit, cum de Locorum & Æquationum Conſtructione tum de Problematum inde pendentium Reſolutione: Atque hæc omnia (quæ in lucem edere in animo habeo) ab ipſis fundamentis ea ratione pertractavit, quæ facilitate ſua expectationem hominum non impleat modo ſed vincat.*

Cum

Cum in Scientiarum finibus undeque proferendis Auctoris nostri felicissimam diligentiam satis (ut arbitror) tibi perspectam dederim; quidni & hæc quoque leviter attingam, quæ ad laudum suarum historiam ulterius pertineant. Itaque cum Vir Reverendus THOMAS PLUME, *S. T. P. & Archidiaconus* Roffensis *Astronomicam Professionem nostram quibusdam optimis Viris instituendam commendaret; illi continuo, negotio illo vix dum peracto, ex tot præclaris ingeniis quot universus exhibuit orbis literatus, tanto tum munere digniorem censuerunt neminem quam* COTESIUM *nostrum; quanquam admodum juvenem atque Artium tantum Baccalaureum. Id quod suo quoque calculo comprobavit Philosophorum omnium princeps, celeberrimus* NEWTONUS.

Cotesius *itaque unanimi omnium suffragio ad hunc honorem evectus, in Philosophia naturali* Experimentorum Cursus *pro mandato* Plumiano *continuo instituit; & de his quoque Lectiones literis mandavit perquam elegantes & copiosas: quas & ego, hac dignitate Illi succedens, Auditoribus meis Academicis subinde expono. Multa quoque in Astronomia practica peragere constituerat; (Tabulas enim* Flamstedianas *&* Cassinianas *ad Solis & Satellitum motus computandos commodiores redidit; novasque componere aggressus est ad Lunæ Theoriam* Newtonianam;) *& peregerat certe, si Observatorium non defuisset. Quod quidem præfati Fundatoris jussu in Academia nostra tum primum apparari cœpit; & in eo curando* Cotesius *noster neque ulli labori nec sumptibus suis pepercit. Sed cum tanto operi perficiendo facultates nec dum satis suppeterent; velut alter Fundator ultro sub-*

venit

venit magnus noſter BENTLEIUS *Collegii S.* Trinitatis *Præfectus: Vir & de omni Literarum genere præclare meritus, & inter primaria ſæculi ſui ornamenta numerandus. Qui ex propenſo ſuo Aſtronomiæ promovendæ ſtudio, cum ad optima Inſtrumenta comparanda, tum ad Ædificium extruendum amplificandumque magnificos ſumptus contulit, tam de ſuo expenſos quam ab aliis impetratos. Is porro de Auctore noſtro (quem ſibi ſemper amiciſſimum habuit) Epitaphium ſequens compoſuit.*

EPITAPHIUM

H. S. E.

ROGERUS ROBERTI FILIUS COTES,
COLLEGI HUJUS S. TRINITATIS SOCIUS,
ASTRONOMIAE ET EXPERIMENTALIS
PHILOSOPHIAE PROFESSOR PLUMIANUS;

QUI IMMATURA MORTE PRAEREPTUS,
PAUCA QUIDEM INGENII SUI
PIGNORA RELIQUIT;
SED EGREGIA, SED ADMIRANDA,
EX INACCESSIS MATHESEOS PENETRALIBUS
FELICI SOLERTIA TUM PRIMUM ERUTA:

POST MAGNUM ILLUM NEWTONUM
SOCIETATIS HUJUS SPES ALTERA,
ET DECUS GEMELLUM:
CUI AD SUMMAM DOCTRINAE LAUDEM
OMNES MORUM VIRTUTUMQUE DOTES
IN CUMULUM ACCESSERUNT;
EO MAGIS SPECTABILES AMABILESQUE,
QUOD IN FORMOSO CORPORE
GRATIORES VENIRENT.

NATUS BURBAGII IN AGRO LEICESTRIENSI
JUL. X, 1682. OBIIT JUN. V. 1716.

* * *

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HARMONIA MENSURARUM

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ROGERI

ROGERI COTES
HARMONIA MENSURARUM.

PARS PRIMA.

LOGOMETRIA.

Eruditissimo Viro

EDMUNDO HALLEIO

Societatis Regalis Secretario

ROGERUS COTES, S. P.

Mitto tibi, hortatu Illustrissimi Præsidis NEWTONI, *quæ aliquot abhinc annis conscripseram de Rationibus dimetiendis. Tu vero, quum & Ipse dudum in eodem Argumento præclare versatus fueris, pro solito tuo candore, tentamen hoc qualecunque benigne accipies. Vale.*

AGITUR in hoc Tractatu de *Mensuris Rationum*. Hæ Mensuræ sunt quantitates cujuscunque generis, quarum magnitudines magnitudinibus rationum sunt analogæ. In dato itaque Systemate, rationis ejusdem eadem est mensura, duplicatæ dupla, triplicatæ tripla, subduplicatæ subdupla, sesquiplicatæ sesquialtera: denique quocunque modo per compositionem vel resolutionem auctæ vel diminutæ rationis, similiter aucta est vel diminuta mensura. Æqua-

PARS PRIMA. litatis ratio nullam habet magnitudinem, quia nullam addita vel detracta mutationem inducit; rationes quæ dicuntur majoris & minoris inæqualitatis contrarias habent magnitudinum suarum affectiones, quoniam in compositione & resolutione contraria semper efficiunt: itaque si mensura rationis quam habet terminus major ad minorem positiva censeatur, mensura rationis quam habet terminus minor ad majorem erit negativa, mensura vero rationis inter æquales terminos nullius erit magnitudinis. Porro diversa mensurarum oriuntur *Systemata*, prout modis diversis exponitur analogia illa determinata & immutabilis quæ est inter magnitudines rationum. Inde vero patet, exhiberi posse numero infinita Systemata, minuendo vel augendo Systematis cujusvis dati mensuras omnes in eadem data quacunque proportione, aut etiam pro mensuris adhibendo quantitates diversi generis. In tanta autem varietate confusionem aliquam oboriri necesse est, ni probe constiterit ad quodnam Systema referendæ sint mensuræ singulæ de quibus contingat sermonem institui. Huic malo remedium optime parari potest si mensura datæ alicujus rationis, quæ commodissima videbitur, pro *Modulo* habeatur ad quem constanter in omni Systemate mensuræ reliquarum rationum exigantur. Id enim si fiat, statim ex dato illo Modulo determinabitur Systema totum: nam ex mensuris constabit quæ Modulo erunt homogeneæ, quæque eo majores habebunt magnitudines vel minores quo major ille fuerit vel minor, ut ita mensurandarum rationum invariata magnitudinum servetur analogia inter ipsas mensuras. Patebit igitur in sequentibus rationem quandam dari, dupli inter & tripli rationes intermediam, ad rationem vero tripli aliquanto propius accedentem, quæ proposito nostro non immerito aptissima judicetur, siquidem ipsa rei natura hujus usum suadere ac non incertis indiciis efflagitare quodammodo videatur. Hanc ego, ex officio ejus desumpto nomine, *Modularem Rationem* appellabo; quo autem pacto ipsa sit accuratius definienda, ostendetur inferius, nunc enim de Logarithmis pauca sunt addenda.

Logarithmi sunt rationum mensuræ Numerales: solent autem in Canone sic disponi, ut singulis numeris naturali ordine crescentibus, & in serie continua positis adscribatur Logarithmus, non quidem ipsius numeri uti vulgo dicitur, sed rationis quam habet numerus ad Unitatem. Exinde vero rationis per quoscunque terminos designatæ facilis est inventio Logarithmi. Nam cum ratio antecedentis ad consequentem sit excessus rationis antecedentis ad Unitatem supra rationem consequentis ad Unitatem: Logarithmus ejus similiter erit excessus Logarithmi rationis quam habet antecedens ad Unitatem supra Logarithmum rationis quam

 conse-

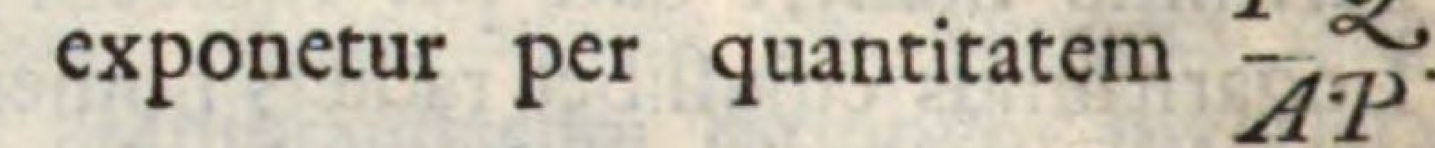
consequens habet ad Unitatem; hoc est, ut vulgari sermone utamur, excessus Logarithmi antecedentis supra Logarithmum consequentis; neutiquam enim displicet loquendi modus jam à multis annis receptus, si recte intelligatur. Exinde porro peregregium enascitur compendium ad operationes Arithmeticas. Datis enim duobus quibuscunque numeris in se multiplicandis, si quæratur numerus ex multiplicatione productus; quoniam rationes numerorum datorum ad Unitatem, conficiunt simul additæ rationem producti ad Unitatem, & rationum componendarum mensuræ simul additæ conficiunt rationis compositæ mensuram: Logarithmus producti æquabitur Logarithmis numerorum datorum simul sumptis. Ad eundem modum si quæratur numerus ex divisione ortus; quoniam ratio divisoris ad Unitatem è ratione dividendi ad Unitatem detracta relinquit rationem quoti ad Unitatem: habebitur quoti Logarithmus subducendo Logarithmum divisoris è Logarithmo dividendi. Et eodem argumento, si quæratur dati cujusvis numeri quælibet potestas; quoniam ratio dati numeri ad Unitatem per Indicem potestatis multiplicata rationem efficit quam habet numeri potestas ad Unitatem, & mensura prioris rationis multiplicata per eundem Indicem efficit pariter mensuram rationis posterioris: Logarithmus potestatis æquabitur Logarithmo numeri dati per Indicem potestatis multiplicato. Et similiter Logarithmus cujuslibet radicis numeri dati æquabitur Logarithmo numeri dati per Indicem radicis diviso. Igitur ope Canonis peragetur inventio potestatum & radicum per multiplicationem & divisionem, multiplicatio autem & divisio per additionem & subductionem. Ceterum de hisce vulgo notis Logarithmorum usibus non est mei instituti fusius disserere: missis ergo ambagibus, ad alia nunc me confero & rem ipsam protinus aggredior.

LOGOMETRIA

PROPOSITIO I.

Invenire Menſuram Rationis cujuſcunque propoſitæ.

PRoponatur Ratio inter AC & AB, cujus Menſuram oportet invenire. Terminorum differentia BC diviſa concipiatur in particulas innumeras quam minimas PQ, atque ratio inter AC & AB in totidem rationes quam minimas inter AQ & AP: & ſi detur magnitudo rationis inter AQ & AP, dividendo dabitur ratio quam habet PQ ad AP; atque adeo data illa magnitudo rationis inter AQ & AP, per datam quantitatem $\frac{PQ}{AP}$ exponi poteſt. Manente AP, augeri vel minui intelligatur particula PQ in proportione quavis; & in eadem proportione augebitur vel minuetur magnitudo rationis inter AQ & AP: capiatur particula dupla vel tripla, ſubdupla vel ſubtripla, & evadet ratio duplicata vel triplicata, ſubduplicata vel ſubtriplicata; etiamnum igitur exponetur per quantitatem $\frac{PQ}{AP}$. Sed &, aſſumpta determinata quavis quantitate M, exponi poteſt per $M \times \frac{PQ}{AP}$: erit ergo quantitas $M \times \frac{PQ}{AP}$ menſura rationis inter AQ & AP. Hæc vero menſura diverſam habebit magnitudinem, & ad Syſtema diverſum accommodabitur, pro diverſa magnitudine quantitatis aſſumptæ M, quæ adeo vocetur Syſtematis *Modulus*. Jam quemadmodum ſumma rationum omnium inter AQ & AP æqualis eſt propoſitæ rationi, quam utique habet AC ad AB: ita ſumma menſurarum omnium $M \times \frac{PQ}{AP}$ (per Methodos ſatis notas invenienda) æqualis erit ejuſdem propoſitæ rationis menſuræ quæſitæ. *Q.E.I.*



Corol. 1. Terminis AP, AQ ita ad æqualitatem accedentibus, ut quam minima ſit eorundem differentia PQ: erit $M \times \frac{PQ}{AP}$ vel $M \times \frac{PQ}{AQ}$ æqualis menſuræ rationis inter AQ & AP ad Modulum M.

Corol.

Corol. 2. Unde Modulus ille M est ad mensuram rationis inter terminos AQ & AP, ut terminorum alteruter AP vel AQ ad terminorum differentiam PQ. LOGOMETRIA

Corol. 3. Data ratione inter AC & AB, datur summa omnium $\frac{PQ}{AP}$, & summa omnium $M \times \frac{PQ}{AP}$ est ut M. Itaque mensura datæ cujuscunque rationis est ut Modulus Systematis ex quo desumitur.

Corol. 4. Modulus ergo, in omni mensurarum Systemate, semper æqualis fit mensuræ rationis cujusdam determinatæ atque immutabilis: Quam proinde *Rationem Modularem* vocabo.

Scholium 1.

Problematis solutio per Exemplum illustrabitur. Sit z quantitas determinata quævis & permanens, sit vero x quantitas indeterminata fluxuque perpetuo variabilis, ejusque fluxio sit $\dot{x}$; & quæratur mensura rationis inter $z+x$ & $z-x$. Statuatur hæc ratio æqualis rationi inter y & 1, exponatur autem numerus y per AP, fluxio ejus $\dot{y}$ per PQ, 1 per AB: & ex Corollario primo colligetur fluxionem quæsitæ mensuræ rationis inter y & 1 esse $M \times \frac{\dot{y}}{y}$. Reponatur jam pro y valor ejus $\frac{z+x}{z-x}$, itemque pro $\dot{y}$ valoris fluxio $\frac{2z\dot{x}}{\overline{z-x}|^2}$: & fluxio mensuræ evadet $2M \times \frac{z\dot{x}}{zz-xx}$ vel $2M \times \frac{\dot{x}}{z-\frac{xx}{z}}$ sive $2M$ in $\frac{\dot{x}}{z} + \frac{\dot{x}x^2}{z^3} + \frac{\dot{x}x^4}{z^5} + \&c.$ Atque adeo mensura illa fiet $2M$ in $\frac{x}{z} + \frac{x^3}{3z^3} + \frac{x^5}{5z^5} + \&c.$ Unde patet Corollarium sequens.

Corol. 5. Si duarum quantitatum summa sit z & differentia sit x; & sumatur $2M\frac{x}{z} = A$, $A\frac{xx}{zz} = B$, $B\frac{xx}{zz} = C$, $C\frac{xx}{zz} = D$, &c: Mensura rationis quam habet quantitas major ad quantitatem minorem, erit $A + \frac{1}{3}B + \frac{1}{5}C + \frac{1}{7}D + \&c.$

Pars Prima.

Scholium 2.

Non abſimili computo menſura rationis inter $1+v$ & 1 erit M in $v-\frac{1}{2}v^2+\frac{1}{3}v^3-\frac{1}{4}v^4+\frac{1}{5}v^5-$&c. Unde ſi menſura illa vocetur m, erit $\frac{m}{M}=v-\frac{1}{2}vv+\frac{1}{3}v^3-\frac{1}{4}v^4+\frac{1}{5}v^5$, &c: ac proinde $\frac{mm}{MM}=vv-v^3+\frac{11}{12}v^4-\frac{5}{6}v^5$, &c; ſimiliterque $\frac{m^3}{M^3}=v^3-\frac{3}{2}v^4+\frac{7}{4}v^5$, &c; quinetiam $\frac{m^4}{M^4}=v^4-2v^5$, &c; ac denique $\frac{m^5}{M^5}=v^5$, &c. Ut igitur viciſſim, ex data menſura m, inveniatur ratio quam metitur; addendo æqualia æqualibus habebitur $\frac{m}{M}+\frac{mm}{2MM}=v*-\frac{1}{6}v^3+\frac{5}{24}v^4-\frac{13}{60}v^5$, &c; atque iterum $\frac{m}{M}+\frac{mm}{2MM}+\frac{m^3}{6M^3}=v**-\frac{1}{24}v^4+\frac{3}{40}v^5$, &c; rurſumque $\frac{m}{M}+\frac{mm}{2MM}+\frac{m^3}{6M^3}+\frac{m^4}{24M^4}=v***-\frac{1}{120}v^5$, &c; atque tandem $\frac{m}{M}+\frac{mm}{2MM}+\frac{m^3}{6M^3}+\frac{m^4}{24M^4}+\frac{m^5}{120M^5}=v****$, &c; id eſt, $\frac{m}{M}+\frac{mm}{2MM}+\frac{m^3}{6M^3}+\frac{m^4}{24M^4}+\frac{m^5}{120M^5}+\&c.=v$. Itaque ratio quæſita inter $1+v$ & 1, eſt ea quam habet $1+\frac{m}{M}+\frac{mm}{2MM}+\frac{m^3}{6M^3}+\frac{m^4}{24M^4}+\frac{m^5}{120M^5}+\&c.$ ad 1. Ponatur $m=M$, ſive $\frac{m}{M}=1$; & exinde Ratio Modularis erit ea quam habet $1+\frac{1}{1}+\frac{1}{2}+\frac{1}{6}+\frac{1}{24}+\frac{1}{120}+\&c.$ ad 1.

Eodem modo, ſi detur ratio inter 1 & $1-v$, menſura hujus rationis erit M in $v+\frac{1}{2}v^2+\frac{1}{3}v^3+\frac{1}{4}v^4+\frac{1}{5}v^5$, &c. Et viciſſim ſi detur rationis menſura m, ratio erit ea quam habet 1 ad $1-\frac{m}{M}+\frac{mm}{2MM}-\frac{m^3}{6M^3}+\frac{m^4}{24M^4}-\frac{m^5}{120M^5}+\&c.$ Ponatur $m=M$, ſive $\frac{m}{M}=1$; & exinde Ratio Modularis erit ea quam habet 1 ad $1-\frac{1}{1}+\frac{1}{2}-\frac{1}{6}+\frac{1}{24}-\frac{1}{120}+\&c.$ Ex hiſce vero patet Corollarium ſequens.

Corol.

Corol. 6. Exposito termino R, si sumatur $\frac{1}{1}R = A$, $\frac{1}{2}A = B$, $\frac{1}{3}B = C$, $\frac{1}{4}C = D$, $\frac{1}{5}D = E$, &c. in infinitum; & capiatur $S = R + A + B + C + D + E +$ &c: Ratio Modularis erit ea quæ est inter terminum minorem expositum R & majorem inventum S. Vel exposito termino S, si sumatur $\frac{1}{1}S = A$, $\frac{1}{2}A = B$, $\frac{1}{3}B = C$, $\frac{1}{4}C = D$, $\frac{1}{5}D = E$, &c. in infinitum; & capiatur $R = S - A + B - C + D - E +$ &c: Ratio Modularis erit ea quæ est inter terminum majorem expositum S & minorem inventum R. Porro eadem ratio est inter 2,718281828459 &c. et 1, vel inter 1 & 0,367879441171 &c. LOGOMETRIA

Scholium 3.

Si forte termini minores desiderentur, qui eandem proxime Rationem Modularem ita exhibeant, ut nulli ipsis non majores propius: instituenda erit operatio ad modum sequentem. Dividatur terminus major

Rationes Vera Majores.		*Rationes Vera Minores.*	
1	0 × 2	0	1
2	1	2	0
3	1 × 2	2	1 × 1
8	3	6	2
11	4 × 1	8	3 × 1
76	28	11	4
87	32 × 1	19	7 × 4
106	39	87	32
193	71 × 6	106	39 × 1
1264	465	1158	426
1457	536 × 1	1264	465 × 1
21768	8008	1457	536
23225	8544 × 1	2721	1001 × 8
25946	9545	23225	8544
49171	18089 × 10	25946	9545 × 1
&c.	&c.	&c.	&c.

2,71828 &c. per minorem 1, vel etiam major 1 per minorem 0,367879 &c. & rursus minor per numerum qui reliquus est, & hic rursus per ultimum residuum, atque ita porro pergatur: & prodibunt quotientes 2, 1, 2, 1, 1, 4, 1, 1, 6, 1, 1, 8, 1, 1, 10, 1, 1, 12. 1, 1, 14, 1, 1, 16, 1, 1, &c.

His

His inventis, perficiendæ ſunt binæ rationum columnæ, quarum altera terminos continet rationem habentes vera majorem, altera terminos quorum ratio eſt vera minor; ineundo computationem à rationibus 1 ad 0, 0 ad 1, quæ remotiſſimæ ſunt à vera; inde autem exorſam deducendo ad rationes reliquas, quæ continue ad veram propius accedunt. Multiplicentur itaque termini 1 & 0 per quotientem primum 2, & ſcribantur facti 2 & 0 infra terminos 0 & 1; & addendo prodibit ratio 2 + 0 ad 0 + 1, ſive 2 ad 1. Hujus termini multiplicentur per quotientem ſecundum 1, factique 2 & 1 addantur terminis 1 & 0; & habebitur ratio 2 + 1 ad 1 + 0, ſive 3 ad 1. Hujus termini multiplicentur per quotientem tertium 2, factique 6 & 2 addantur terminis præcedentibus 2 & 1; & habebitur ratio 8 ad 3. Hujus termini multiplicentur per quotientem quartum 1, factique 8 & 3 addantur terminis præcedentibus 3 & 1; & habebitur ratio 11 ad 4. Hujus termini multiplicentur per quotientem quintum 1, factique 11 & 4 addantur præcedentibus 8 & 3; & habebitur ratio 19 ad 7. Hujus termini rurſus multiplicentur per quotientem ſextum 4, factique 76 & 28 addantur præcedentibus 11 & 4, ad inveniendam rationem 87 ad 32; & ſic porro pergendum quouſque libuerit, tranſitu alternis facto in alteram columnam. Hiſce peractis, habebuntur rationes vera majores 3 ad 1, 11 ad 4, 87 ad 32, 193 ad 71, 1457 ad 536, 23225 ad 8544, 49171 ad 18089, &c. Vera autem minores erunt 2 ad 1, 8 ad 3, 19 ad 7, 106 ad 39, 1264 ad 465, 2721 ad 1001, 25946 ad 9545, &c. Atque hæ quidem ſunt præcipuæ & primariæ rationes, quibus ad rationem propoſitam continue appropinquatur.

Quod ſi exquiratur integra ſeries rationum omnium vera majorum quæ ita dari poſſint, ut nulla minoribus terminis deſignata ratio vera major ad veram propius accedat; & ſimiliter ſeries integra rationum omnium vera minorum quæ ita dari poſſint, ut nulla minoribus terminis deſignata ratio vera minor ad veram propius accedat: inter primarias illas modo inventas inſerendæ ſunt aliæ ſecundariæ rationes. Hæ vero locum habent ubi quotiens unitatem ſuperat. Inveniuntur autem mutata multiplicatione, quæ ſupra per quotientem facta eſt, in continuam additionem terminorum tot vicibus quot ſunt unitates in quotiente. Sic quia quotiens primus erat 2, termini 1 & 0 bis addendi ſunt terminis 0 & 1; & ſummæ dabunt rationes 1 ad 1, 2 ad 1. Hi ultimi termini 2 & 1, quia quotiens ſecundus erat 1, ſemel addendi ſunt terminis 1 & 0; & ſummæ dabunt rationem 3 ad 1. Hi termini 3 & 1, quia quotiens tertius erat 2, bis addendi ſunt terminis 2 & 1; & ſummæ dabunt rationes 5 ad 2, 8 ad 3. Hi ultimi termini 8 & 3, quia quotiens

tiens quartus erat 1, semel addendi sunt terminis 3 & 1; & summæ dabunt rationem 11 ad 4. Hi termini 11 & 4, quia quotiens quintus erat 1, semel addendi sunt terminis 8 & 3; & summæ dabunt rationem 19 ad 7. Hi denique termini 19 & 7, quia quotiens sextus erat 4, quater addendi sunt terminis 11 & 4; & summæ dabunt rationes 30 ad 11, 49 ad 18, 68 ad 25, 87 ad 32. Et sic porro LOGOMETRIA

Rationes Vera Majores.		*Rationes Vera Minores.*	
1	0×2	0	1
2	1	1	0
3	1×2	1	1
8	3	1	0
11	4×1	2	1×1
19	7	3	1
30	11	5	2
19	7	3	1
49	18	8	3×1
19	7	11	4
68	25	19	7×4
19	7	87	32
87	32×1	106	39×1
&c.	&c.	&c.	&c.

procedere licebit quousque commodum videbitur. Ista tandem operatione peracta, series integra rationum omnium vera majorum, erit 1 ad 0, 3 ad 1, 11 ad 4, 30 ad 11, 49 ad 18, 68 ad 25, 87 ad 32, &c. similiterque series integra rationum omnium vera minorum, erit 0 ad 1, 1 ad 1, 2 ad 1, 5 ad 2, 8 ad 3, 19 ad 7, &c.

Harum approximationum utilitas ad alia multa sese diffundit: quapropter earum inventionem aliquanto prolixius expositam dedi, per Methodum quæ mihi simplicissima & facillima videtur. Idem argumentum paulo aliter pertractarunt Viri celeberrimi *Wallisius* & *Hugenius*.

PARS PRIMA.

PROPOSITIO II.

Logarithmorum Canonem Briggianum *construere.*

NUmerorum Compositorum Logarithmi derivantur ex Logarithmis Primorum componentium, per additionem solam; horum autem investigatio pluribus modis institui potest: Exemplum unicum appono.

Per Corollarium quintum Propositionis superioris, scribendo 1 pro M, inveniantur Logarithmi rationum inter 126 & 125, 225 & 224, 2401 & 2400, 4375 & 4374; qui vocentur respective p, q, r, s: & Logarithmus denarii seu rationis decupli erit $239p + 90q - 63r + 103s$, sive 2,302585092994 &c. Itaque cum Logarithmus *Briggianus* denarii sit 1; fiat (per Corol. 3. Prop. 1.) ut denarii Logarithmus modo inventus 2,302585092994 &c. ad Modulum suum 1, ita denarii Logarithmus *Briggianus* 1, ad Modulum *Briggianum*, qui adeo erit 0,434294481903 &c. Ponatur ergo deinceps iste valor pro M, & erunt $M \times \overline{202p + 76q - 53r + 87s}$, $M \times \overline{167p + 63q - 44r + 72s}$, $M \times \overline{114p + 43q - 30r + 49s}$ Logarithmi *Briggiani* numerorum 7, 5, 3. Logarithmus numeri 2 habetur, subducendo Logarithmum numeri 5 à Logarithmo numeri 10. Atque ita dantur & Modulus *Briggianus* & Logarithmi Primorum omnium qui sunt minores denario.

Logarithmi numerorum sequentium Primorum 11, 13, 17, 19, 23, &c. ita computari possunt. Quæratur tum factus à numeris Primo proposito utrinque proxime adjacentibus, tum Primi ipsius quadratum, quod semper unitate factum illud superabit. Logarithmo rationis quadrati ad factum (per Corol. 5. Prop. 1. inveniendo) addatur ipsius facti Logarithmus, qui semper componetur ex datis Logarithmis Primorum qui proposito Primo sunt minores: & semisumma erit Logarithmus Primi quæsitus.

Corol. Canonis *Briggiani* Modulus est 0,434294481903 &c: Hujus vero Reciprocus est 2,302585092994 &c.

Scholium.

Ad hunc itaque modum perfici posset Logarithmorum Tabula amplissima, qualis edita est à *Briggio* vel *Vlacco*. Inventioni autem Numerorum & Logarithmorum sibi invicem congruentium, qui intermedii

intermedii ſunt & ultra Tabulæ limites excurrunt, abunde ſufficiet terminus primus Seriei quæ in Corollario quinto Propoſitionis præcedentis exhibetur. LOGOMETRIA

Si dato Numero intermedio quæratur ejus Logarithmus; pone a & e pro Numero intermedio propoſito atque huic proximo tabulari, ita ut a deſignet majorem, e minorem; ſit eorum ſumma z, differentia x; pone λ pro Logarithmo rationis quam habet a ad e, hoc eſt, pro exceſſu Logarithmi Numeri a ſupra Logarithmum Numeri e: & erit $\lambda = 2\,M\frac{x}{z}$ quamproxime.

Si quæratur Numerus qui congruit Logarithmo intermedio; quoniam eſt $\lambda = \frac{2Mx}{z} = \frac{2Mx}{2a-x}$ vel $\frac{2Mx}{2e+x}$; erit $x = \frac{\lambda}{M+\frac{1}{2}\lambda}a$ vel $\frac{\lambda}{M-\frac{1}{2}\lambda}e$ quamproxime.

PROPOSITIO III.

Syſtematis cujuſvis Logometrici conſtructionem exponere per Canonem Logarithmorum.

Caſ. 1. SI detur, è Syſtemate propoſito, menſura rationis alicujus determinatæ: rationis cujuſvis oblatæ menſura, erit ad menſuram illam datam determinatæ rationis, ut oblatæ rationis Logarithmus, ad Logarithmum rationis ejuſdem determinatæ.

Caſ. 2. Si non detur, è Syſtemate propoſito, menſura rationis alicujus determinatæ: inveniendus erit Modulus propoſiti Syſtematis, per Corollarium ſecundum Propoſitionis primæ. Et menſura cujuſvis oblatæ rationis, erit ad Modulum inventum, ut oblatæ rationis Logarithmus, ad Canonis Modulum.

Caſus hujus ultimi habentur Exempla in ſequentibus.

PROPOSITIO IV.

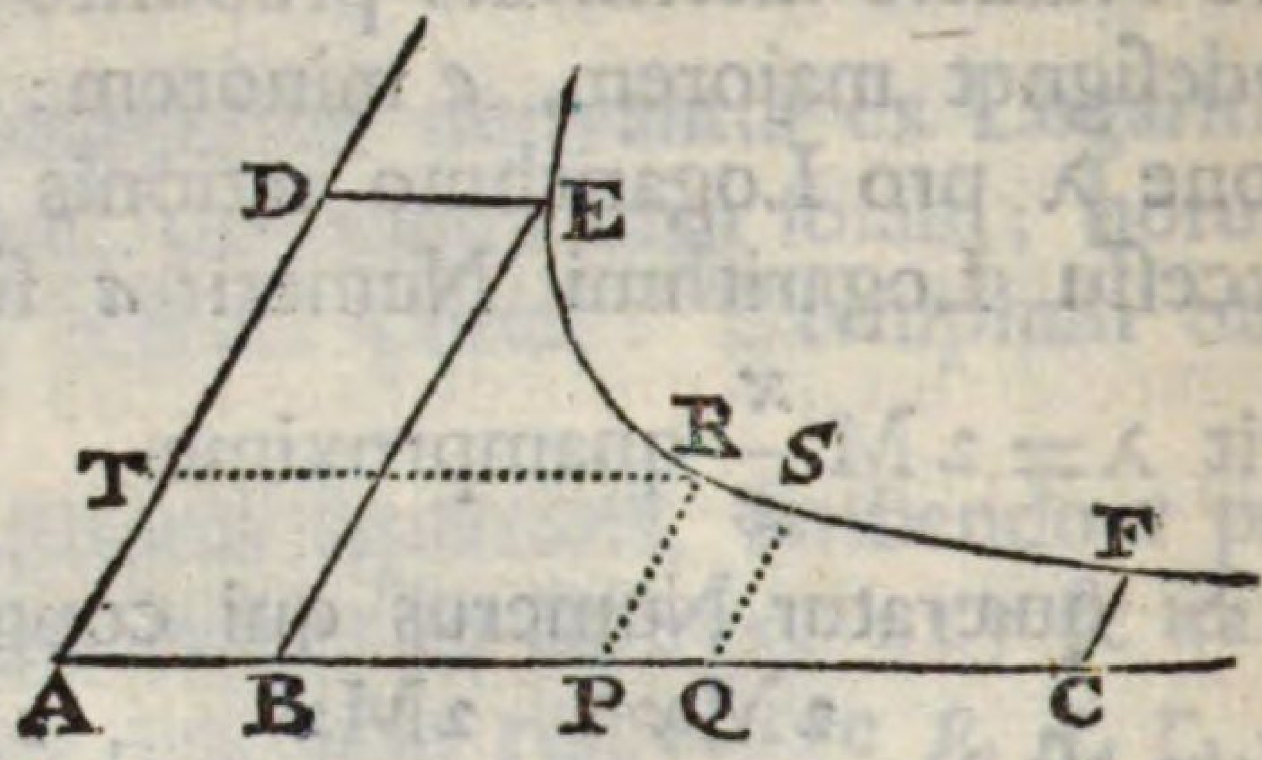
Spatium quodvis Hyperbolicum quare [dra] *per Canonem Logarithmorum.*

SIT Hyperbola quævis $ERSF$ centro A, Aſymptotis ABC, AD deſcripta; & quæratur area $BEFC$ quam claudunt rectæ BE, CF ad Aſymptoton AD parallelæ. Compleatur parallelogrammum $ABED$, & ad hunc Modulum inveniatur (per Propo-

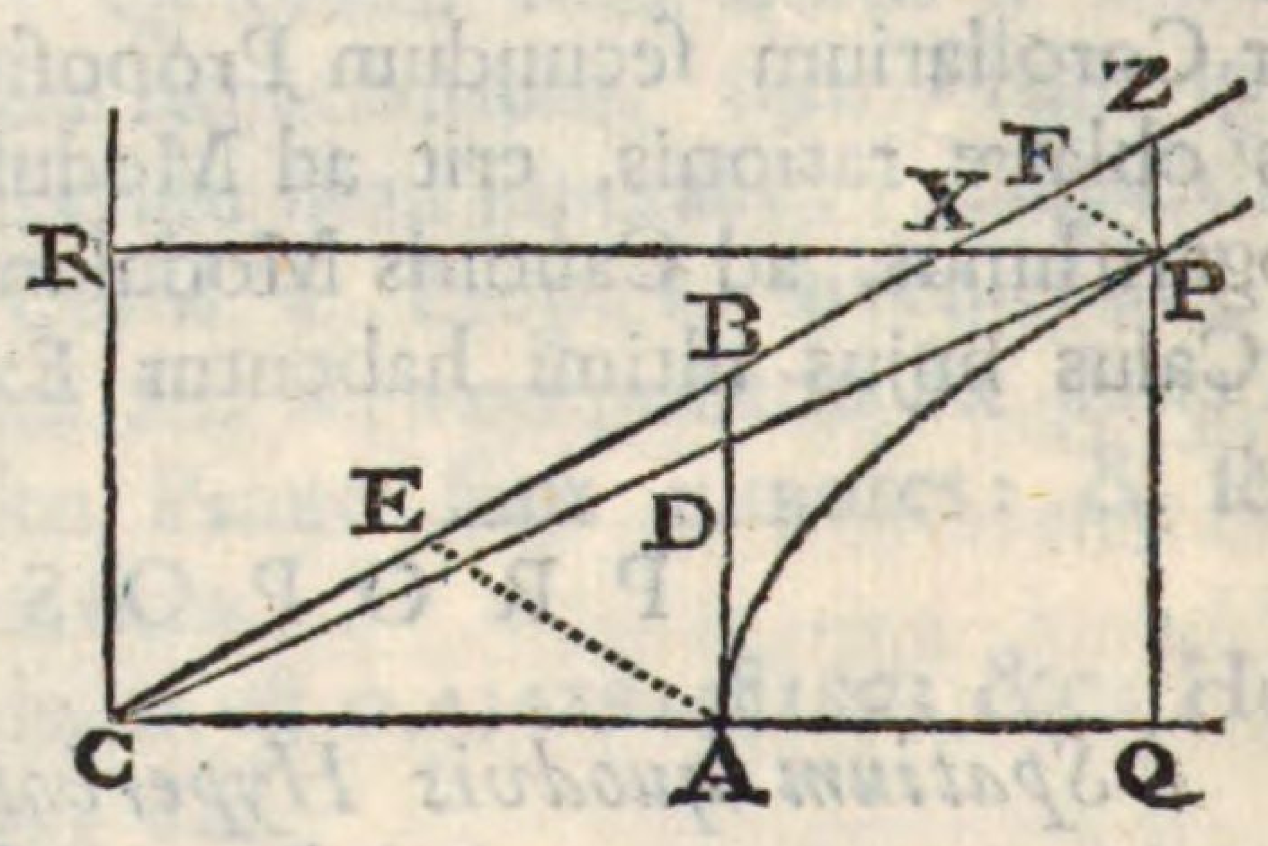
 ſitionem

PARS PRIMA.

ſitionem tertiam) menſura rationis inter *AC* & *AB* vel inter *BE* & *CF*: Dico menſuram inventam æqualem fore magnitudini areæ quæſitæ *BEFC*. Nam diviſa concipiatur hujus areæ baſis *BC* in particulas innumeras quam minimas *PQ*, ea lege, ut ubique detur ratio illa quæ eſt inter *AQ* & *AP*, & ducantur Aſymptoto *AD* parallelæ *PR*, *QS*. Quoniam itaque eſt *AQ* ut *AP*; erit diviſim *PQ* ut *AP*, hoc eſt, ut *PR* reciproce. Unde data eſt area *PRSQ*, quæ proinde poteſt haberi pro menſura rationis datæ quæ eſt inter *AQ* & *AP*. Hujus autem menſuræ Modulus erit parallelogrammum *ABED*, per Corol. 2. Prop. 1. Nam ſi compleatur æquale parallelogrammum *APRT*; ſtatim intelligetur, ita illud ſe habere ad aream *PRSQ*, ut ſe habet *AP* ad *PQ*. Similes ergo ſummas arearum atque rationum utrinque colligendo; area tota *BEFC* erit menſura rationis totius quæ eſt inter *AC* & *AB*, vel inter *BE* & *CF*, ad eundem Modulum *ABED*.



Aliter. Sit rurſus Hyperbola quævis *AP*, centro *C* atque Aſymptoto *CB* deſcripta; & quæratur area Sectoris cujuſlibet *CAP*, ſemidiametris *CA*, *CP* curvæque *AP* interjecti. Producta ſemidiametro utravis *CAQ* ultra verticem *A*, ducatur illius conjugata *CR*; & ad ipſas *CQ*, *CR* ordinatim applicentur à puncto *P* rectæ *PQ*, *PR*, quæ Aſymptoto *CB* occurrant in *Z* & *X*; deinde agatur *AB* quæ Hyperbolam tangat in *A*, Aſymptoton ſecet in *B* rectamque *CP* in *D*: & Triangulo *ABC* exiſtente Modulo, area quæſita ſectoris *CAP* erit menſura rationis inter *QZ* + *QP* & *AB*, ſive rationis inter *AB* & *QZ* − *QP*, ſive ſubduplicatæ rationis inter *QZ* + *QP*, & *QZ* − *QP*, ſive ſubduplicatæ rationis inter *AB* + *AD* & *AB*

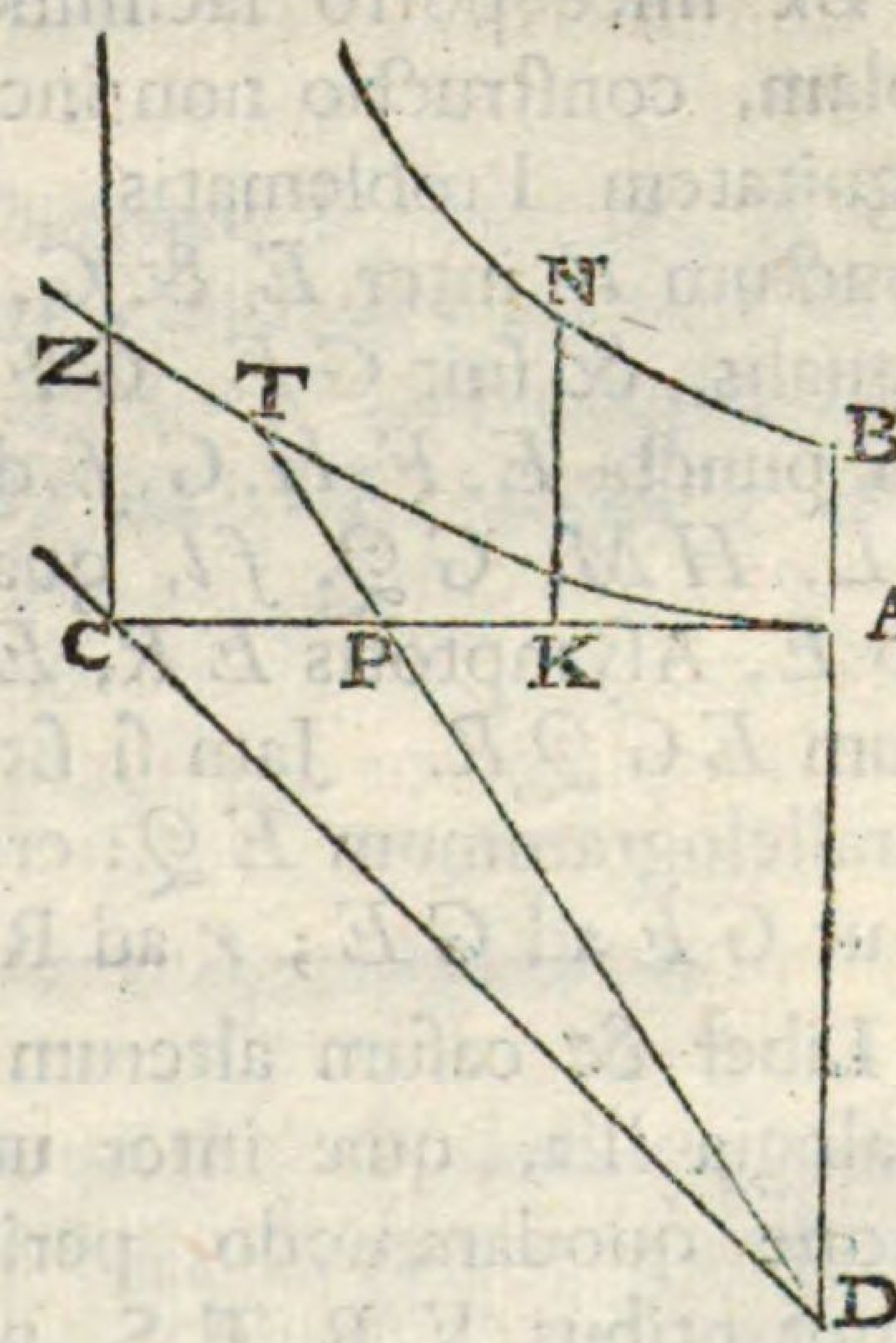


$AB - AD$; vel erit menſura rationis inter $RP + RX$ & CA, vel rationis inter CA & $RP - RX$, vel ſubduplicatæ rationis inter $RP + RX$ & $RP - RX$. Nam ſi ducantur rectæ AE, PF quæ ſecent Aſymptoton CB in E & F, alterique Aſymptoto parallelæ ſint: æquales erunt hæ omnes rationes rationi quam habet AE ad PF, vel CF ad CE; erit & ſector CAP areæ $EAPF$ æqualis; ſimiliterque triangulum ABC duplicato triangulo AEC, ſive parallelogrammo Aſymptotis & Hyperbolæ inſcripto æquabitur. Quare patet propoſitum ex ſupra demonſtratis. LOGOMETRIA.

Data vero per modum priorem area $BEFC$, vel per modum poſteriorem area CAP; dabitur alia quævis area Hyperbolica ad arcum EF, vel ad arcum AP terminata: quippe quæ ſemper eſt areæ modo inventæ & areæ alicujus rectilineæ vel ſumma vel differentia. *Q. E. I.*

Scholium.

Hinc facilem habent ſolutionem Problemata omnia, quæcunque pendent ab Hyperbolæ quadratura. Exemplum ſatis luculentum præbebit deſcenſus gravium in Mediis, quorum reſiſtentia eſt in duplicata ratione velocitatis corporis moti. Sit V velocitas maxima quam corpus in hujuſmodi Medio, infinite deſcendendo, poteſt acquirere; T dimidium temporis quo corpus idem in eodem Medio, vi ſola ponderis ſui relativi, abſque reſiſtentia cadendo velocitatem illam acquiret; S ſpatium hocce caſu deſcriptum; R pondus relativum corporis in Medio reſiſtente: & quæratur ſpatium s quod corpus deſcendens, tempore quovis t, deſcribet in Medio reſiſtente; & reſiſtentia r quam patitur in fine illius temporis; & velocitas v ex iſto deſcenſu acquiſita.



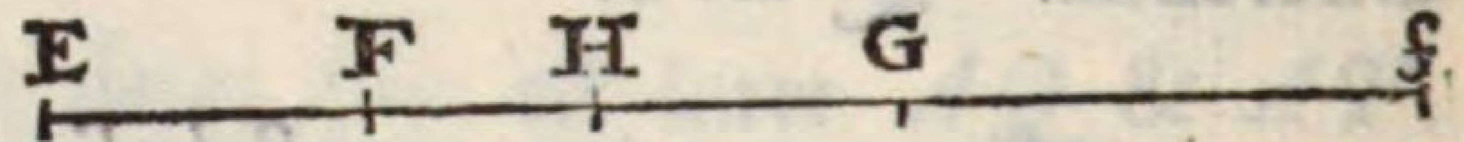
Centro D, vertice A deſcribatur Hyperbola æquilatera AT, cujus una Aſymptotorum eſt DC & ad verticem tangens AC ſemiaxi AD æqualis. Capiatur area DAT ad dimidium trianguli DAC ut t ad T, ſecetque DT tangentem AC in P: & erit v ad V

ut

PARS PRIMA.

ut AP ad AC. Sit AK ipſis AC, AP tertia proportionalis: & erit r ad R ut AK ad AC. Ad tangentem AC erigantur normales CZ, KN, AB; centroque C & Aſymptotis CA, CZ deſcribatur Hyperbola quævis BN: & erit s ad S ut area $ABNK$ ad rectangulum CKN. Patent hæc omnia per Propoſitiones octavam & nonam Libri ſecundi Philoſophiæ *Newtonianæ*.

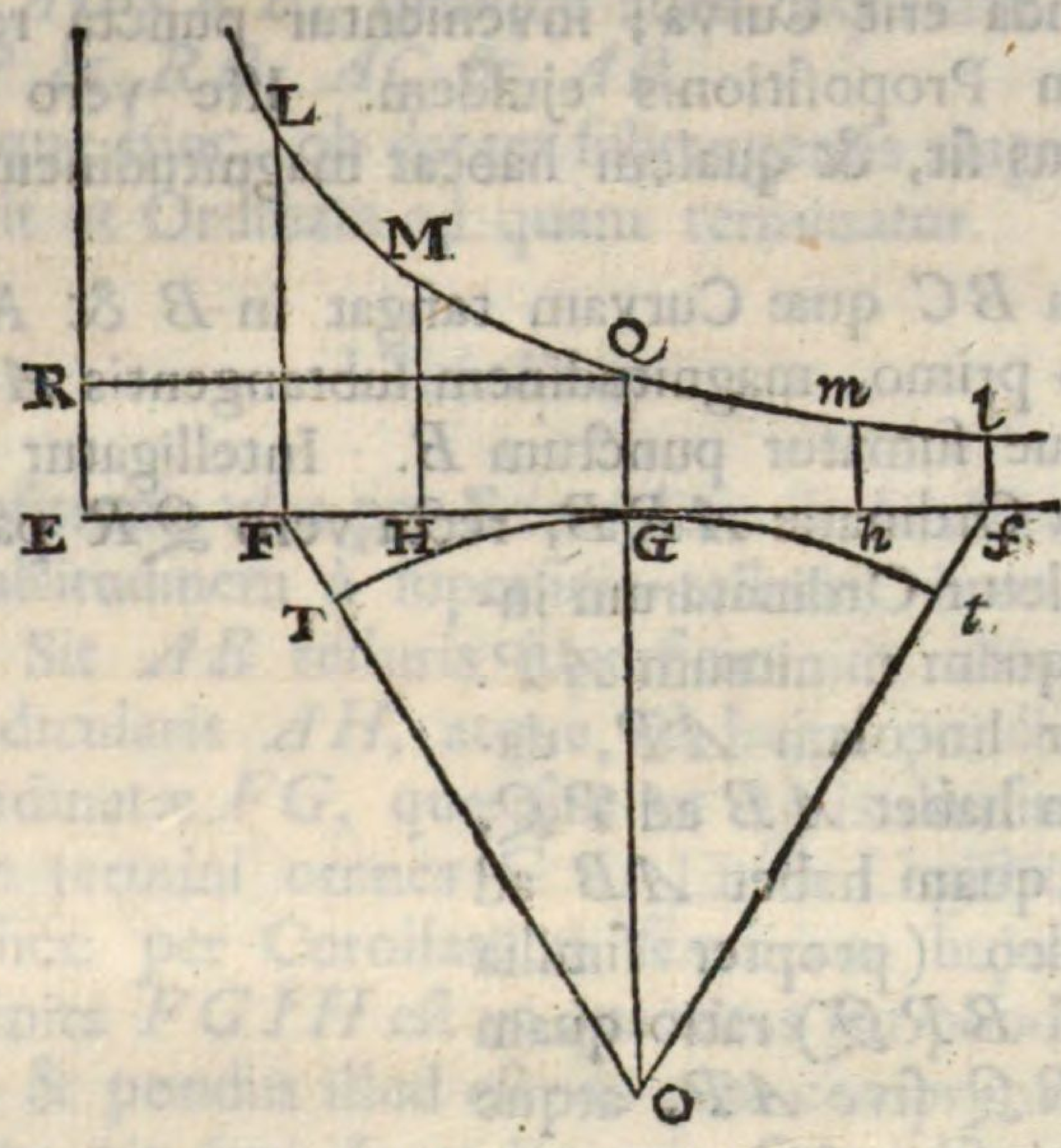
Eſt itaque t ad T ut area Hyperbolica DAT ad dimidium trianguli DAC, hoc eſt, ut dimidiata menſura rationis inter $AC+AP$ & $AC-AP$ ad illius menſuræ dimidiatum Modulum. Ergo ſi recta quævis EF producatur ad f, ita ut t ſit menſura rationis inter Ef & EF ad Modulum T, & biſecetur Ff in G: erit GF ad GE ut AP ad AC, hoc eſt, ut v ad V. Sumantur GE, GF, GH continue proportionales: & erit GH & GE ut AK ad AC, hoc eſt, ut r ad R. Erit inſuper EG ad EH ut CA ad CK; unde cum ſit s ad S ut area $ABNK$ ad rectangulum CKN, hoc eſt, ut menſura rationis inter CA & CK vel inter EG & EH ad menſuræ Modulum: erit s menſura rationis inter EG & EH ad Modulum S, atque inde dabitur.



Ex hiſce porro facillime ſe prodit, per unicam quamvis Hyperbolam, conſtructio non inconcinna; quam & adſcribere viſum eſt ob dignitatem Problematis. In recta quavis GE ſumatur utcunque punctum F inter E & G, & ab altera parte capiatur Gf ipſi GF æqualis, & ſint GE, GF, GH continue proportionales. Deinde per puncta E, F, H, G, f ducantur ſibi invicem parallelæ rectæ ER, FL, HM, GQ, fl, quas ſecet Hyperbola quævis $LMQl$ centro E, Aſymptotis ER, EG deſcripta, & compleatur parallelogrammum $EGQR$. Jam ſi ſit t ad T ut area Hyperbolica $LFfl$ ad parallelogrammum EQ: erit s ad S ut area $MHGQ$ ad EQ; v ad V ut GF ad GE; r ad R ut GH ad GE.

Libet & caſum alterum adjicere ubi corpus aſcendit; ne forte analogia illa, quæ inter utrumque ſervari debet, in allata conſtructione quodammodo perire videatur. Ergo eadem atque prius denotantibus V, R, T, S, ponantur v & r pro velocitate & reſiſtentia ſub aſcenſus initio, s pro ſpatio quod corpus aſcendendo deſcribere poſſit antequam tota velocitas amittatur, t pro tempore hujus aſcenſus. Ad EG erigatur perpendicularis GO ipſi EG æqualis, & ſumendo puncta F, f, ad eaſdem diſtantias hinc inde à puncto G, jun-

jungantur OF, Of, quibus occurrat in T & t circuli arcus TGt centro O descriptus, & sint Gh, Gf, GE continue proportionales, & ducatur ipsi ER parallela hm Hyperbolæ occurrens in m. De- LOGOMETRIA.



inde si t sit mensura anguli FOf ad Modulum T, hoc est, si t sit ad T ut arcus TGt ad radium OG: erit s mensura rationis inter Eh & EG ad Modulum S, vel erit s ad S ut area Hyperbolica $mhGQ$ ad EQ; & v erit ad V ut Gf ad GE; atque r ad R ut Gh ad GE.

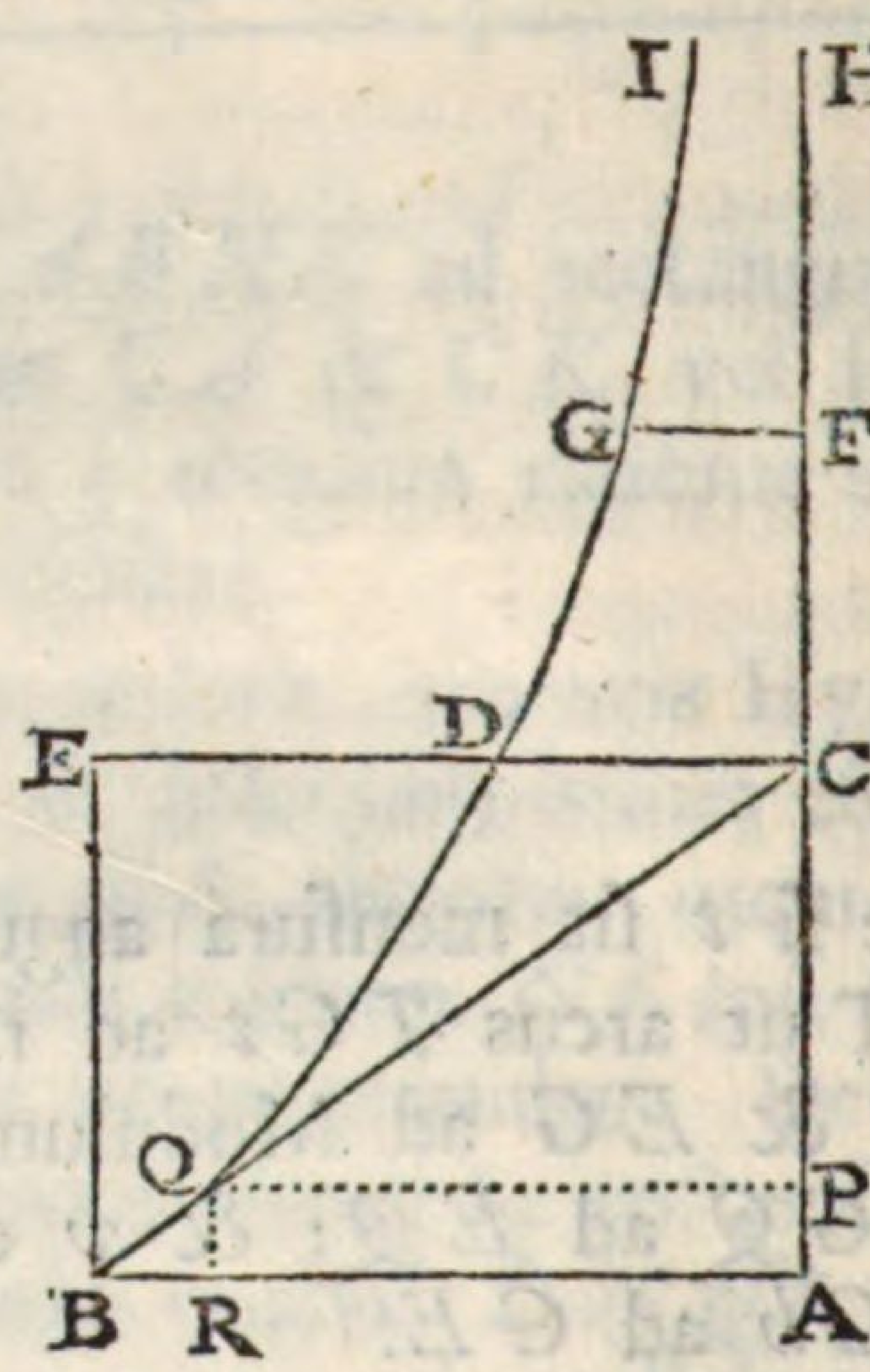
PROPOSITIO V.

Logisticam describere per Canonem Logarithmorum.

SI ad Logisticæ $BQDG$ Asymptoton $APCF$ ordinatim applicentur binæ quævis rectæ AB, FG intercludentes Asymptoti portionem quamvis AF: erit illa portio mensura rationis quam ad invicem habent ordinatæ; hæc utique est natura Curvæ notissima. Integrum ergo & perfectum Systema Logometricum per hanc Lineam exhibetur: id quod etiam de Hyperbola dici potest per Propositionem præcedentem, de Spirali Æquiangula per subsequentem; nam

nam omitto complures alias Figuras, quæ & ipſæ dudum ſunt in Geometriam receptæ. Itaque ſi detur Aſymptoti poſitio & ſimul duo puncta per quæ Curva tranſire debet, dabuntur puncta reliqua per caſum priorem Propoſitionis tertiæ. Quod ſi data poſitione Aſymptoti, detur inſuper Syſtematis Modulus atque unicum punctum per quod ducenda erit Curva; invenientur puncta reliqua per Caſum poſteriorem Propoſitionis ejuſdem. Iſte vero Modulus quo pacto definiendus ſit, & qualem habeat magnitudinem, jam oportet exponere.

Ducatur recta *BC* quæ Curvam tangat in *B* & Aſymptoton ſecet in *C*. Dico primo, magnitudinem ſubtangentis *AC* eandem permanere ubicunque ſumatur punctum *B*. Intelligatur enim Ordinata *PQ* viciniſſima Ordinatæ *ARB*, recta vero *QR* parallela Aſymptoto *AC*, ac detur Ordinatarum intervallum illud quam minimum *AP*. Ob datam igitur lineolam *AP*, dabitur ratio quam habet *AB* ad *PQ*, & diviſim ratio quam habet *AB* ad *RB*, atque adeo (propter ſimilia triangula *BAC*, *BRQ*) ratio quam habet *AC* ad *RQ* ſive *AP*, atque inde magnitudo ipſius *AC*.



Dico ſecundo, determinatam hanc & immutabilem ſubtangentem *AC*, eſſe Modulum ad quem exigendæ ſunt menſuræ illæ interceptæ *AF*. Patet hoc per Corollarium ſecundum Propoſitionis primæ: nam dum termini *AB* & *PQ* ad æqualitatem proxime accedunt, erit *AC* ad *AP*, quæ metitur rationem inter *AB* & *PQ*, ut terminus *AB* ad terminorum differentiam *BR*. Unde data ſubtangente, facilis eſt deſcriptio Curvæ & ſolutio Problematum omnium quæ exhinc pendent.

Si Curva jam deſcripta habeatur, ſubtangentis magnitudo ſic determinabitur. Producatur Ordinata quævis *CD* ad *E*, ita ut *CE* ad *CD* rationem habeat Modularem, per Corollarium ſextum Propoſitionis primæ definitam; & recta *EB* quæ à puncto *E* parallela ducitur Aſymptoto, quæque Curvæ occurrit in puncto *B*, æqualis erit ſubtangenti quæſitæ.

Corol.

Corol. 1. Area *ABIH*, quæ inter Curvam *BDI* & Aſymptoton ejus *ACH* infinite verſus *HI* extenditur, & ad alteram partem ab Ordinata *AB* terminatur, æqualis eſt parallelogrammo *ABEC* ab Ordinata eadem *AB* & ſubtangente *AC* comprehenſo. Componuntur enim area & parallelogrammum ex elementis quæ ſunt ut $AP \times AB$ & $AC \times RB$, quæque adeo æquantur propter analogiam inter *AP* & *RB*, *AC* & *AB*. LOGOMETRIA

Corol. 2. Atque hinc, ob datam ſubtangentis magnitudinem, area illa indefinita erit ut Ordinata ad quam terminatur.

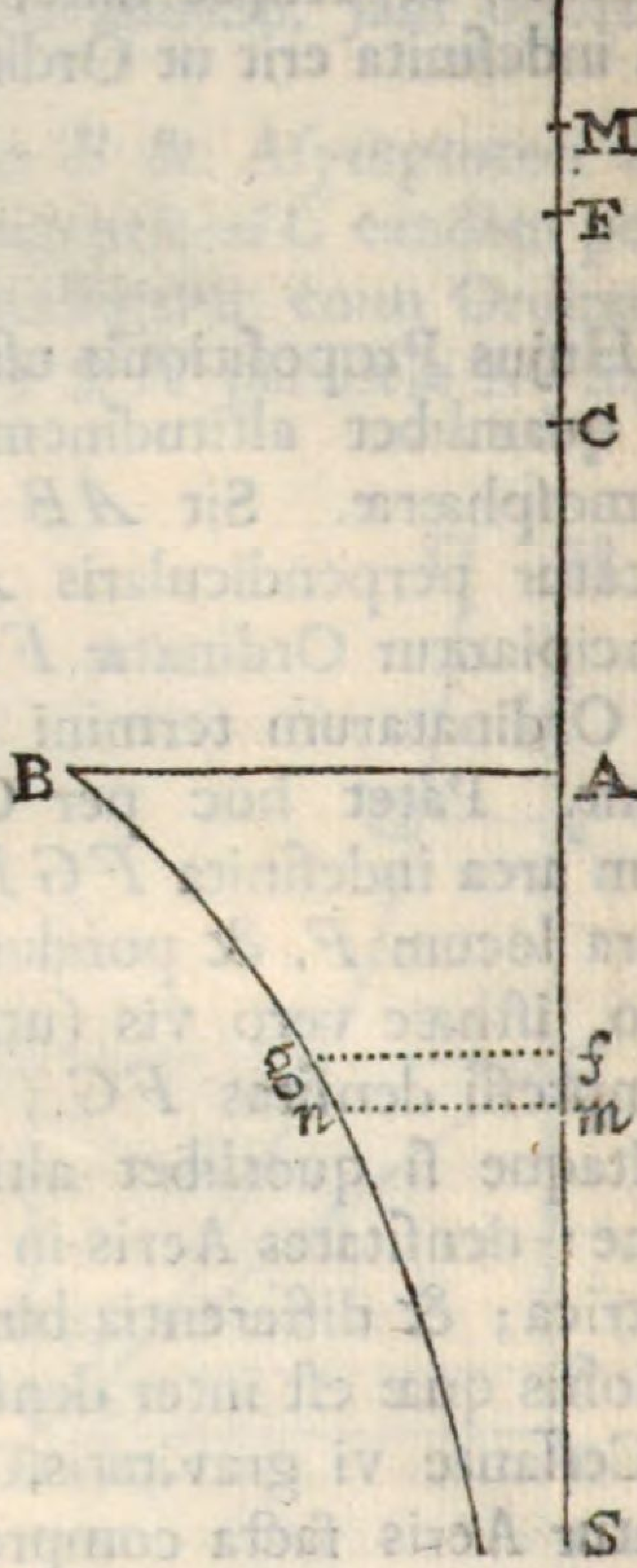
Scholium.

Hujus Propoſitionis uſus per Exemplum declarabitur. Proponatur ad quamlibet altitudinem à ſuperficie telluris, invenire denſitatem Atmoſphæræ. Sit *AB* telluris ſuperficies, & abinde ſurſum producatur perpendicularis *AH*, atque ad hujus puncta ſingula ductæ concipiantur Ordinatæ *FG*, quæ ſint ut Aeris denſitates in locis *F*; & Ordinatarum termini omnes *G* in Linea Logiſtica *BDGI* ſiti erunt. Patet hoc per Corollarium ſecundum hujus Propoſitionis. Nam area indefinita *FGIH* eſt ut quantitas ſeu pondus Atmoſphæræ ſupra locum *F*, & pondus illud eſt vis quæ comprimit Aerem in hoc loco, iſthæc vero vis (uti docet Experientia multiplex) eſt ut Aeris compreſſi denſitas *FG*.

Itaque ſi quotlibet altitudines ſumantur in Arithmetica progreſſione: denſitates Aeris in his altitudinibus erunt in progreſſione Geometrica; & differentia binarum quarumvis altitudinum, erit menſura rationis quæ eſt inter denſitates Aeris in iſtis altitudinibus.

Ceſſante vi gravitatis, ita jam per vim aliquam extraneam intelligatur Aeris facta compreſſio, ut eandem habeat ubique denſitatem quam ad terræ ſuperficiem; & quantitas ejus, quæ modo erat expoſita per aream indefinitam *HABI*, nunc per æquale rectangulum *ABEC* exhibebitur. Atmoſphæræ hujus homogeneæ altitudo *AC*, eſt ad altitudinem Hydrargyri in tubo *Torricellii*, ut gravitas Hydrargyri ad gravitatem Aeris; atque inde datur. Huic autem datæ altitudini æquatur (per Corol. 1.) ſubtangens Curvæ *BDGI*, atque adeo Modulus Syſtematis menſurarum omnium *AF*. Eſt ergo Logarithmus rationis inter denſitates Aeris in binis quibuſvis altitudinibus, ad Modulum Canonis, ut altitudinum earundem differentia, ad Atmoſphæræ prædictæ homogeneæ altitudinem illam datam *AC*.

Hæc ita se habent ex Hypothesi, quod vis gravitatis eadem sit ad omnes altitudines. Ceterum ex Philosophia *Newtoniana* constat eam diminui, in recessu à centro telluris, in duplicata ratione distantiæ: conclusio itaque paulo aliter se habebit. Sit S centrum telluris, & AB superficies ejusdem; sumatur ipsis SF, SA tertia proportionalis Sf, erigatur ordinata fg quæ sit ut Aeris densitas in F: & Curva Bgn quam punctum g perpetuo tangit, erit eadem atque prius Logistica, sed inverso situ. Augeatur enim altitudo AF particula quam minima FM, capiatur Sm ad SA ut SA ad SM, ducatur Ordinata mn quæ sit ut Aeris densitas in M; & erit Sm ad Sf ut SF ad SM, & divisim fm ad FM ut Sf ad SM, sive ut Sf ad SF, hoc est, ut SAq ad SFq. Unde fm est ut SFq inverse & FM directe, id est, ut gravitatio & moles Aeris inter F & M conjunctim; adeoque $fm \times fg$ sive area $fgnm$ est ut gravitatio, moles & densitas ejusdem Aeris conjunctim, hoc est, ut pressio illius in Aerem inferiorem: & summa similium omnium arearum infra fg est ut summa pressionum omnium supra F, id est, ut Aeris in F densitas fg: & summarum differentia $fgnm$ ut densitatum differentia $fg - mn$. Detur lineola fm; & erit fg ut area $fgnm$, adeoque ut $fg - mn$, atque inde (dividendo) ut mn. Ergo data lineola fm erit mensura datæ illius rationis quæ est inter fg & mn: atque hinc patet Curvam Bgn esse Logisticam. Sed & eandem esse cum supra descripta Logistica, facile abinde colligitur, quod ordinatæ basi AB vicinissimæ & ad æqualia intervalla quam minima dispositæ, respective sunt æquales in utraque Curva; ac proinde eadem curvatura, eadem inclinatio tangentis ad punctum B, eademque subtangentis magnitudo.



Ergo

Ergo ſi diſtantiæ SF à centro telluris, capiantur in Muſica progreſſione; harum reciprocæ, nempe diſtantiæ Sf, erunt in progreſſione Arithmetica; & Aeris denſitates fg erunt in progreſſione Geometrica. LOGOMETRIA

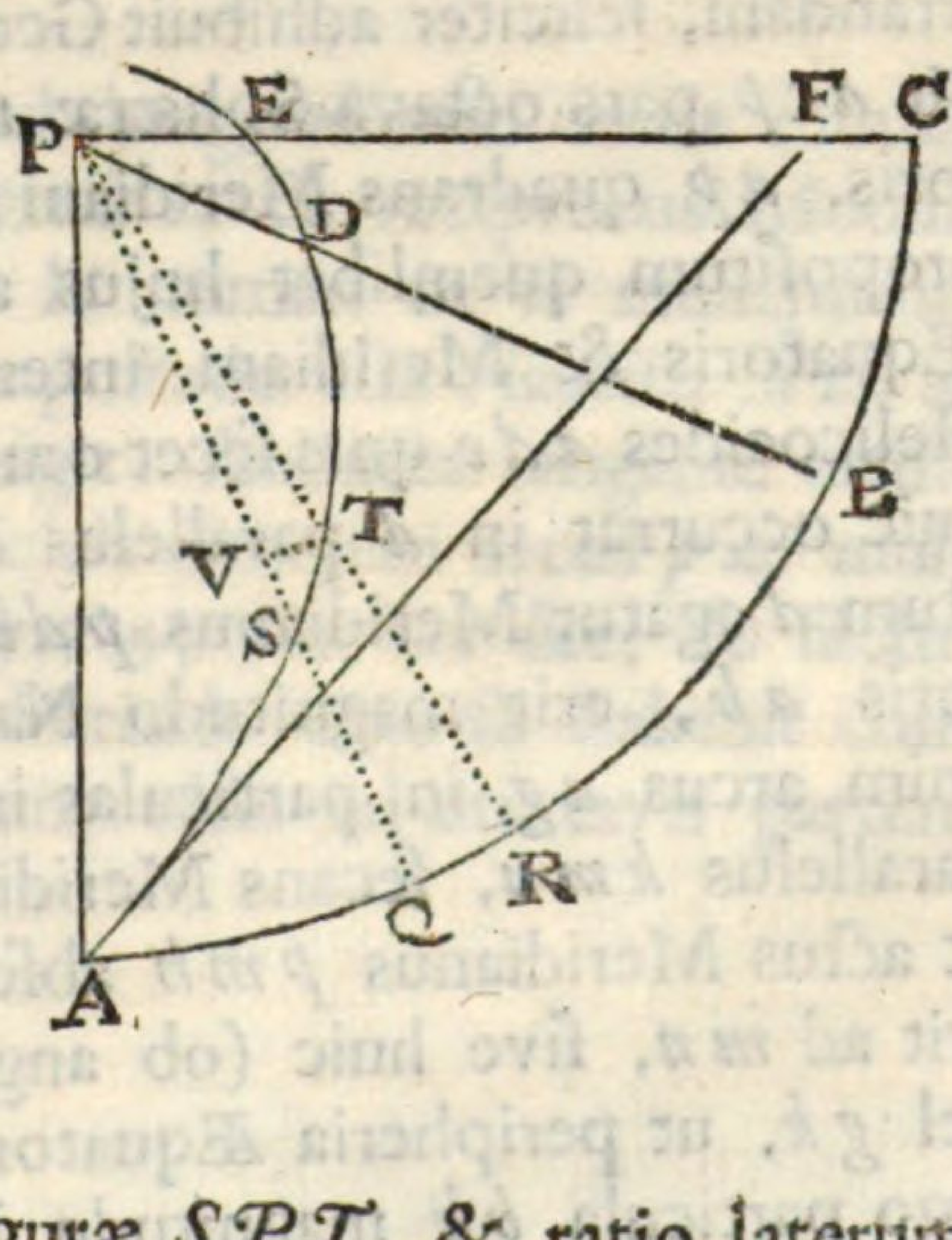
Ad inveniendam itaque denſitatem in loco quovis F, minuenda eſt altitudo AF in ratione diſtantiæ SF ad telluris ſemidiametrum SA: & Logarithmus rationis inter denſitates Aeris in A & F, erit ad Modulum Canonis, ut altitudo illa diminuta Af, ad Atmoſphæræ homogeneæ altitudinem AC.

Quæ ſupra demonſtrata ſunt, accurate obtinebunt, ſi modo Atmoſphæra ex Aere pariter Elaſtico tota conſtet: rationes igitur allatas paululum conturbabunt admiſti vapores atque exhalationes, quibus etiam accedet Caloris Frigoriſque diverſa temperies ad altitudines diverſas.

PROPOSITIO VI.

Logarithmorum Canonem ad Spiralem Æquiangulam accomodare.

ÆQiangula Spiralis appellatur Linea illa curva ADE, quæ polo P deſcripta, in eodem dato angulo ſecat exeuntes à polo radios PA, PD, PE, &c.

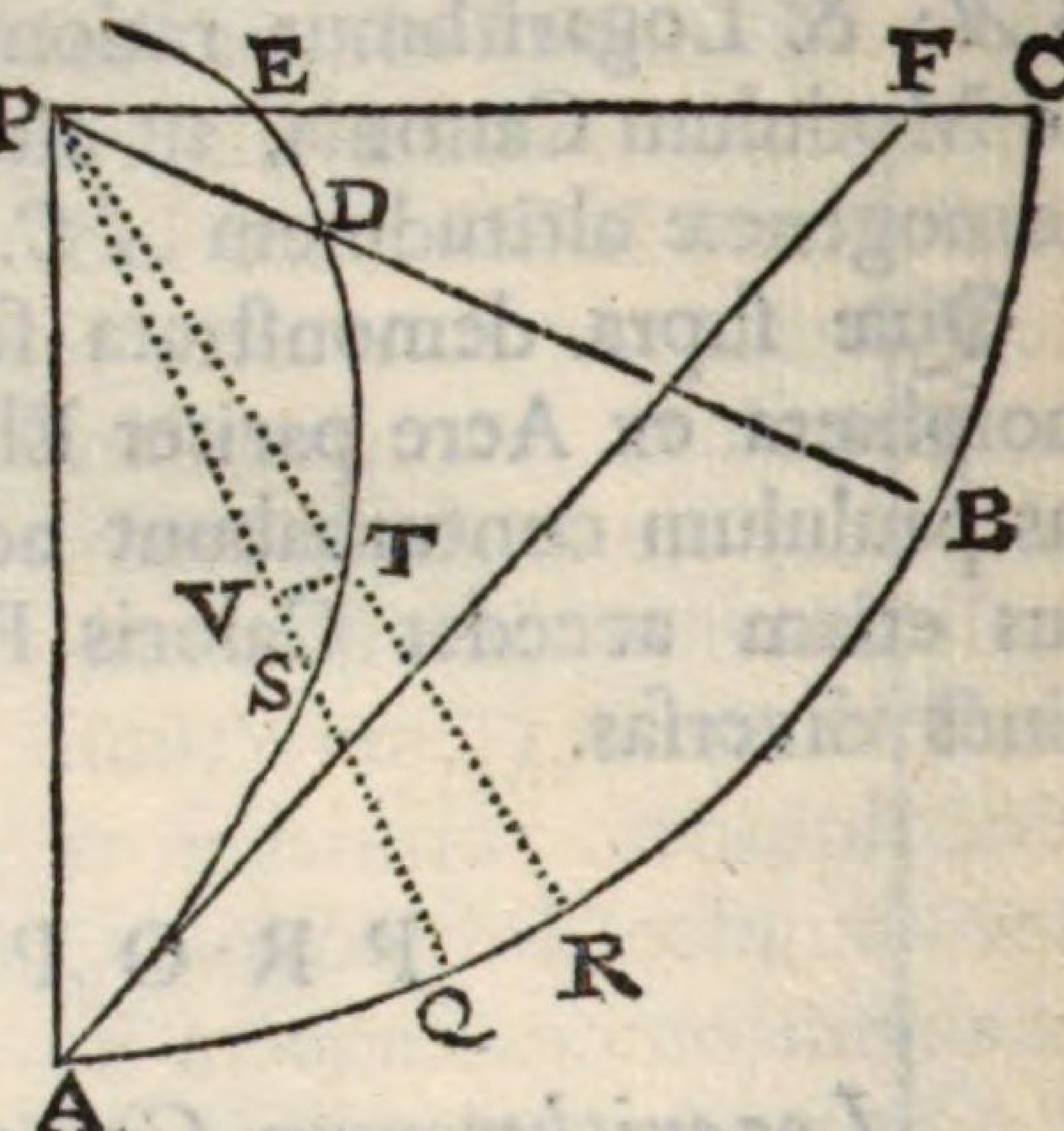


Si centro P & intervallo quovis PA deſcribatur circulus ABC, qui radiis PA, PD, PE occurrat in A, B, C: Dico interceptum arcum BC menſuram fore rationis quam habet PD ad PE, & interceptum arcum AB menſuram rationis quam habet PA ad PD. Dividatur enim arcus AB in particulas quam minimas & æquales QR, & jungantur PQ, PR ſecantes Spiralem ad S & T in angulis datis PST, PTS: & ob datam particulam QR, dabitur angulus QPR, atque adeo ſpecies Figuræ SPT, & ratio laterum PS, PT. Data ergo particula QR menſura erit rationis datæ quam

 habet

PARS PRIMA. habet *PS* ad *PT*; & ſumma particularum, nempe arcus *AB*, menſura erit ſummæ ſimilis rationum, hoc eſt, rationis quam habet *PA* ad *PD*. Et eodem argumento erit arcus *BC* menſura rationis quam habet *PD* ad *PE*.

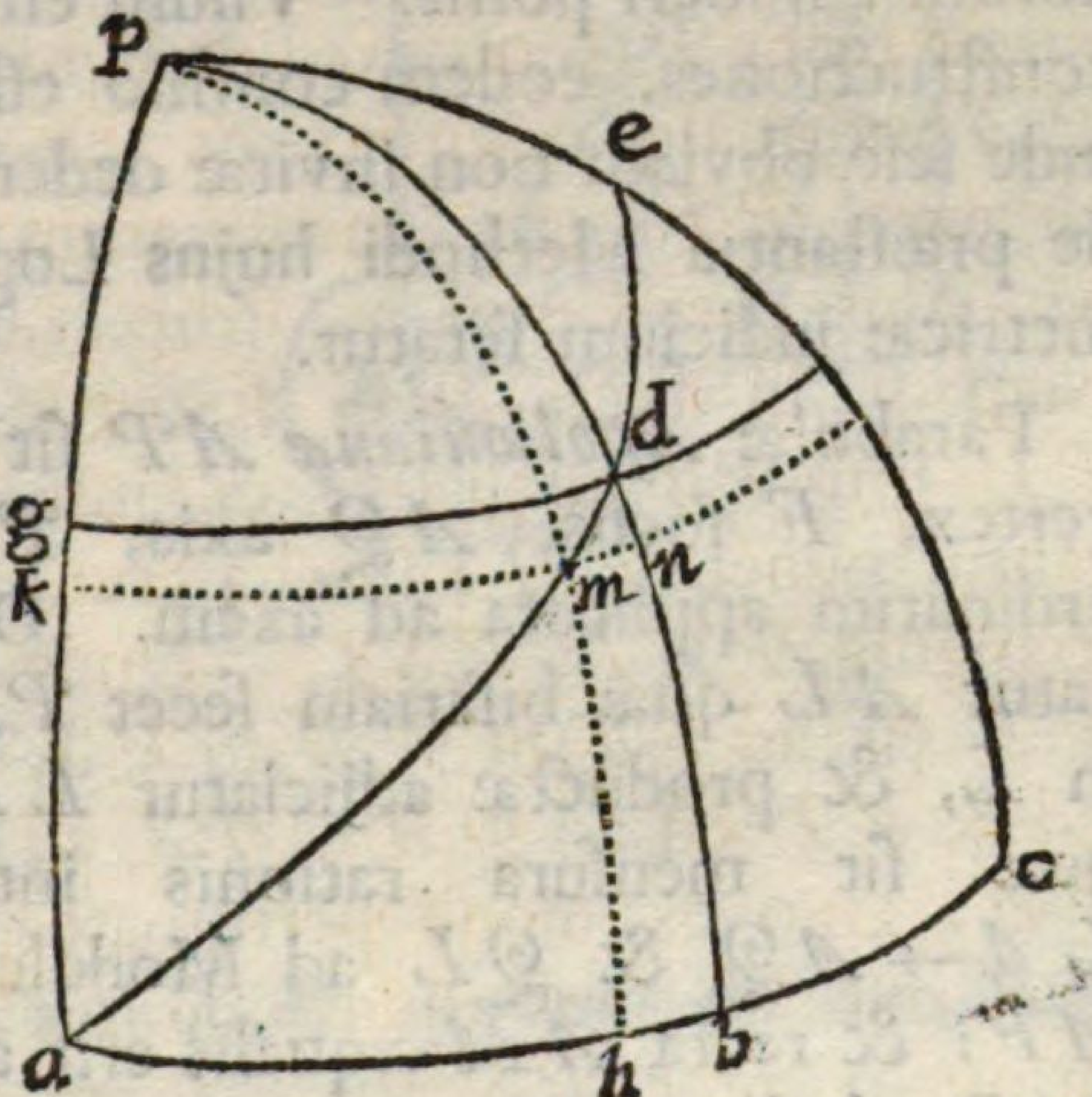
Ducatur *AF* Spiralem tangens ad Circuli & Spiralis interſectionem *A*, huic vero in *F* occurrat recta *PC* quæ ad radium *PA* normalis erigitur: & ſubtangens *PF* erit menſurarum Modulus, per Corol. 2. Prop. 1. Nam ſi in recta *PS* ſumatur *PV* ipſi *PT* æqualis, & jungantur puncta *V*, *T*; ſimilia erunt triangula *PAF*, *VST*. Unde *PF* eſt ad *VT* ut *PA* ad *VS*, ſed & *VT* eſt ad *QR* ut *PT* ad *PA*: ergo ex æquo perturbate, *PF* eſt ad *QR* quæ metitur rationem inter *PS* ad *PT*, ut terminus *PT* ad terminorum differentiam *VS*.



Scholium.

Spiralem æquiangulam, ad Meridianæ Nauticæ diviſionem demonſtrandam, feliciter adhibuit Geometra clariſſimus *Edmundus Halleius*. Sit *acp* pars octava Sphæræ terreſtris, *p* Polus, *ac* quadrans Æquatoris, *ap* quadrans Meridiani; & quæratur magnitudo rectæ, quæ propoſitum quemlibet hujus arcum deſignet in Planiſphærio. Per Æquatoris & Meridiani interſectionem *a*, ducta intelligatur linea Helicoeides *ade* quæ ſecet omnes Meridianos ad angulum ſemirectum, huic occurrat in *d* parallelus Æquatori circulus *gd*, per idem punctum *d* agatur Meridianus *pdb*; & longitudo intercepti arcus Æquatoris *ab*, erit magnitudo Nautica quæſita arcus *ag*. Reſolvatur enim arcus *ag* in particulas innumeras quam minimas *gk*, ducatur parallelus *kmn*, ſecans Meridianum *pdb* in *n*, Lineam *ade* in *m*; & actus Meridianus *pmh* abſcindet Æquatoris particulam *bh*, quæ erit ad *mn*, ſive huic (ob angulum ſemirectum *mdn*) æqualem *dn* vel *gk*, ut peripheria Æquatoris ad peripheriam paralleli *kmn*. Eſt ergo particula *bh* magnitudo Nautica particulæ *gk*; & ſumma particularum omnium *bh*, nempe longitudo arcus *ab*, magnitudo Nautica

tica ſummæ particularum omnium *gk*, id eſt, arcus *ag*. Manente jam Æquatore *abc* vel *ABC*, concipiatur Sphærica ſuperficies in plano ejus Stereographice depingi; & Polo *p* occupante centrum *P*, projicientur Meridiani *pga*, *pdb*, *pec* in totidem rectas *PA*, *PDB*, *PEC* à centro *P* exeuntes, ita ut diſtantia abinde puncti cujuſvis *D* vel *A*, tangens ſit arcus dimidiati *pd* vel *pa* quem diſtantia illa repræſentat. Linea vero Helicoeides *ade* convertet ſe in Spiralem æquiangulam *ADE*, polo *P* deſcriptam, & ſecantem radios ſuos omnes ad angulum ſemirectum. Hoc ſiquidem poſtulat nota Lex hujuſce Projectionis, ut anguli omnes eandem in Plano ac in Sphærica ſuperficie magnitudinem ſervent. Arcus itaque propoſiti *ag* magnitudo Nautica *ab* vel *AB*, eſt ad ſubtangentem *PF* vel huic jam æqualem Sphæræ radium *PC*, ut Logarithmus rationis inter *PA* & *PD*, hoc eſt, inter tangentes dimidiatorum arcuum *pa* & *pd*, vel *pa* & *pg*, ad Modulum Canonis.



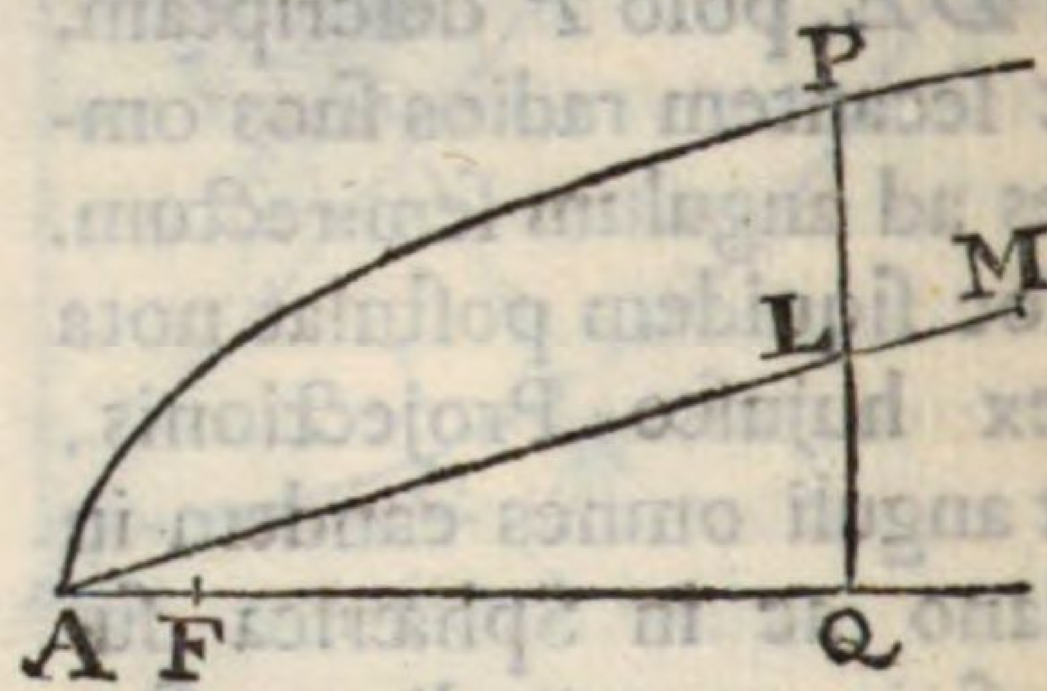
Hinc quoniam longitudo Radii eſt ad longitudinem arcus minuti unius primi, ut 3437,74677078493 9 &c ad 1, & reciprocus Moduli Canonis eſt 2,302585092994 &c, atque hi numeri in ſe multiplicati efficiunt 7915,7044678978 19 &c: ſi magnitudo illa Nautica *AB* in minutis primis exhibenda ſit, uti mos exigit: ſubducta tangente artificiali dimidiati arcus *pg* à tangente artificiali dimidiati arcus *pa*, multiplicetur reſiduum per numerum 7915,704467897819 &c, & factus dabit partes Meridionales deſideratas. Perinde vero ſe habebit concluſio, ſive in Æquatore, ſive extra hunc alibi ad utramvis partem locetur punctum *a*.

SCHO-

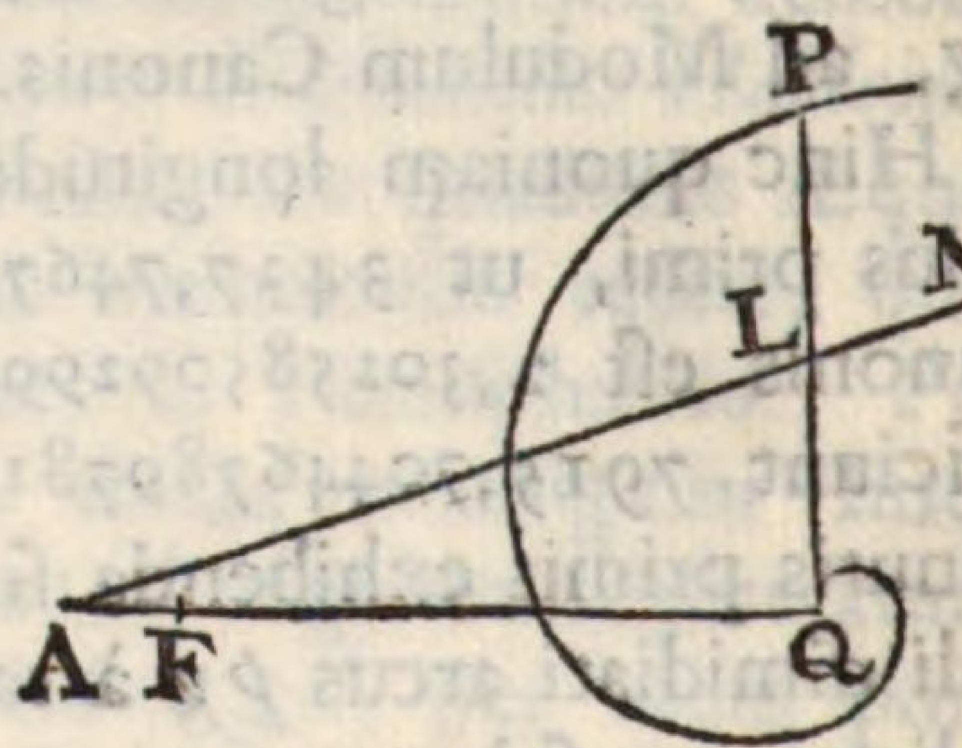
SCHOLIUM GENERALE.

IN eum potissimum finem præcedentia conscripsi, ut allatis aliquot Exemplis ostenderem, qua commodissima ratione Logarithmorum usus in Geometriam recipi, & ad resolutionem Problematum difficiliorum adhiberi possit. Visum est hoc loco nonnullas adjicere porro constructiones, eodem consilio effectas, quæ mihi ista tractanti subinde sese obviam non invitæ dederunt: ut ita, ex uberiore specimine, de præstantia Methodi hujus Logometricæ judicium feratur.

Parabolæ *Apollonianæ* AP sit A vertex, F focus, AQ axis, PQ ordinatim applicata ad axem. Ducatur AL quæ bifariam secet PQ in L, & productæ adjiciatur LM quæ sit mensura rationis inter $LA + AQ$ & QL ad Modulum AF: & recta AM æqualis erit arcui Parabolico AP.

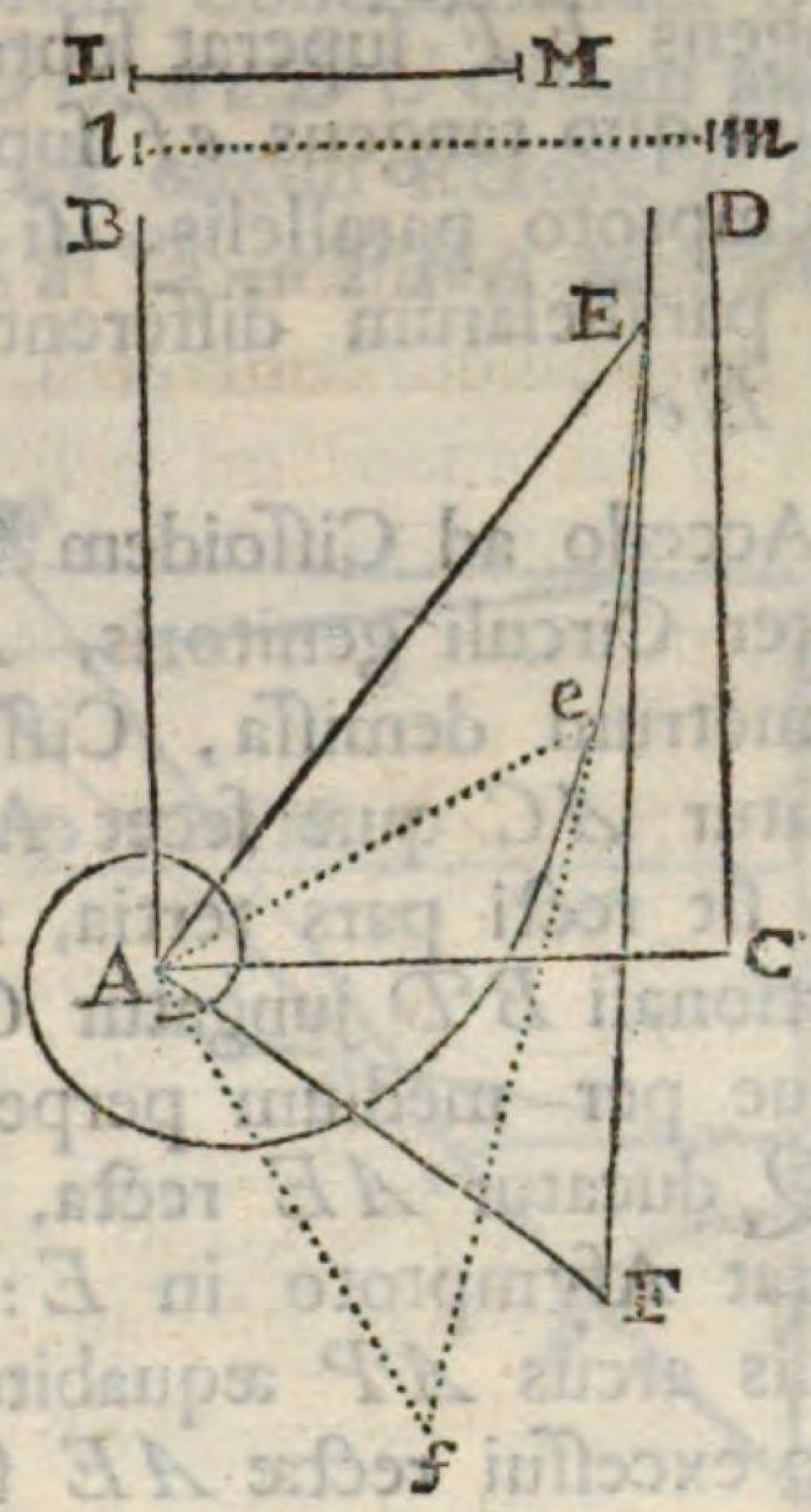


Spiralis *Archimedea* PQ similem habet extensionem in rectam. Sit Q polus ejus, QP radius à polo ductus ad Curvæ quodlibet punctum P, & ad eum radium normalis QA. Ducatur LA parallela tangenti Spiralem in P, quæ radium PQ bifariam secet in L; & ponendo AF ad QL ut QL ad QA, ipsi AL adjiciatur LM quæ sit mensura rationis inter $LA + AQ$ & QL ad Modulum AF: & recta AM æquabitur Spiralis arcui PQ.

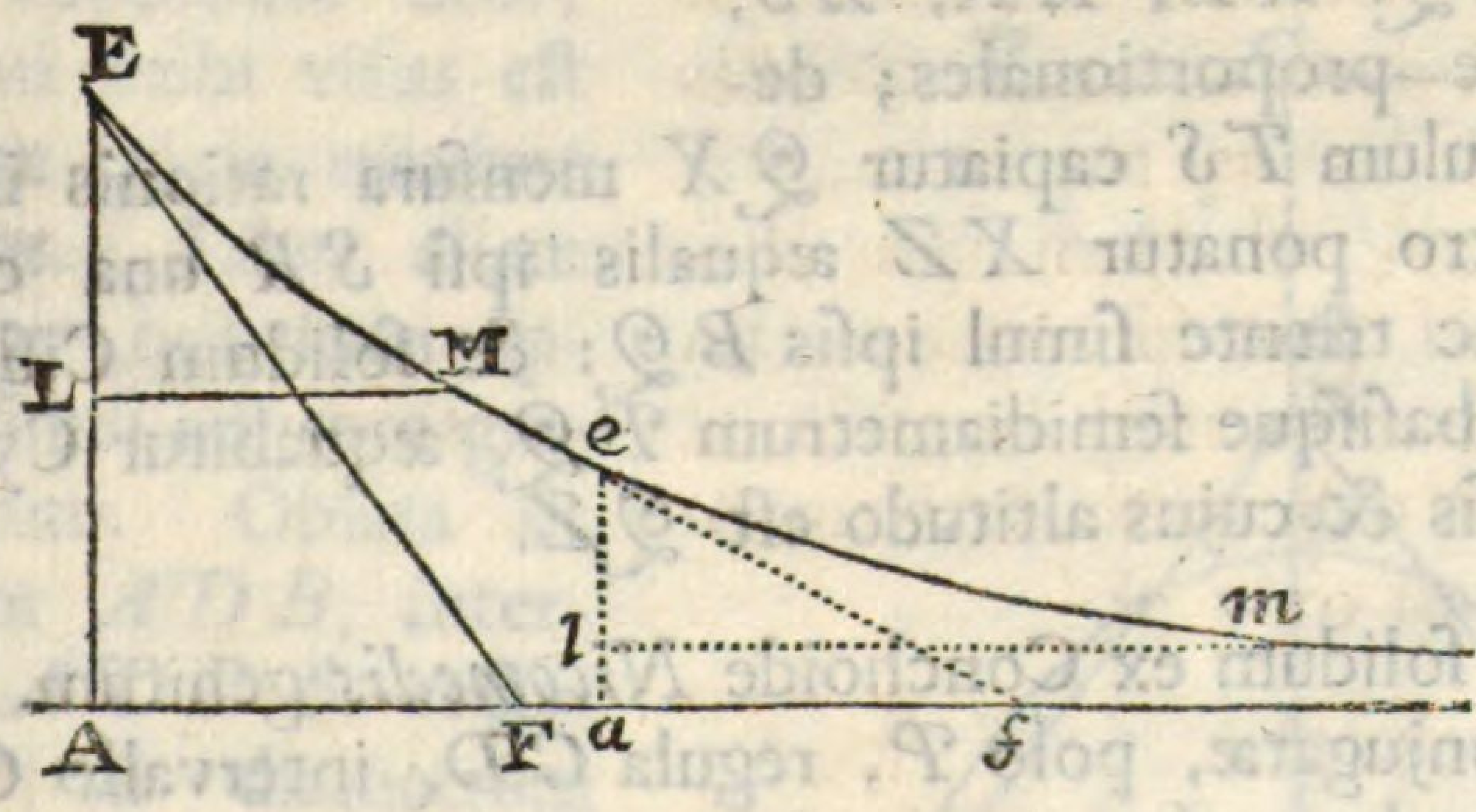


Spiralis Reciprocæ AeE sit A polus, AB radius primus & infinitus, CD asymptotos radio primo parallela ad distantiam AC; & invenienda proponatur hujusce Curvæ longitudo. Inter Spiralem illam vulgarem *Archimedis* atque hanc, quam Reciprocam appello, isthæc intercedit differentia, quod cum illius radii sint ut anguli quos faciunt cum radio suo primo, hujus radii è contrario sunt

ſunt reciproce ut iidem anguli: eandem utique proportionem habet radius AE ad radium Ae quam habet angulus eAB ad angulum EAB. Unde facile colligitur, ſi ad puncta E & e ducantur tangentes EF, ef, & ad radios AE, Ae erigantur normales AF, Af, fore normales iſtas ſibi invicem & Aſymptoti intervallo AC æquales. Invenietur autem longitudo cujuſvis arcus Ee, ponendo LM menſuram rationis inter AE & $EF-AF$ ad Modulum AF, & ſimiliter lm menſuram rationis inter Ae & $ef-Af$ ad æqualem Modulum Af. Nam ſi tangentium differentiæ $EF-ef$ adjiciatur menſurarum differentia $lm-LM$, aggregatum æquabitur arcui Ee.



Linea illa Logiſtica, cujus aliquas expoſuimus affectiones in Propoſitione quinta, non abſimilem habet longitudinis ſuæ determinationem; quam & hoc loco apponam in eorum gratiam qui hujuſ-



modi contemplationibus delectantur. Oblata ſit igitur Logiſtica $EMem$, cujus Aſymptotos $AFaf$: & quæratur longitudo cujuſvis arcus Ee. Demiſſis in Aſymptoton perpendiculis ELA, ela, &

PARS PRIMA. & ductis tangentibus EF, ef, capiatur AL æqualis excessui quo tangens EF superat subtangentem AF, & similiter al æqualis excessui quo tangens ef superat subtangentem af: & actis LM, lm Asymptoto parallelis, si tangentium differentiæ $EF-ef$ adjiciatur parallelarum differentia $lm-LM$, aggregatum æquabitur arcui Ee.

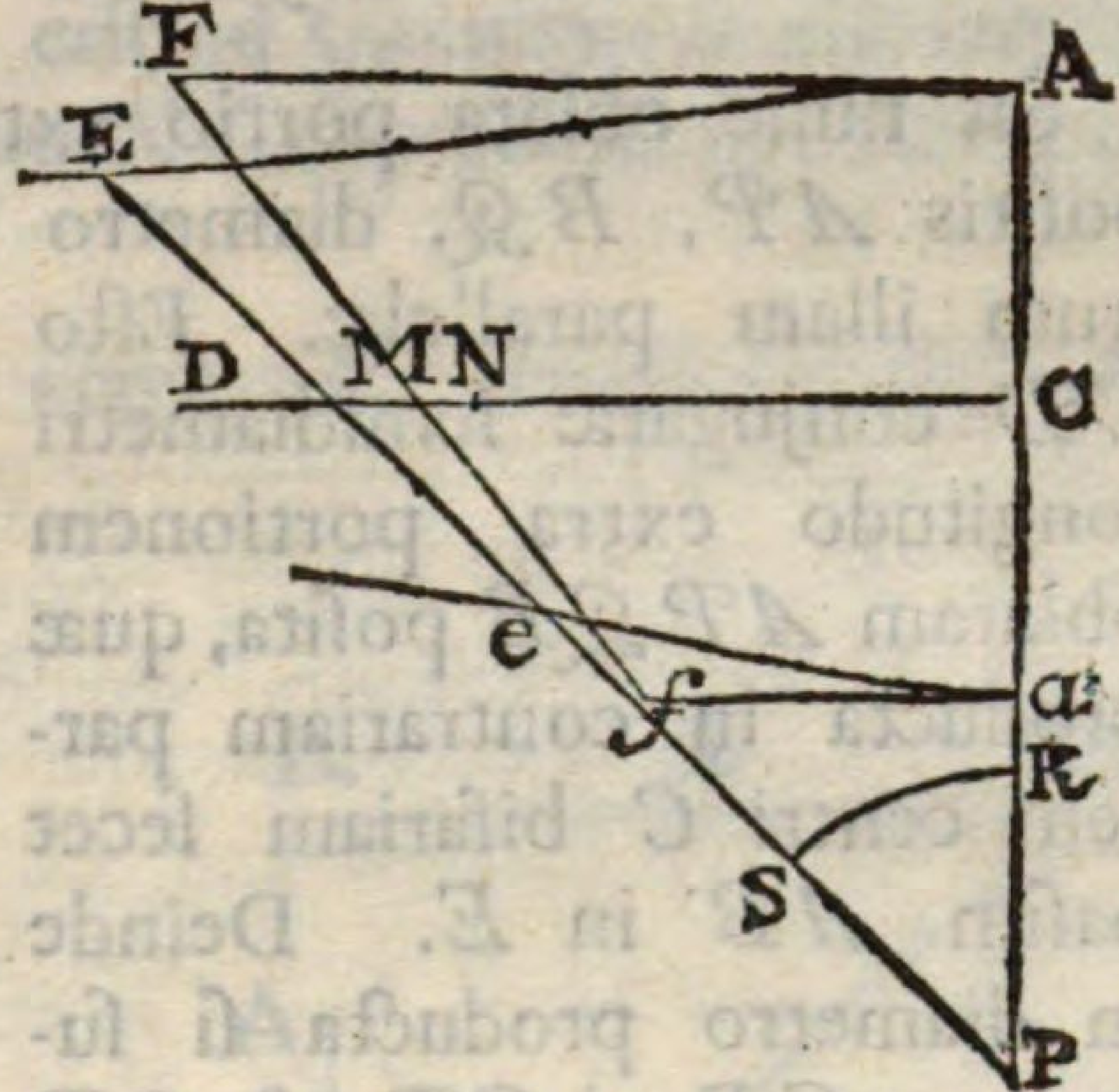
Accedo ad Cissoidem *Diocleam*. Sit A vertex ejus, AB diameter Circuli genitoris, BC Asymptotos, PQ perpendicularis in diametrum demissa, Cissoidi in P & diametro in Q occurrens. Agatur AC quæ secet Asymptoton in C ac faciat angulum BAC qui sit recti pars tertia, sumptaque inter BQ & BA media proportionali BD jungatur CD; denique per medium perpendiculum PQ ducatur AE recta, quæ occurrat Asymptoto in E: & Cissoidis arcus AP æquabitur duplicato excessui rectæ AE supra diametrum AB, & simul triplicatæ mensuræ rationis inter $BA+AC$ & $BD+DC$ ad Modulum BC.



Si Cissoidis area APQ convertatur circum axem AQ; generabitur solidum cujus dimensio pendet à Logometria, & sic construitur. Sint AQ, AB, AR, AS, AT continue proportionales; deinde ad Modulum TS capiatur QX mensura rationis inter AB & BQ, & retro ponatur XZ æqualis ipsi SR una cum dimidio ipsius RB ac triente simul ipsis BQ: & solidum Cissoidale axem habens AQ basisque semidiametrum PQ, æquabitur Cylindro cujus eadem est basis & cujus altitudo est QZ.

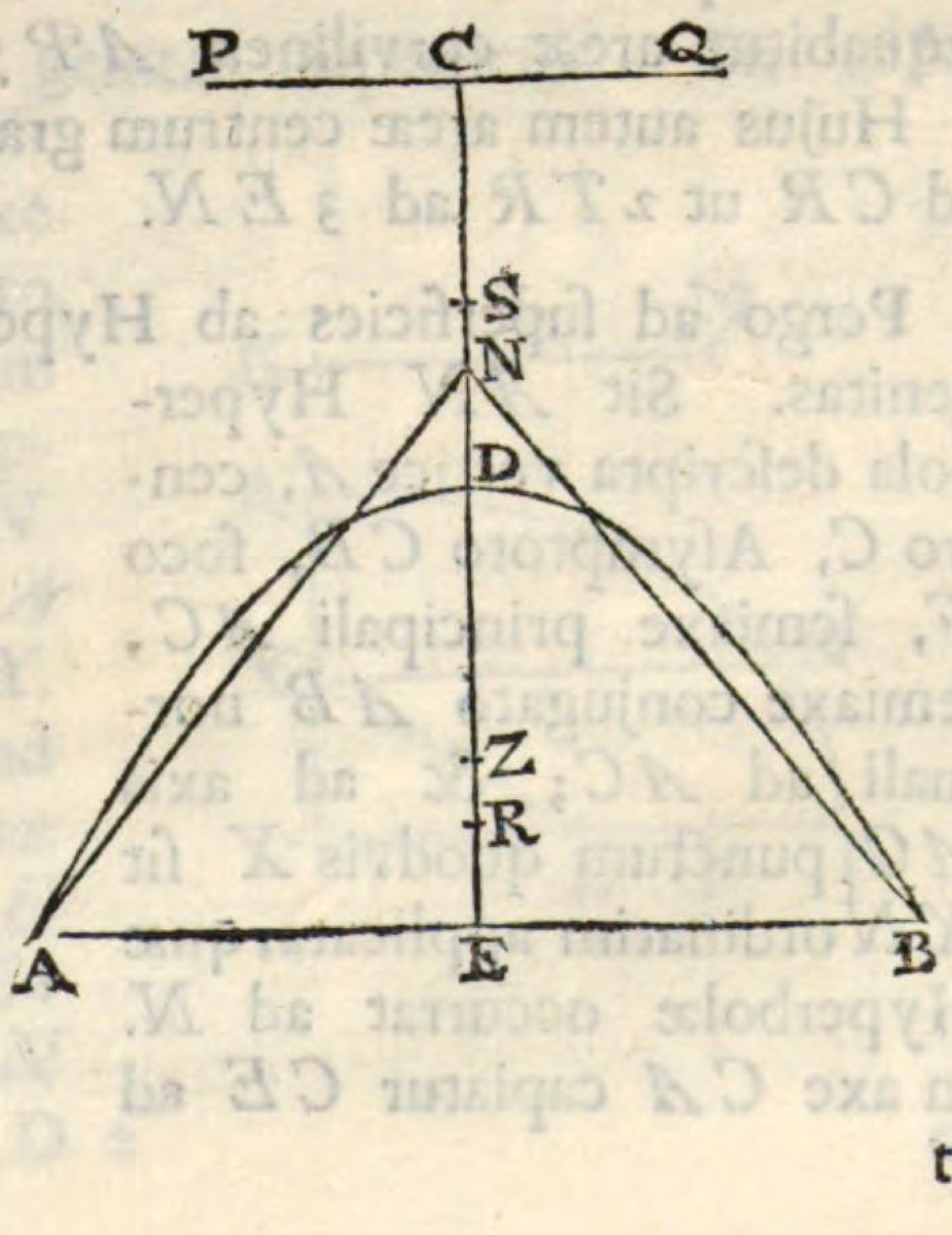
Adjungam solidum ex Conchoide *Nicomedis* genitum. Sint AE, ae Curvæ conjugatæ, polo P, regula CD, intervallo CA vel Ca, axe $PaCA$ ad regulam normali, verticibusque A & a descriptæ. Per polum P ducatur ad libitum recta $PeDE$, regulæ occurrens in D, Lineæ vero in E & e: & ex natura Conchoidis, erunt segmenta DE, De intervallo CA vel Ca æqualia. Eodem intervallo centroque P describatur circuli arcus RS secans axem PC in R & rectam

rectam PD in S: & semisumma solidorum Conchoidalium quæ generantur ex conversione Figurarum $AEDC$, $aeDC$ circum axem AaP, erit ad sectorem Sphæræ genitum ex circuli sectore PRS circum axem eundem converso, ut $3\,PC \times PD + PRq$ ad PRq. Eorundem vero semidifferentia Cylindro æquatur, cujus basis est circulus diametro Aa descriptus, & cujus altitudo est mensura duplicata rationis inter PD & PC ad Modulum PC. LOGOMETRIA



Area vero Figuræ totius $AEea$ æquatur rectangulo cujus basis est Aa, & cujus altitudo CM est mensura rationis inter $PD + DC$ & PC ad Modulum PC. Quod si desideretur quadratura partium $AEDC$, $aeDC$; ductis ad axem normalibus AF, af, in regula CD sumenda est CN quæ sit anguli CPD mensura ad eundem Modulum PC: & acta per punctum M recta FMf quæ parallela sit rectæ jungenti puncta P, N, quæque occurrat normalibus in F & f; erit area $AEDC$ æqualis Trapezio $AFMC$, & area $aeDC$ æqualis Trapezio $afMC$.

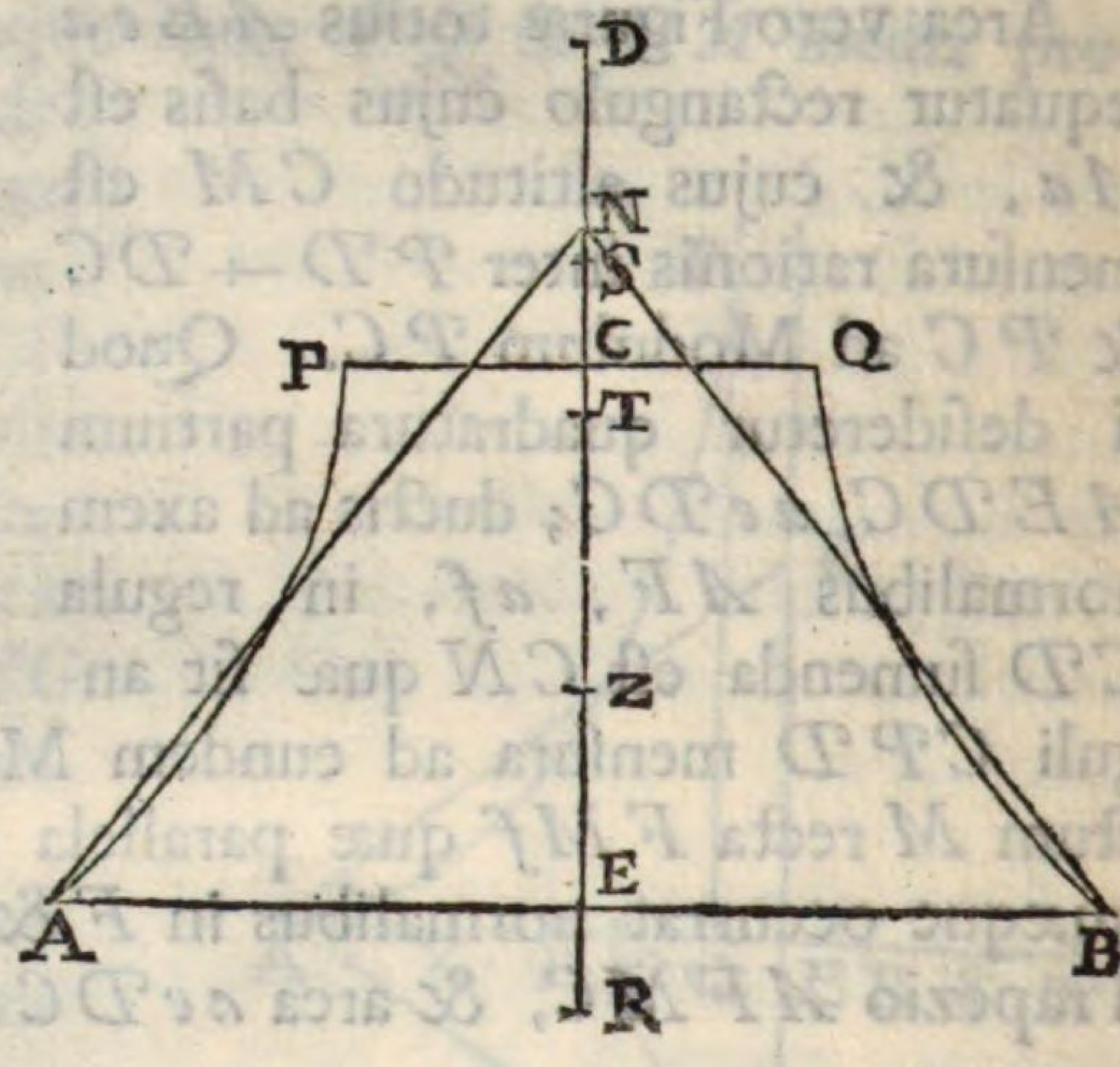
Hyperbolæ quadraturam in superioribus expositam dedi, eo modo, qui mihi visus est ad propositum quam maxime accommodatus. Libet aliam constructionem hoc loco apponere, & simul adjicere gravitatis centrum. Oblata sit portio interior ADB, interclusa curvæ ADB & rectæ cuivis AB ad diametrum PQ parallelæ. A Figuræ centro C producatur diameter CDE, quæ basin AB bifariam secet in E; deinde si in diametro producta suman-



PARS PRIMA. tur CR ad CD, & CD ad CS, ut basis AB ad diametrum PQ. & ad Modulum CS fiat CN mensura rationis quam habet CD ad ER: triangulum rectilineum ANB æquabitur areæ curvilineæ ADB.

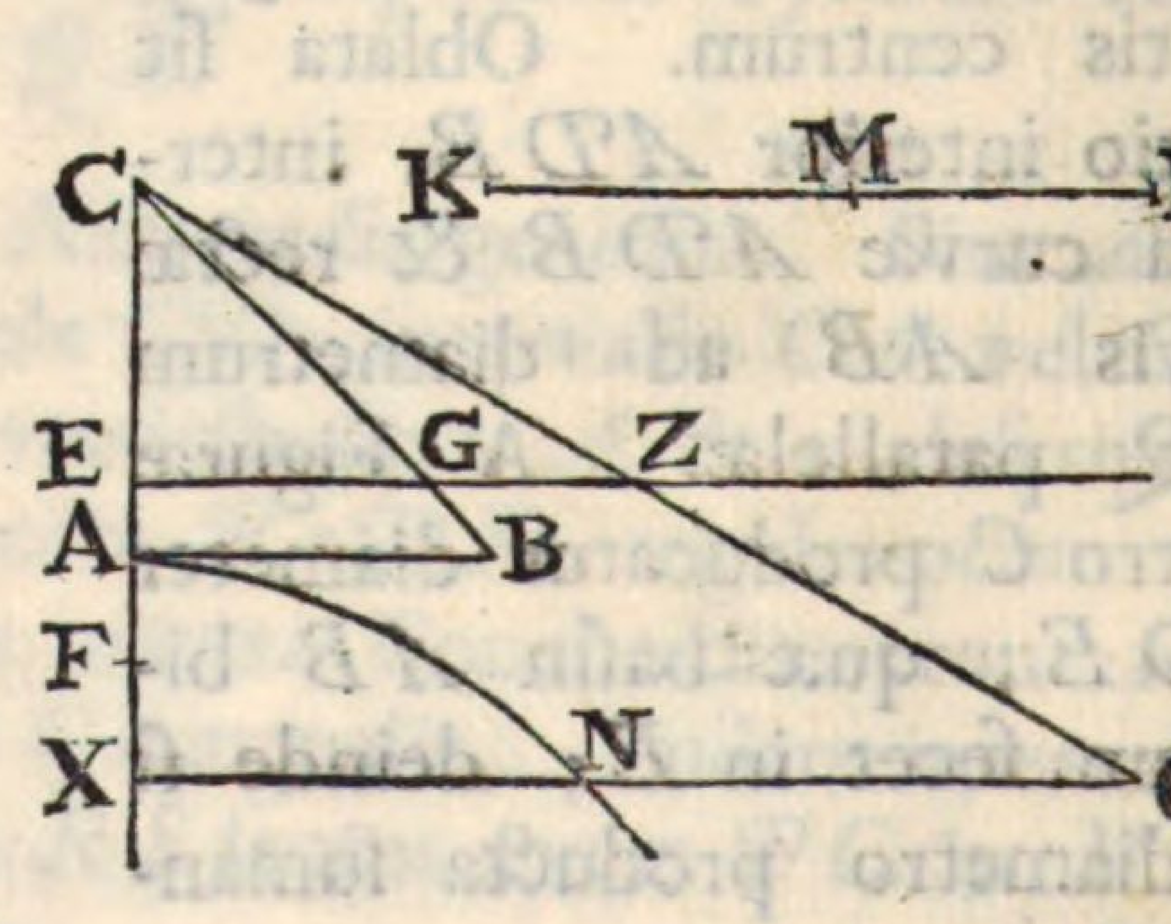
Hujus autem areæ centrum gravitatis Z invenietur, capiendo CZ ad CR ut $2CR$ ad $3EN$.

Sit nunc oblata portio exterior $APQB$, interclusa curvis oppositis AP, BQ, diametro PQ, & rectæ cuivis AB ad diametrum illam parallelæ. Esto CD conjugatæ semidiametri longitudo extra portionem oblatam $APQB$ posita, quæ producta in contrariam partem centri C bifariam secet basim AB in E. Deinde in diametro producta si sumantur CR ad CD, & CD ad CS, & CS ad CT, ut basis AB ad diametrum PQ, ponantur vero CR & CT ad eandem centri partem cum basi AB; & ad Modulum CS, in contrariam centri partem, sumatur CN mensura rationis quam habet CD ad ER: triangulum rectilineum ANB æquabitur areæ curvilineæ $APQB$.



Hujus autem areæ centrum gravitatis Z invenietur, capiendo CZ ad CR ut $2TR$ ad $3EN$.

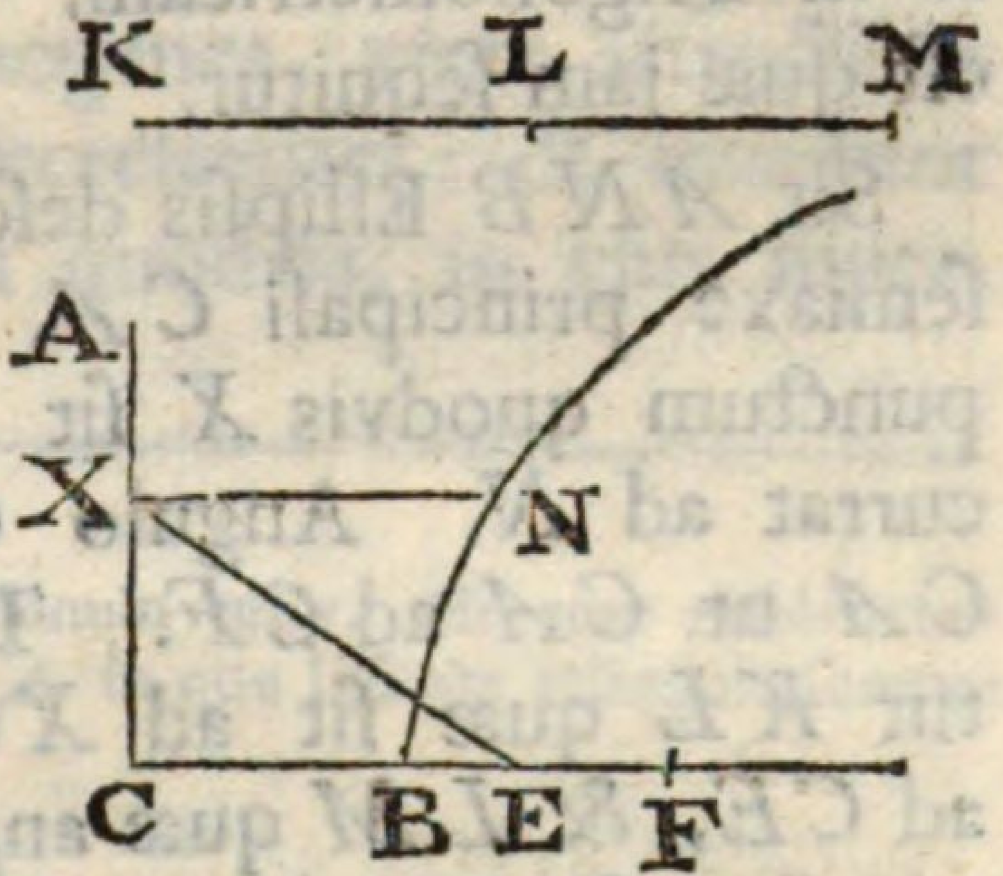
Pergo ad superficies ab Hyperbola circum axes suos convoluta genitas. Sit AN Hyperbola descripta vertice A, centro C, Asymptoto CB, foco F, semiaxe principali AC, semiaxe conjugato AB normali ad AC; & ad axis AC punctum quodvis X sit XN ordinatim applicata, quæ Hyperbolæ occurrat ad N. In axe CA capiatur CE ad



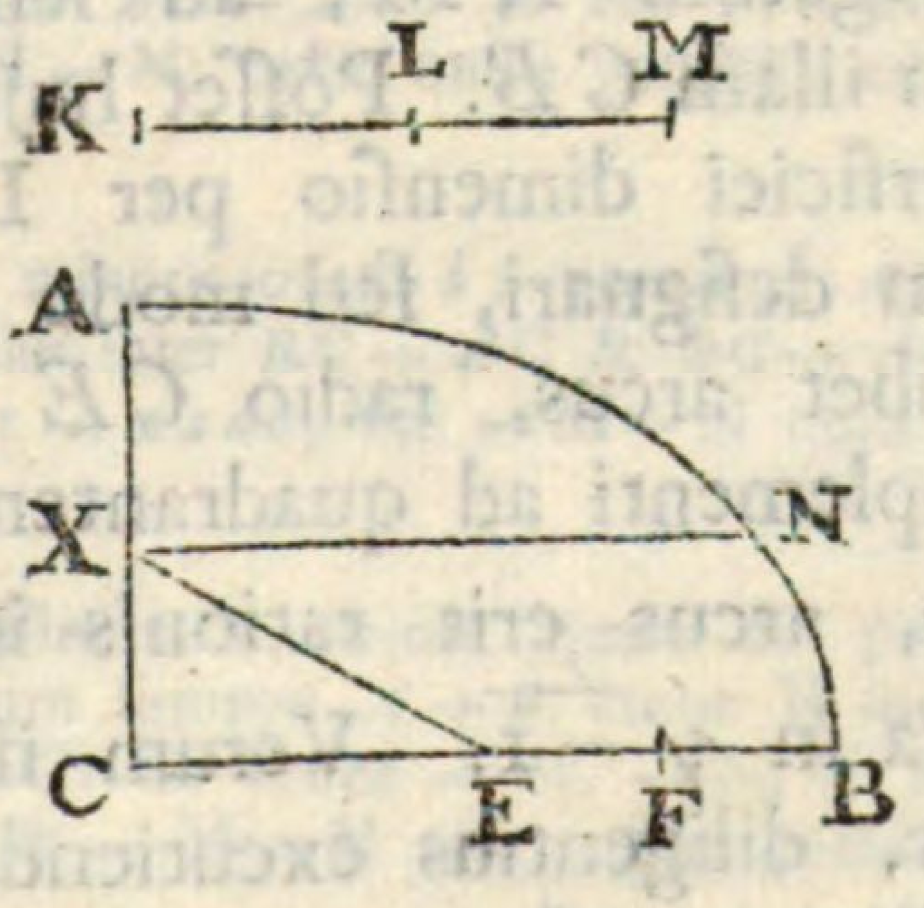
CA ut

CA ut *CA* ad *CF*; & ad eundem axem erecta perpendiculari *EZ*, quæ Asymptoto occurrat in *G*, angulo *CEZ* inscribatur æqualis ipsi *CX* recta *CZ*, quæ porro producta secet ordinatim applicatam *XN* ad *O*. Tum sumatur *KL* quæ sit æqualis excessui quo *XO* superat *AB*, atque *LM* quæ sit mensura rationis inter *CZ* + *ZE* & *CG* + *GE* ad Modulum *CE*: & superficies genita ex arcus *AN* conversione circum axem *AX*, erit ad Circulum semidiametro *AB* descriptum, ut excessus *KM* quo *KL* superat *LM*, ad semidiametrum illam *AB*. LOGOMETRIA

Sit rursus *BN* Hyperbola descripta vertice *B*, centro *C*, foco *F*, semiaxe principali *CB*, semiaxe conjugato *CA* normali ad *CB*; & ad axis *AC* punctum quodvis *X* sit *XN* ordinatim applicata, quæ Hyperbolæ occurrat ad *N*. In axe *CB* capiatur *CE* ad *CA* ut *CA* ad *CF*, & jungatur *EX*. Tum sumatur *KL* quæ sit ad *XC* ut *XE* ad *CE*, & *LM* quæ rationis inter *EX* + *XC* & *CE* mensura sit ad Modulum *CE*: & superficies genita ex arcus *BN* conversione circum axem *CX*, erit ad Circulum semidiametro *CB* descriptum, ut linearum *KL* & *LM* aggregatum *KM*, ad semidiametrum illam *CB*.

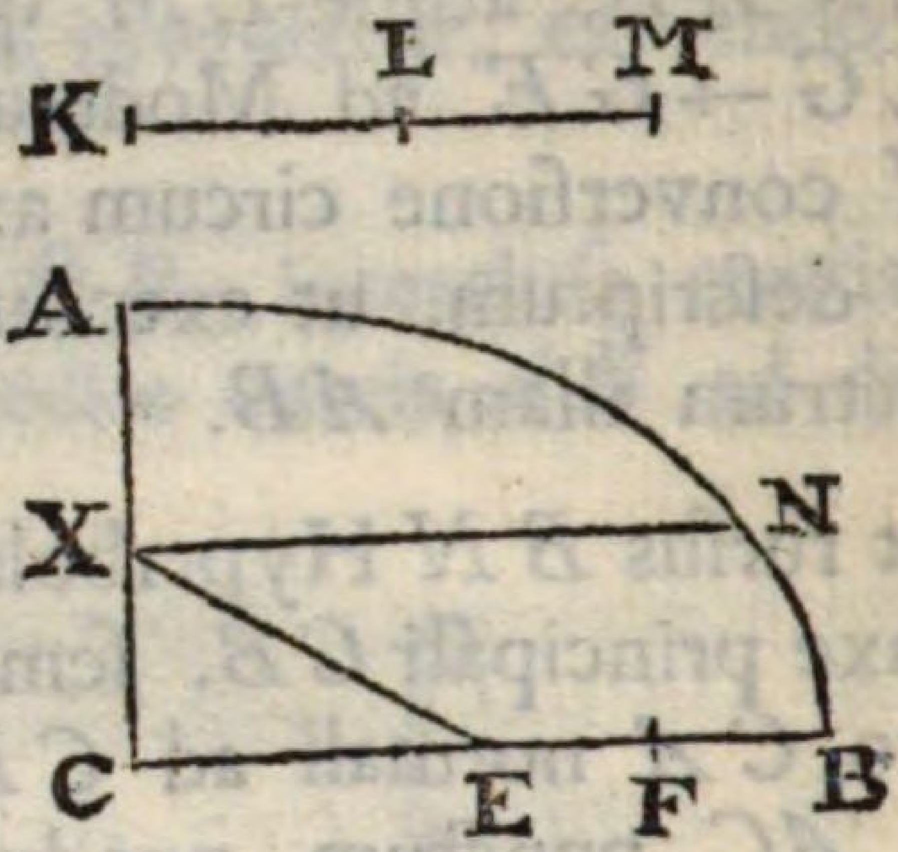


His addere licebit ab Ellipsi genitas superficies. Sit *ANB* Ellipsis descripta centro *C*, verticibus *A* & *B*, foco *F*, semiaxe principali *CB*, semiaxe conjugato *CA*; & ad axis *CA* punctum quodvis *X* sit *XN* ordinatim applicata, quæ Ellipsi occurrat ad *N*. In axe *CB* capiatur *CE* ad *CA* ut *CA* ad *CF*, & jungatur *EX*. Tum sumatur *KL* quæ sit ad *XC* ut *XE* ad *CE*, & *LM* quæ rationis inter *EX* + *XC* & *CE* mensura sit ad Modulum *CE*: & superficies genita ex arcus *BN*

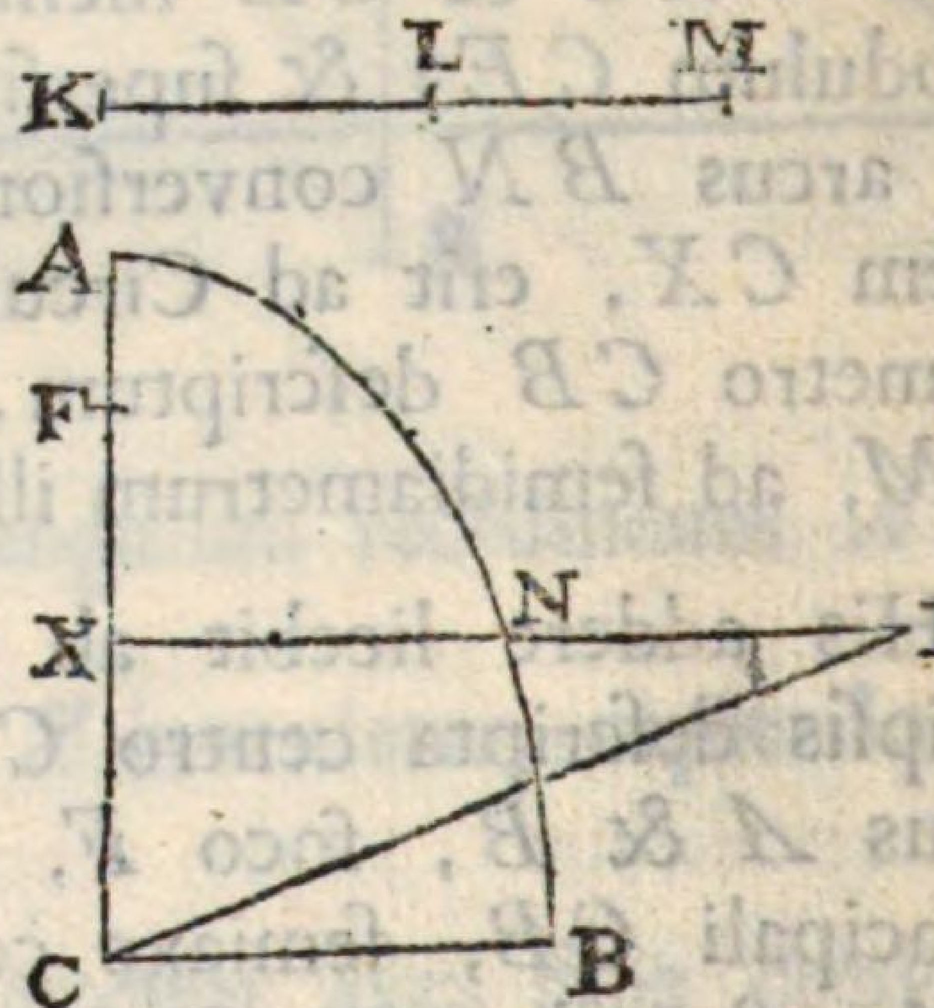


PARS PRIMA.

converſione circum axem CX, erit ad Circulum ſemidiametro CB deſcriptum, ut linearum KL & LM aggregatum KM, ad ſemidiametrum illam CB. Ut hæc ultima conſtructio locum habeat, oportet ſemiaxem CA circa quem converſio facta eſt, minorem eſſe altero ſemiaxe CB; aliter enim Moduli CE quantitas $\frac{CAq}{\sqrt{CBq - CAq}}$ evadet impoſſibilis, & conſtructio illa Logometrica (quod in hujuſmodi caſibus fieri ſolet) convertet ſe in Trigonometricam, qualis illa eſt quæ jam ſequitur.



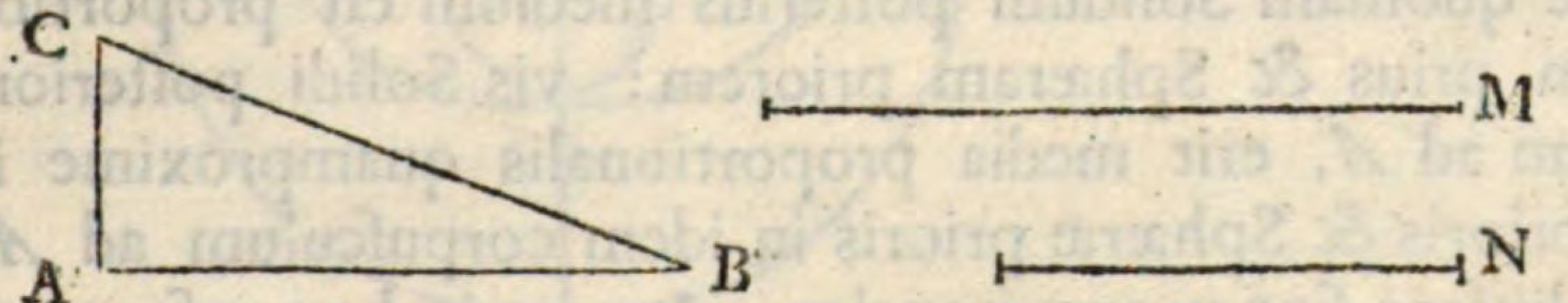
Sit ANB Ellipſis deſcripta centro C, verticibus A & B, foco F, ſemiaxe principali CA, ſemiaxe conjugato CB; & ad axis CA punctum quodvis X ſit XN ordinatim applicata, quæ Ellipſi occurrat ad N. Angulo CXN inſcribatur recta CE, quæ ſit ad CA ut CA ad CF. Tum ſumatur KL quæ ſit ad XC ut XE ad CE, & LM quæ anguli XEC menſura ſit ad Modulum CE, hoc eſt, quæ ſit æqualis arcui cujus ſinus eſt XC ad radium CE: & ſuperficies genita ex arcus BN converſione circum axem CX, erit ad Circulum ſemidiametro CB deſcriptum, ut linearum KL & LM aggregatum KM, ad ſemidiametrum illam CB. Poſſet hujus etiam ſuperficiei dimenſio per Logometriam deſignari, ſed modo inexplicabili. Nam ſi quadrantis circuli quilibet arcus, radio CE deſcriptus, ſinum habeat CX ſinumque complementi ad quadrantem XE: ſumendo radium CE pro Modulo, arcus erit rationis inter $EX + XC\sqrt{-1}$ & CE menſura ducta in $\sqrt{-1}$. Verum iſthæc aliis, quibus operæ pretium videbitur, diligentius excutienda relinquo. Ceterum ex præcedentibus intelligi poteſt, quanta ſit cognatio inter angulorum atque rationum menſuras, quamque levi mutatione in ſe invicem facillime convertantur



tantur pro variis ejusdem Problematis casibus. De Cubicarum æquationum radicibus dudum ab Analystis observatum est; vel eas exprimi posse per *Cardani* regulas, atque adeo per duarum mediarum proportionalium inventionem; vel per divisionem arcus circularis in tres æquales partes, si forte fuerint inexplicabiles per memoratas regulas. * Hoc animadvertit *Cartesius*, sed & ante *Cartesium* idem observavit *Franciscus Vieta* sub finem Supplementi Geometriæ. Exhinc autem aperte colligitur, qualis sit ordo Naturæ transeuntis ad Anguli trisectionem à trisectione Rationis. LOGOMETRIA.

Mirabilem illam Harmoniam ulterius declarare lubet, Exemplo desumpto ab eadem Figura circum axes suos convoluta. Sit igitur *APBQ* Ellipsis, axis ejus major *AB*, minor *PQ*, centrum *C*, focus *F*. Hæc circum axem utrumvis convoluta Solidum generet, cujus particulæ constantes ex materia homogenea, vires attractivas habeant in duplicata distantiarum ratione decrescentes: & quæratur vis qua Solidum illud attrahit corpusculum quodvis, in ejus superficie:

* Sublato etenim termino secundo, tres habentur Æquationum casus. Hi vero resolvuntur ope trianguli rectanguli *ABC*, rectum habentis angulum ad *A*, in quo insuper triangulo semper data sunt duo latera.



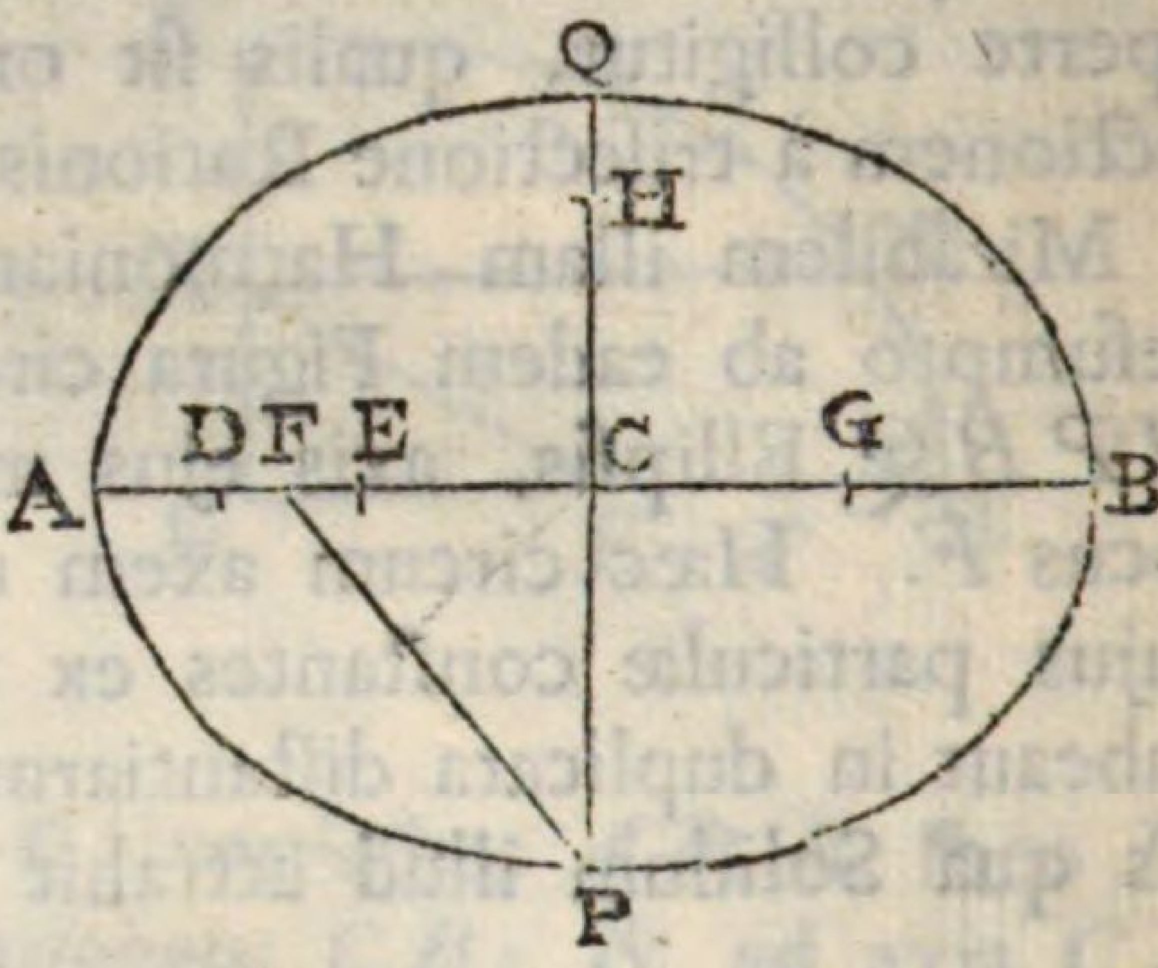
Cas. 1. Nam si sit $x^3 + 3aax = \pm 2aab$: ponantur $AB = a$, $AC = b$; & sumantur M & N binæ mediæ proportionales inter $BC + AC$ & $BC - AC$: & erit $M - N$ radix unica possibilis affirmativa, si habeatur $+ 2aab$; vel $N - M$ radix unica possibilis negativa, si habeatur $- 2aab$.

Cas. 2. Si sit $x^3 - 3aax = \pm 2aab$, existente a minore quam b: ponantur $AB = a$, $BC = b$; & sumantur M & N binæ mediæ proportionales inter $BC + AC$ & $BC - AC$: & erit $M + N$ radix unica possibilis affirmativa, si habeatur $+ 2aab$; vel $- M - N$ radix unica possibilis negativa, si habeatur $- 2aab$.

Cas. 3. Denique si sit $x^3 - 3aax = \pm 2aab$, existente a majore quam b: ponantur $AB = b$, $BC = a$; & sumatur M sinus trientis angulorum summæ $A + B$, atque N sinus trientis angulorum differentiæ $A - B$, existente radio $2BC$: & erunt $-M$, $-N$, & $M + N$ tres radices possibiles, si habeatur $+ 2aab$; vel M, N, & $- M - N$ tres radices possibiles, si habeatur $- 2aab$.

Atque ita Problemata omnia Solida solutionem facilem recipiunt, vel per Canonem Logarithmicum, vel per Canonem Trigonometricum.

PARS PRIMA. ficie locatum ad axis illius terminum. Jungantur puncta P, F, ac sumatur CD quæ sit mensura rationis inter $PF + FC$ & CP ad Modulum CA, pariterque sumatur CE quæ sit anguli CPF mensura ad Modulum CP; sitque FD excessus mensuræ CD supra CF, atque FE excessus ipsius CF supra mensuram CE: & Solidi convolutione circum axem majorem AB geniti vis in corpusculum ad A locatum, erit ad Sphæræ homogeneæ & eodem axe descriptæ vim in idem corpusculum, ut $3FD \times CPq$ ad $CF\,cub$; Solidi autem conversione circum axem minorem PQ geniti vis in corpusculum ad P locatum, erit ad Sphæræ homogeneæ & eodem axe descriptæ vim in idem corpusculum, ut $3FE \times CAq$ ad $CF\,cub$. Unde cum vis Sphæræ prioris in corpusculum ad A, sit ad vim Sphæræ posterioris in corpusculum ad P, ut CA ad CP: erit vis Solidi prioris in corpusculum ad A, ad vim Solidi posterioris in corpusculum ad P, ut $FD \times CP$ ad $FE \times CA$.

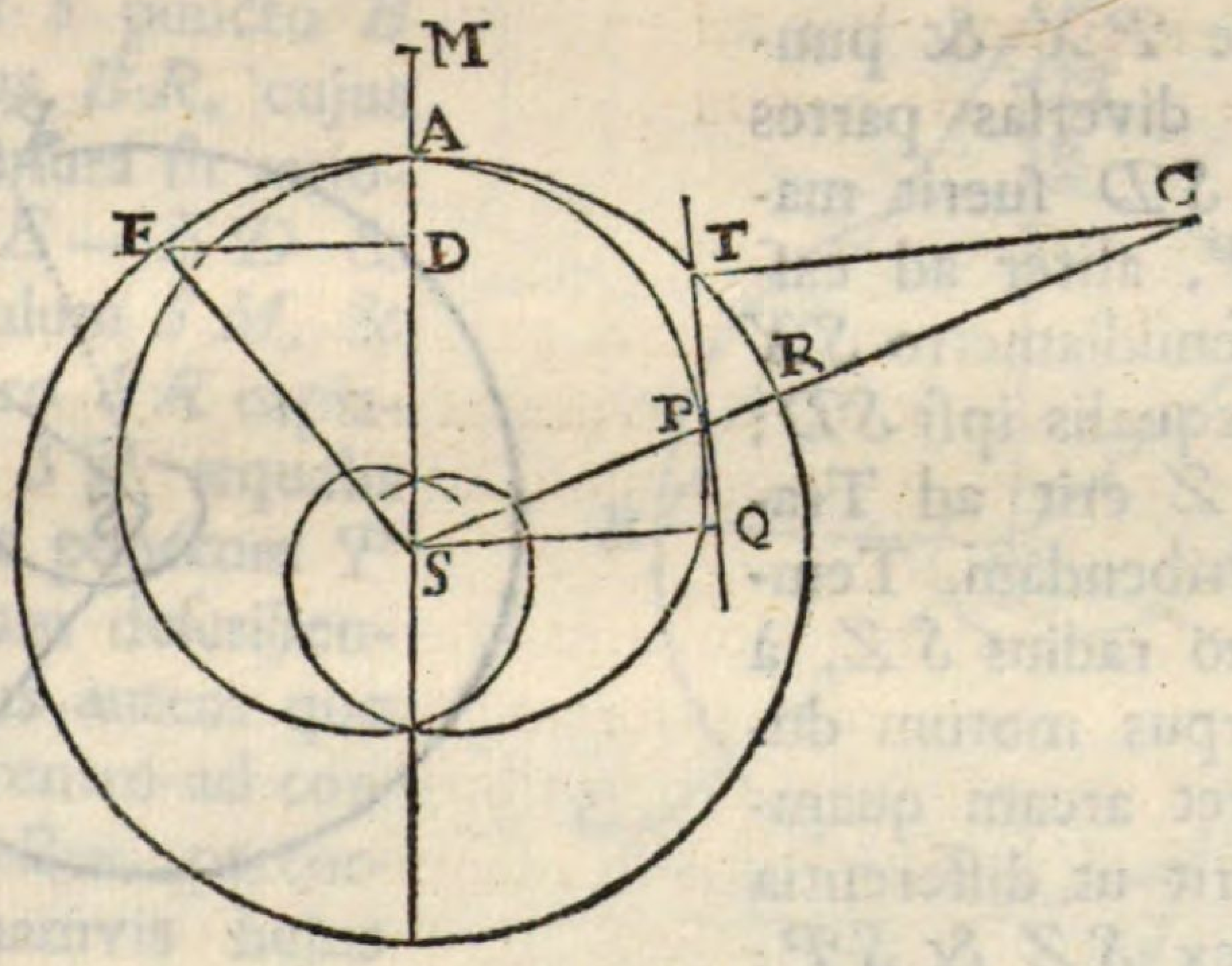


Hinc quoniam Solidum posterius medium est proportionale inter Solidum prius & Sphæram priorem: vis Solidi posterioris in corpusculum ad A, erit media proportionalis quamproxime inter vires Solidi prioris & Sphæræ prioris in idem corpusculum ad A, si modo axes Ellipseos sint prope æquales. Itaque in hoc casu, ponendo CG mediam proportionalem inter CF & $3FD$, & capiendo CH ad $3FE$ ut CA ad CF; posterioris Solidi vires ad A & P, vel ad B & Q, erunt ad invicem quamproxime ut CG ad CH. Id quod non inutile præbet compendium ad inventionem Figuræ Telluris, qualem eam subtiliter instituit celeberrimus *Newtonus*, summus ille Philosophiæ sanioris Instaurator.

Consideratio virium centripetarum aliud porro mihi suggerit Exemplum, in quo satis ampla se prodit mutationum varietas. Proponatur Trajectoriarum species enumerare, in quibus corpora moveri possunt, quæ à viribus centripetis in ratione distantiarum triplicata decrescentibus agitantur, quæque de loco dato, cum data velocitate, secundum datam rectam egrediuntur.

Cas. 1.

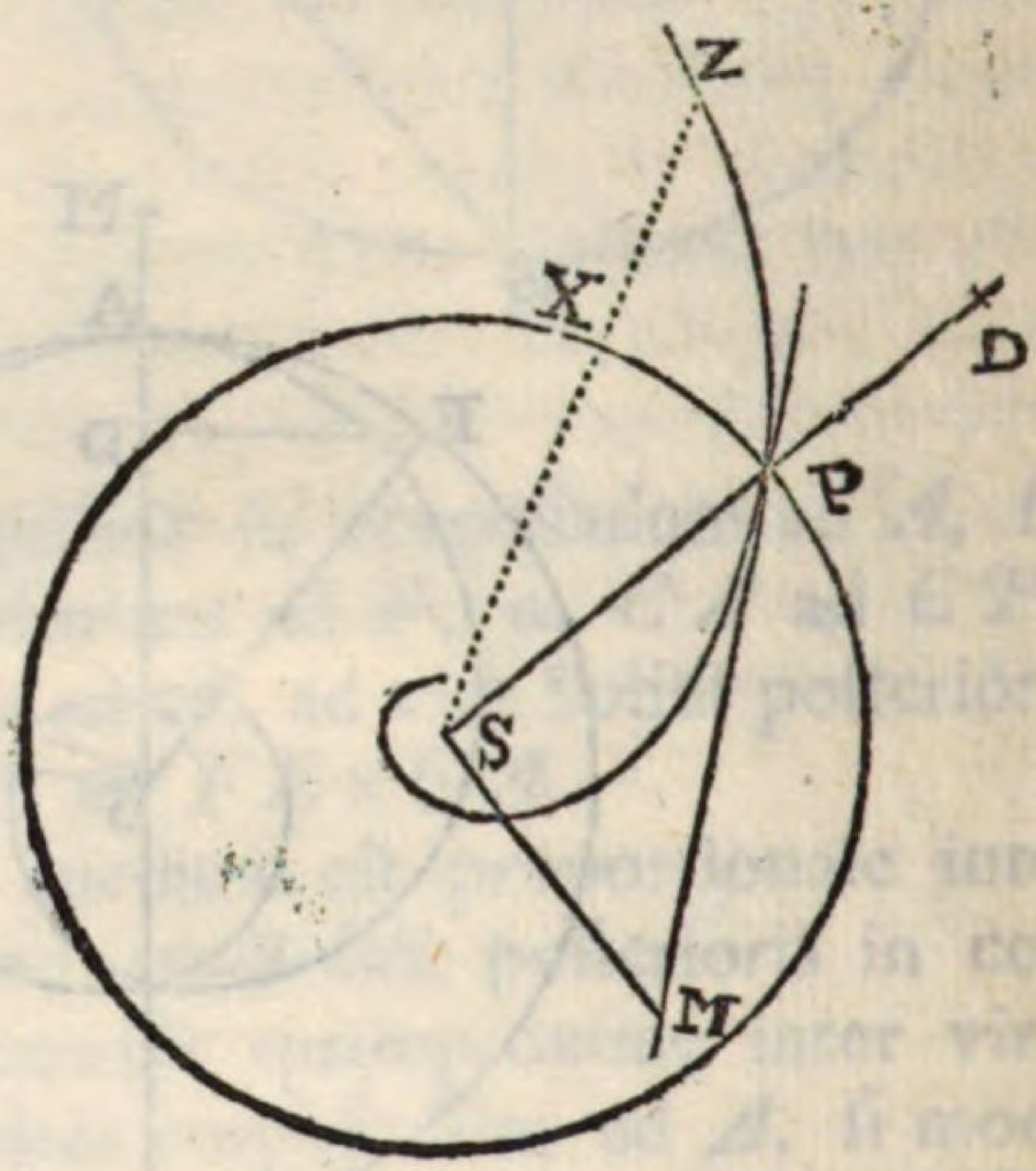
Caſ. 1. Sit S centrum virium, exeatque corpus de loco P ſecundum rectam PQ vel QP, ea cum velocitate quam acquirere poſſet ab iiſdem viribus, libere cadendo verſus centrum S de loco C, & caſu ſuo deſcribendo altitudinem CP. In datam rectam QPT demittantur perpendicula SQ, CT, centroque S & intervallo $\sqrt{SQq + QTq}$ deſcribatur circulus RTA, rectæ SPC occurrens in R: deinde ad Modulum $\sqrt{SCq - SRq}$ ſit arcus RA menſura rationis inter $SR \pm \sqrt{SRq - SPq}$ & SP, jaceant autem arcus ille RA & punctum Q ad diverſas partes rectæ SR; & punctum A erit Apſis ſumma Trajectoriæ. Exhinc vero Trajectoria dabitur, ſumendo SM æqualem ipſi $\sqrt{SCq - SRq}$, deinde in recta SA LOGOMETRIA



capiendo longitudinem quamvis SD quæ ſit minor quam SA, ad eandem erigendo perpendiculum DE ſecans circulum in E, & jungendo SE. Nam ſi ad utraſque partes puncti A ponatur arcus circularis AR, cujus longitudo ſit menſura rationis inter $SE + ED$ & SD ad Modulum SM, & in ſemidiametris SR capiantur diſtantiæ SP æquales ipſi SD: erunt puncta P ad Trajectoriam deſcribendam. Tempus autem quo radius SP, à centro ad corpus motum ductus, percurret aream quamvis SAP, erit ut recta DE: nam area percurſa æquatur ipſi DE in Modulum dimidiatum $\frac{1}{2}SM$ ductæ. Velocitas vero corporis in loco quovis P, erit ad velocitatem qua in Circulo, ad eandem diſtantiam SP, cum iiſdem viribus revolvi

revolvi posset, ut $\sqrt{SCq - SPq}$ ad SC. Ex ipsa constructione patet, hanc Spirialem primam infinitis gyris circa centrum virium contorqueri, quin & seipsam infinitis vicibus decussare, & siti erunt Nodi omnes ad Apsidis lineam AS.

Cas. 2. Recedat punctum C ad infinitam distantiam à centro S; & corporis de loco P secundum rectam PM vel MP exeuntis ea sit velocitas, quam acquirere posset cadendo libere ad eundem locum P ab infinita distantia. Ad rectam SP ducatur normalis SM, quæ secet PM in M; deinde centro S & intervallo SP describatur circulus, & in ejus circumferentia capiatur arcus PX, cujus longitudo sit mensura rationis inter distantiam quamvis SD & distantiam datam SP ad Modulum SM, jaceant autem arcus ille PX & punctum M ad diversas partes rectæ SP si SD fuerit major quam SP, aliter ad easdem, inque semidiametro SX ponatur SZ æqualis ipsi SD; & punctum Z erit ad Trajectoriam describendam. Tempus autem quo radius SZ, à centro ad corpus motum ductus, percurret aream quamvis SPZ, erit ut differentia quadratorum ex SZ & SP:

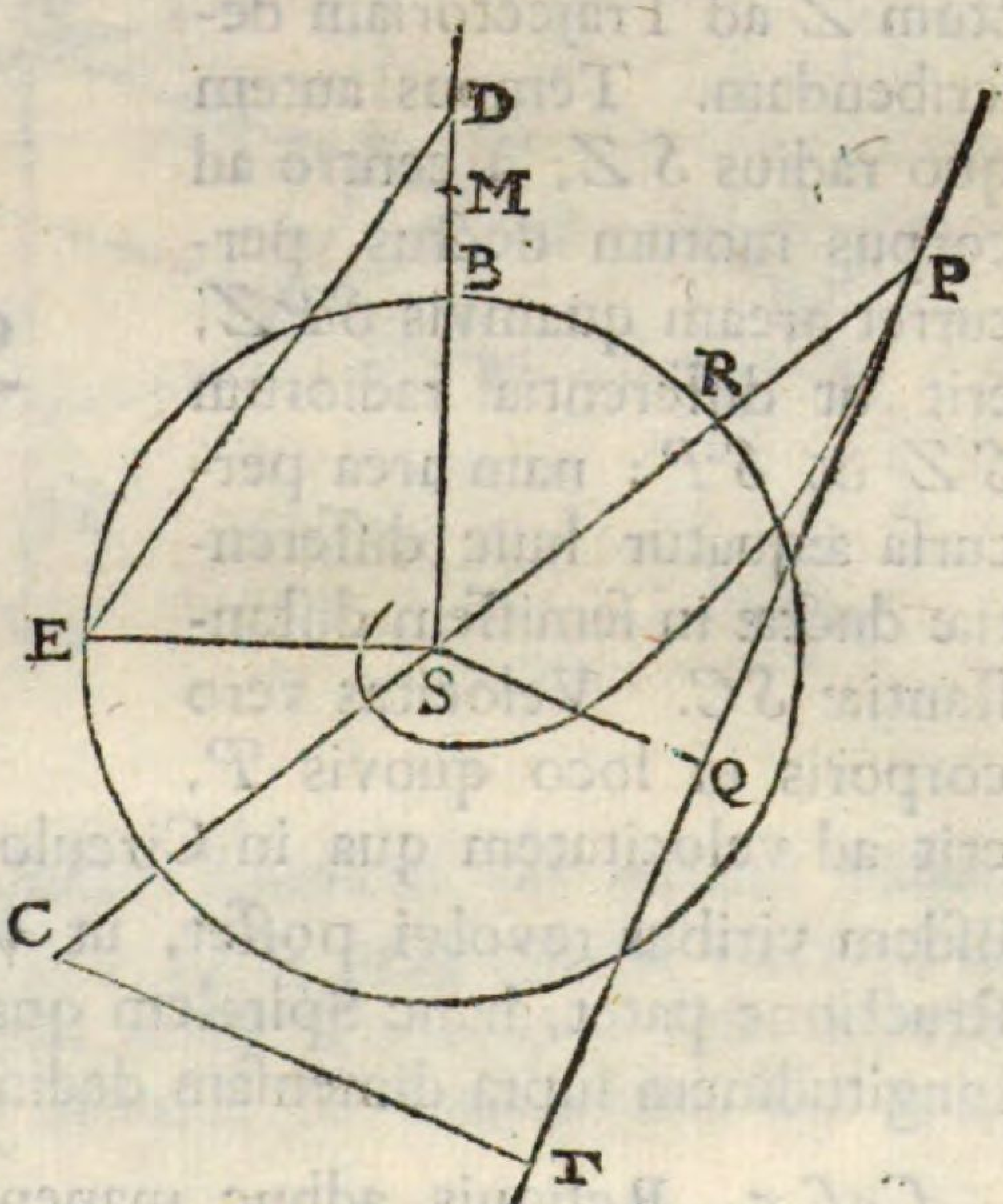


Nam area percursa, est ad dimidiatam illam differentiam, in data ratione Moduli dimidiati $\frac{1}{2}SM$ ad SP. Velocitas vero corporis in loco quovis P, æqualis erit velocitati qua in Circulo, ad eandem distantiam SP, cum iisdem viribus revolvi posset. Ex constructione patet hanc secundam Spiralem esse Æquiangulam illam Propositionis sextæ; ea vero migrabit in Circulum ubi angulus SPM fit rectus.

Cas. 3. Ut velocitas sit adhuc major, abeat jam punctum C ad distantiam plusquam infinitam à centro S, vel (quod perinde est) accedat à parte contraria eidem centro, ad finitam distantiam; & corporis de loco P secundum rectam PQ vel QP exeuntis, ea sit velocitas, quam acquirere posset ascendendo libere de loco C ad infinitam distantiam, & deinde ab infinita distantia ex altera centri

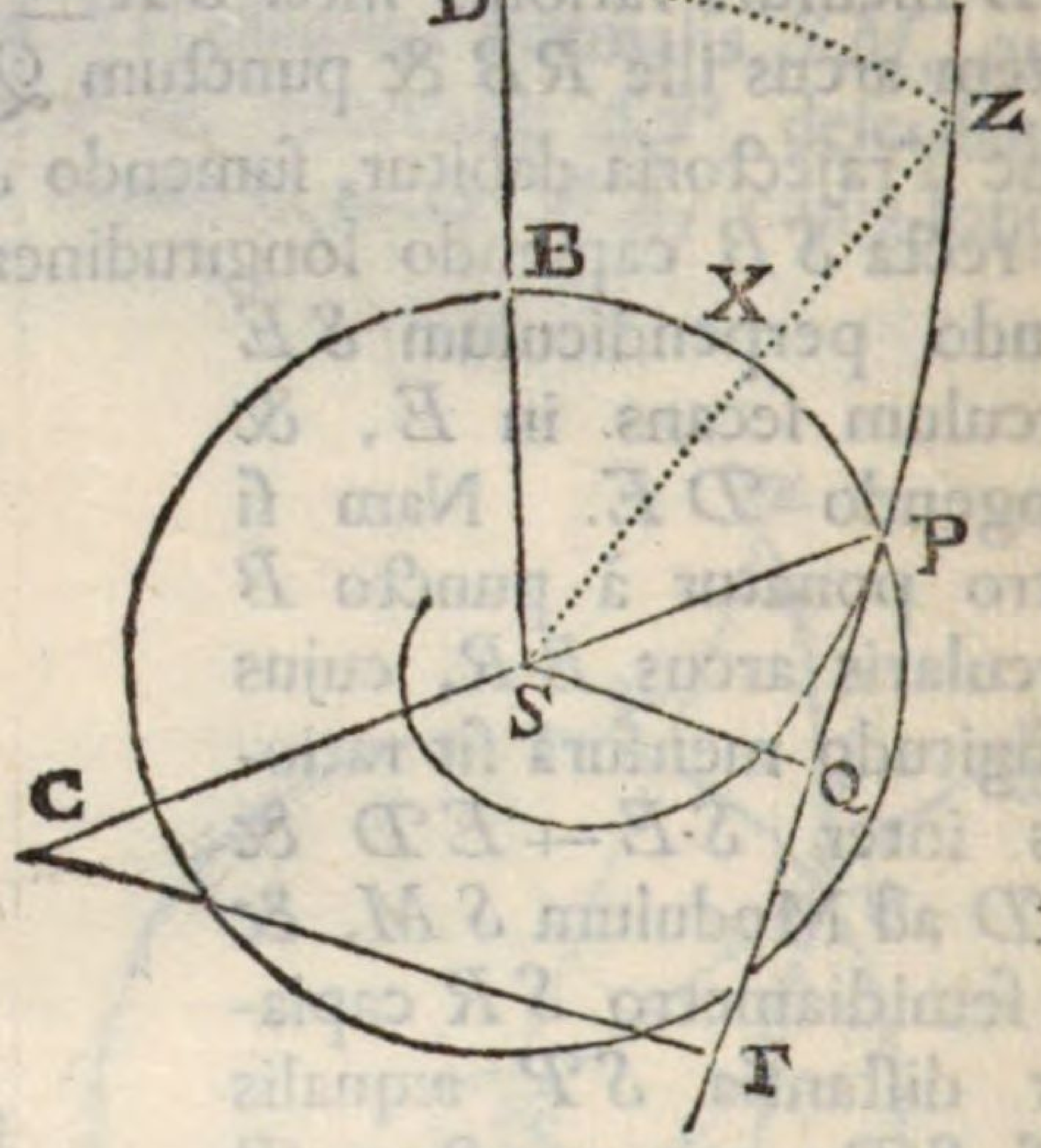
parte

parte descendendo ad locum P, viribus centripetis inter ascendendum in æquales vires centrifugas conversis. In datam rectam PQT demittantur perpendicula SQ, CT; & erit TQ vel major, vel æqualis, vel minor quam SQ. Si TQ fuerit major quam SQ; centro S & intervallo $\sqrt{TQq - SQq}$ describatur circulus RBE rectæ SP occurrens in R, deinde ad Modulum $\sqrt{SCq - SRq}$ sit arcus RB mensura rationis inter $SR \pm \sqrt{SRq + SPq}$ & SP, jaceant autem arcus ille RB & punctum Q ad partes diversas rectæ SP. Exhinc Trajectoria dabitur, sumendo SM æqualem ipsi $\sqrt{SCq - SRq}$, in recta SB capiendo longitudinem quamvis SD, ad eandem erigendo perpendiculum SE circulum secans in E, & jungendo DE. Nam si retro ponatur à puncto B circularis arcus BR, cujus longitudo mensura sit rationis inter $SE + ED$ & SD ad Modulum SM, & in semidiametro SR capiatur distantia SP æqualis ipsi SD: erit punctum P ad Trajectoriam describendam. Tempus autem quo radius SP, à centro ad corpus motum ductus, percurret aream quamvis hujus Trajectoriæ, erit ut incrementum vel decrementum rectæ DE per tempus illud factum: nam area percursa æquatur huic incremento vel decremento in Modulum dimidiatum $\frac{1}{2}SM$ ducto. Velocitas vero corporis in loco quovis P, erit ad velocitatem qua in Circulo, ad eandem distantiam SP, cum iisdem viribus revolvi posset, ut $\sqrt{SCq + SPq}$ ad SC. Ex constructione patet, hanc Spiralem tertiam infinitis gyris centrum cingere infra punctum datum P; at supra idem punctum vel non undique cinget, si arcus RB minor fuerit quam circumferentia tota $RBER$; vel toties cinget, quoties arcus ille circumferentiam excedit.



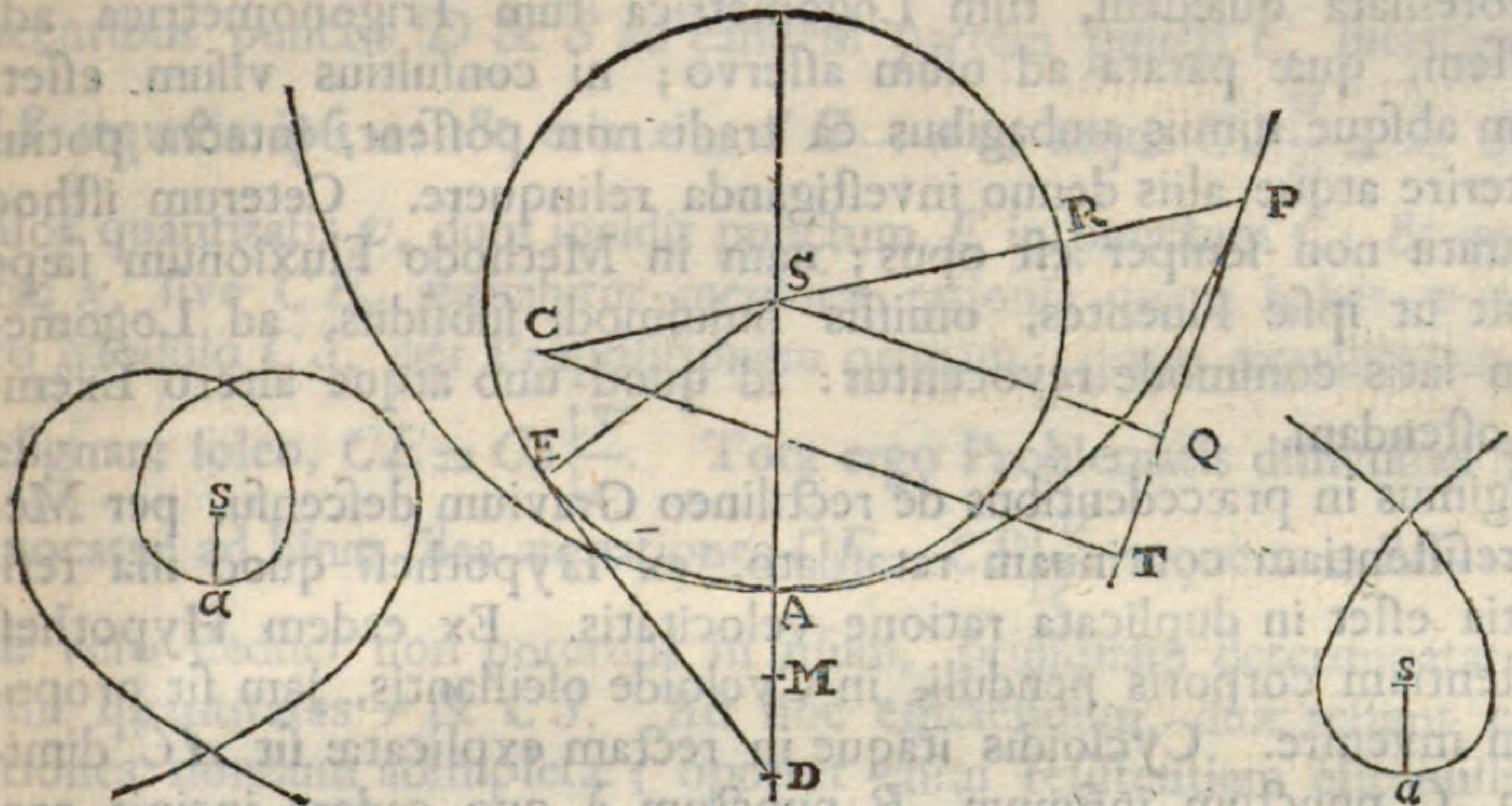
LOGOMETRIA

PARS PRIMA. *Cas.* 4. Reliquis manentibus, sint jam TQ & SQ æquales. Centro S & intervallo SP describatur circulus PXB, & sit arcus PB æqualis ipsi SC, jaceant autem arcus PB & punctum Q ad partes diversas rectæ SP. Exhinc Trajectoria dabitur, sumendo in recta SB longitudinem quamvis SD, centroque S & intervallo SD describendo circuli arcum DZ æqualem ipsi SC. Nam si ordine circulari contrario ponantur arcus PB à puncto P & arcus DZ à puncto D: erit punctum Z ad Trajectoriam describendam. Tempus autem quo radius SZ, à centro ad corpus motum ductus, percurret aream quamvis SPZ, erit ut differentia radiorum SZ & SP: nam area percursa æquatur huic differentiæ ductæ in semissem distanstantiæ SC. Velocitas vero corporis in loco quovis P, erit ad velocitatem qua in Circulo, ad eandem distantiam SP, cum iisdem viribus revolvi posset, ut $\sqrt{SCq + SPq}$ ad SC. Ex constructione patet, hanc Spiralem quartam esse Reciprocam illam, cujus longitudinem supra dimensam dedimus.



Cas. 5. Reliquis adhuc manentibus, sit jam TQ minor quam SQ. Centro S & intervallo $\sqrt{SQq - TQq}$ describatur circulus RAE rectæ SP occurrens in R; deinde sit arcus RA, ad ejusdem circuli arcum cujus secans est SP, ut $\sqrt{SCq + SRq}$ ad SR; ponatur autem arcus ille RA ad easdem partes rectæ SP cum puncto Q: & A erit Apsis ima Trajectoriæ. Exhinc vero Trajectoria dabitur, sumendo SM æqualem ipsi $\sqrt{SCq + SRq}$, in recta SA capiendo longitudinem quamvis SD quæ sit major quam SA, ducendo DE quæ circulum tangat in E, & jungendo SE. Nam si ad utrasque partes puncti A ponatur arcus circularis AR, cujus longitudo mensura sit anguli DSE ad Modulum SM, & in semidiametris SR capiantur distantiæ SP æquales ipsi SD: erunt puncta P ad

P ad Trajectoriam describendam. Tempus autem quo radius SP, à centro ad corpus motum ductus, percurret aream quamvis SAP, erit ut recta DE: nam area percursa æquatur ipsi DE in Modulum dimidiatum $\frac{1}{2} SM$ ductæ. Velocitas vero corporis in loco quovis P, erit ad velocitatem qua in Circulo, ad eandem distantiam SP, cum iisdem viribus revolvi posset, ut $\sqrt{SCq + SPq}$ ad SC. LOGOMETRIA



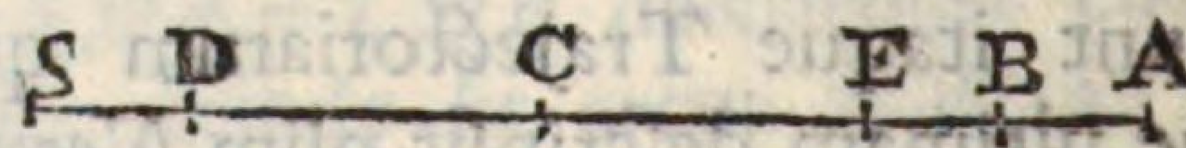
Ex constructione patet, hanc quintam Spiralem vel nullum habere Nodum, vel unicum, vel plures, pro varia proportione rectæ SM ad diametrum circuli EAR: toties enim Trajectoria sese decussabit, quoties illa recta diametrum excedit, & Nodi omnes siti erunt ad Apsidis lineam AS.

Sunt itaque Trajectoriarum quinque Species. Harum primam atque ultimam descripsit olim *Newtonus*, per Hyperbolæ & Ellipseos quadraturam.

Geometris integrum erit, ex adductis hactenus Exemplis de Methodo nostra judicare; quam quidem, si proba fuerit, ulterius excolere pergent & excolendo latius promovebunt. Patet utique campus amplissimus in quo vires suas experiri poterunt, præsertim si Logometriæ Trigonometriam insuper adjungant, quibus miram quandam affinitatem in se invicem euntibus intercedere notabam. Hisce quidem Principiis haud facile crediderim generaliora dari posse; cum

tota Matheſis vix quicquam in univerſo ſuo ambitu complectatur, præter angulorum & rationum Theoriam. Neque ſane commodiora ſperabit, qui animadvertit Effectionis facilitatem per ampliſſimas illas, omnibuſque ſuis numeris abſolutas, tum Logarithmorum tum Sinuum & Tangentium Tabulas, quas anteceſſorum noſtrorum laudatiſſimæ ſolertiæ debemus acceptas. Ut vero tanti beneficii uberior nobis exſurgat fructus, id nunc exponendum reſtat, quibus artibus ad iſtiuſmodi concluſiones rectiſſima perveniatur. In hunc finem Theoremata quædam, tum Logometrica tum Trigonometrica adjeciſſem, quæ parata ad uſum aſſervo; ni conſultius viſum eſſet, quum abſque nimiis ambagibus ea tradi non poſſent, intacta potius præterire atque aliis denuo inveſtiganda relinquere. Ceterum iſthoc apparatu non ſemper eſt opus; nam in Methodo Fluxionum ſæpe evenit ut ipſæ Fluentes, omiſſis hujuſmodi ſubſidiis, ad Logometriam ſatis commode revocentur: id quod uno atque altero Exemplo oſtendam.

Egimus in præcedentibus de rectilineo Gravium deſcenſu, per Medii reſiſtentiam continuam retardato, ex Hypotheſi quod illa reſiſtentia eſſet in duplicata ratione velocitatis. Ex eadem Hypotheſi reſiſtentiam corporis penduli, in Cycloide oſcillantis, jam ſit propoſitum invenire. Cycloidis itaque in rectam explicatæ ſit AC dimidium, C punctum infimum, B punctum à quo cadere incipit corpus pendulum, BC, CD arcus deſcenſu ejus & ſubſequente aſcenſu deſcripti. Hiſce poſitis, exquirenda eſt ratio quam habet reſiſtentia corporis in loco quovis E, ad pondus ejus relativum in Medio reſiſtente. Exponatur pondus illud per AC; & vis ab eodem oriunda, qua pendulum acceleratur ad E, exponetur per CE: quæ ſi dicatur x, & momentum ejus $-+\dot{x}$; momentum arcus jam deſcripti BE erit $-\dot{x}$. Exponatur vis reſiſtentiæ per z; & vis qua pendulum vere acceleratur, erit ut exceſſus vis prioris ſupra reſiſtentiam, hoc



eſt, ut $x-z$. Itaque cum reſiſtentia ſit ut quadratum velocitatis, reſiſtentiæ momentum $\dot{z}$ erit ut velocitas & velocitatis momentum, hoc eſt, ut $-\dot{x}$ & $x-z$, ſive ut $z\dot{x}-x\dot{x}$. Nam ſi tempus in particulas æquales dividatur, erit velocitas ut arcus deſcripti momentum $-\dot{x}$, & velocitatis momentum ut vis acceleratrix $x-z$ quæ momentum illud generat. Quoniam ergo $\dot{z}$ eſt ut $z\dot{x}-x\dot{x}$, ſi capiatur quantitas invariabilis a, quæ ſit idoneæ magnitudinis: erit $a\dot{z}=z\dot{x}-x\dot{x}$.

Ad

Ad hanc æquationem conſtruendam, aſſumatur quantitas v quæ ſit variabilis, & fingatur æquatio $z = p + qx + rv$, in qua notæ p, q, r deſignent alias novas quantitates invariabiles; & erit $\dot{z} = q\dot{x} + r\dot{v}$. Hiſce porro valoribus ipſarum z & $\dot{z}$ ſubſtitutis in æquatione prima $a\dot{z} = z\dot{x} - x\dot{x}$, habebitur $\overline{aq - p}, \dot{x} + ar\dot{v} = \overline{q - 1}, x\dot{x} + rv\dot{x}$. Ut hæc æquatio ſimplicior evadat, ponatur $q - 1 = 0$, & $aq - p = 0$; ſive $q = 1$, & $p = a$: & fiet $a\frac{\dot{v}}{v} = \dot{x}$, ac præterea $z = a + x + rv$.

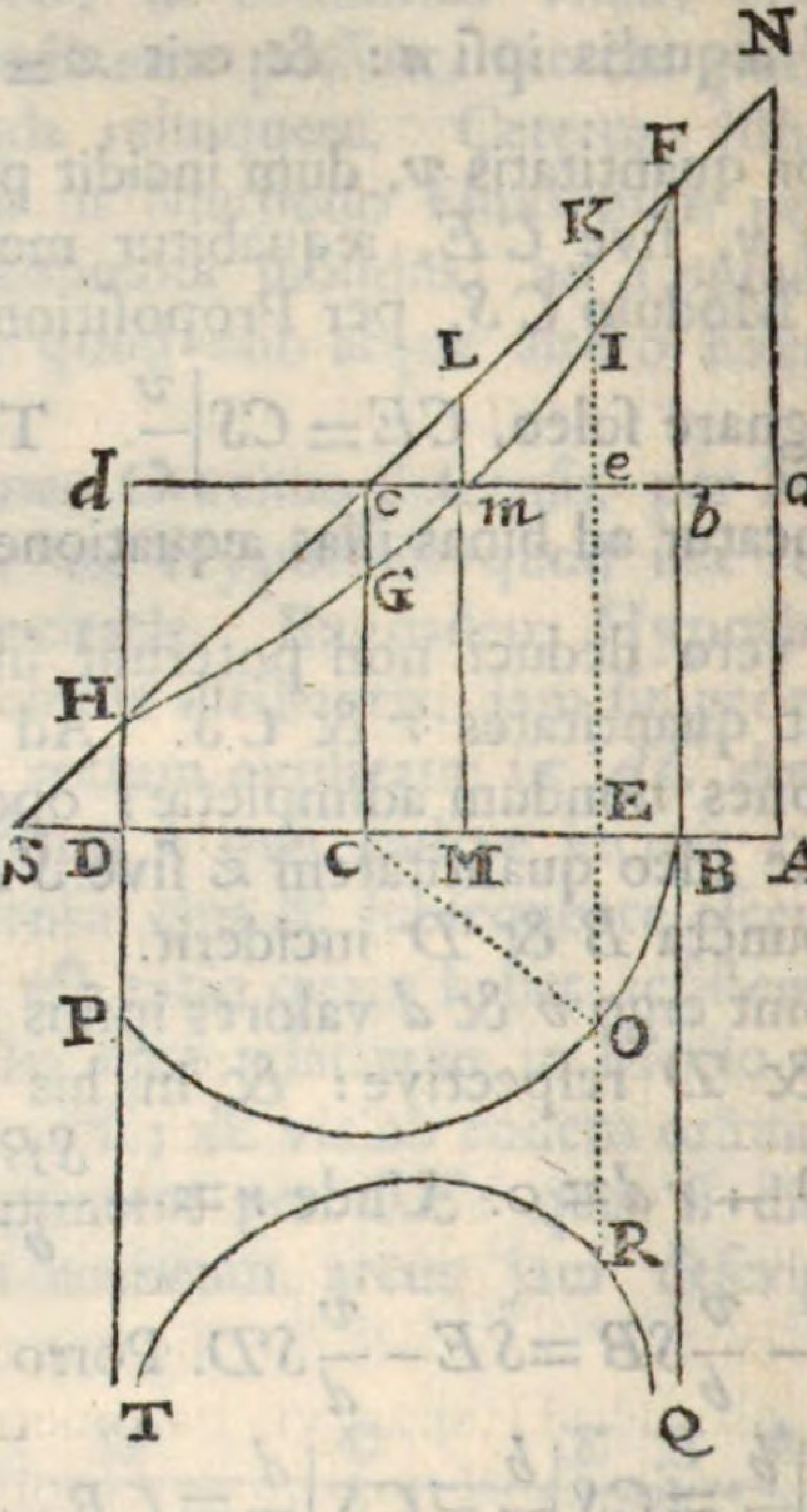
Jacentibus punctis D & S ad eandem partem puncti C, intelligatur CS æqualis ipſi a: & erit $z = SE + rv$, atque $CS\frac{\dot{v}}{v} = \dot{x}$. Sit c valor quantitatis v, dum incidit punctum E in punctum C: & quantitas x, ſive CE, æquabitur menſuræ rationis quam habet v ad c pro Modulo CS, per Propoſitionem primam: quam æqualitatem ſic deſignare ſoleo, $CE = CS\left|\frac{v}{c}\right.$. Tota ergo Problematis difficultas jam revocatur ad binas illas æquationes $CE = CS\left|\frac{v}{c}\right.$, atque $z = SE + rv$: hæ vero deduci non poterunt in uſum, priuſquam determinatæ fuerint quantitates r & CS. Ad hoc efficiendum, duæ reſtant conditiones nondum adimpletæ; oportet enim reſiſtentiam eſſe nullam, atque adeo quantitatem z ſive $SE + rv$ evaneſcere, ubi punctum E in puncta B & D inciderit.

Sint ergo b & d valores ipſius v, dum incidit punctum E in puncta B & D reſpective: & in his caſibus habebuntur $SB + rb = 0$, $SD + rd = 0$. Unde $r = -\frac{SB}{b}$, $r = -\frac{SD}{d}$, atque $z = SE + rv = SE - \frac{v}{b}SB = SE - \frac{v}{d}SD$. Porro erit $\frac{SB}{SD} = \frac{b}{d}$; atque adeo $CS\left|\frac{SB}{SD}\right. = \left(CS\left|\frac{b}{d}\right. = CS\left|\frac{b}{c}\right. - CS\left|\frac{d}{c}\right. = CB + CD = \right) BD$: unde dabitur punctum S.

Componetur itaque Problema hunc in modum. Producatur BD verſus D ad S, eo uſque, donec BD fuerit menſura rationis inter SB & SD ad Modulum CS. Deinde ad arbitrium poſita quantitate c, ita capiantur quantitates b & v, ut eodem Modulo CS, fiat CB menſura rationis quam habet b ad c, fiat quoque CE menſura rationis quam habet v ad c: & erit vis reſiſtentiæ in loco E, ad pondus relativum corporis penduli, ut $SE - \frac{v}{b}SB$, ad CA.

Hujus

Hujus Problematis ſolutio utilitatem habet in Phyſica non contemnendam: quapropter conſtructionem ejuſdem Linearem, ex eadem Analyſi deductam, ſubjungere viſum eſt. Invento uti ſupra puncto *S*; ad rectam *SA* erigantur perpendicula *DH*, *Cc*, *EK*, *BF*, *AN*, rectæ *SN* utcunque per *S* ductæ occurrentia in *H*, *c*, *K*, *F*, *N*. Per punctum *c* ducatur recta *da* parallela rectæ *DA*, quæ iiſdem perpendiculis occurrat in *d*, *c*, *e*, *b*, *a*; & ad Aſymptoton *SA* ducatur Logiſtica *HGIF*, quæ tranſeat per puncta *H* & *F*, ſecetque perpendicula *Cc*, *EK* in *G* & *I*, ac parallelam *da* in *m*: namque his poſitis, erit pondus relativum corporis penduli, ad vim illam qua pendulum acceleratur ad punctum *E* in Medio non reſiſtente, ut *aN* ad *eK*; erit autem ad vim reſiſtentiæ in loco *E*, ut *aN* ad *KI*; atque adeo ad vim qua pendulum acceleratur ad punctum *E* in Medio reſiſtente, ut *aN* ad *eI*. Porro, ſi per punctum *m* ducatur ad rectam *SMA* perpendicularis *LmM*, quæ ſecet *SN* in *L*: erit *M* locus ubi reſiſtentia fit maxima: atque adeo reſiſtentia illa maxima, erit ad pondus relativum penduli, ut *Lm* ad *Na*, hoc eſt, ut *CM* ad *CA*.

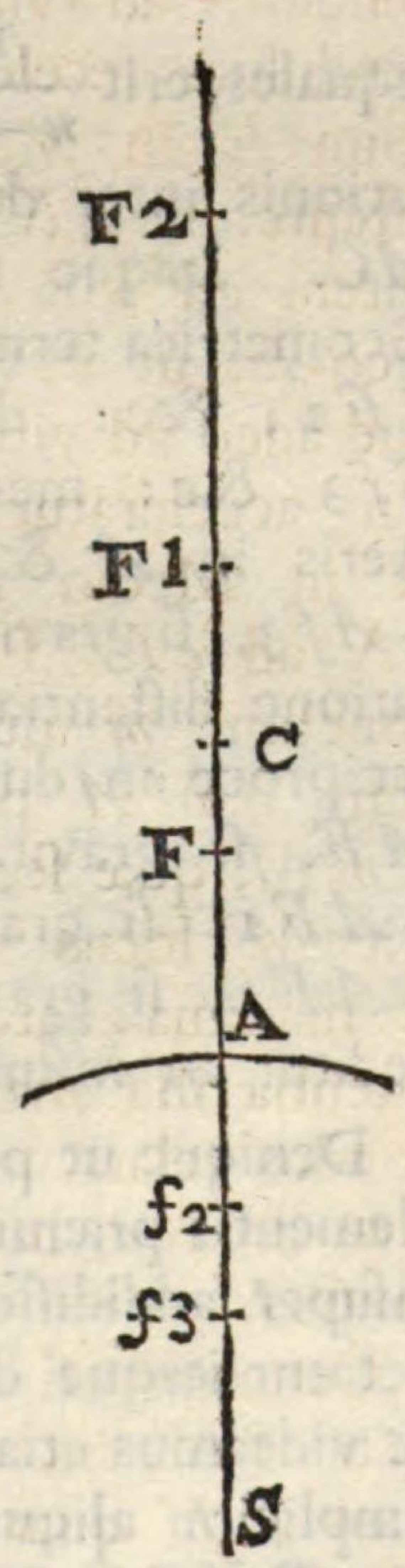


Ceterum ſi ita ducatur recta *SN*, ut abſcindat rectam *DH* quæ ſit dupla ipſius *SD*, centroque *C* & intervallo *CB* deſcribatur Circulus *BOP*, qui occurrat perpendiculo *KE* in *O*: erit penduli in Medio reſiſtente oſcillantis velocitas in loco *E*, ad velocitatem penduli ejuſdem ad eundem locum *E* delati per idem pondus relativum in Medio non reſiſtente, ut media proportionalis inter *CS* & *KI*, ad *EO*.

Adhæc

Adhæc ſi jungatur CO, & in perpendiculo KE ſumatur ER, quæ ſit ad CB ut CB ad mediam proportionalem inter $4CS$ & KI; continuoque ductu rectæ ER in baſim BE generetur area $BQRE$: erit tempus quo Cycloidis arcus BE deſcribitur in Medio reſiſtente, ad tempus quo idem arcus deſcriberetur in Medio non reſiſtente, ut area illa $BQRE$, ad Circuli ſectorem BOC. Pergo nunc ad alia.

Denſitatem Aeris invenimus ad quamvis altitudinem, ubi vis Gravitatis vel erat uniformis, vel decreſcebat in receſſu à centro telluris in duplicata ratione diſtantiæ: libet eandem exquirere denuo, ubi gravitatio vel augetur vel diminuitur in ratione datæ cujuſvis dignitatis diſtantiæ. Sit S centrum telluris, A punctum in ejus ſuperficie vel alibi utcunque ſitum, $SAF2$ recta à centro ad ſummitatem Atmoſphæræ producta: & quærenda ſit ratio denſitatis in loco A, ad denſitatem in loco quovis F, ex Hypotheſi quod vis gravitatis in F ſit ut diſtantiæ SF dignitas quæcunque SF^n, cujus index eſt n. Pro SF ſcribatur x, ac deſignent d & v denſitates Aeris ad A & F; & cum denſitas ſit ubique ut preſſura totius Aeris incumbentis, erit denſitatis momentum ut momentum preſſuræ, hoc eſt, $\dot{v}$ ut $v\dot{x}x^n$, atque adeo $\frac{\dot{v}}{v}$ ut $\dot{x}x^n$. Sit AC altitudo Atmoſphæræ, cujus uniformis denſitas eadem eſſet ac denſitas loci A; vel ſit AC ad altitudinem Hydrargyri barometrici in loco A, ut denſitas Hydrargyri ad denſitatem Aeris in eodem loco A: & ſi punctum F accedere intelligatur ad punctum A, erit altitudo Hydrargyri barometrici in loco A, ad altitudinem Hydrargyri barometrici in loco F, ut AC ad FC. Aeris ergo in loco A denſitas d, eſt ad Aeris in loco F denſitatem v, ut AC ad FC: unde conſequitur ut ſit $d-v$ ſive $\dot{v}$, ad d ſive v, ut AF ſive $\dot{x}$, ad AC.

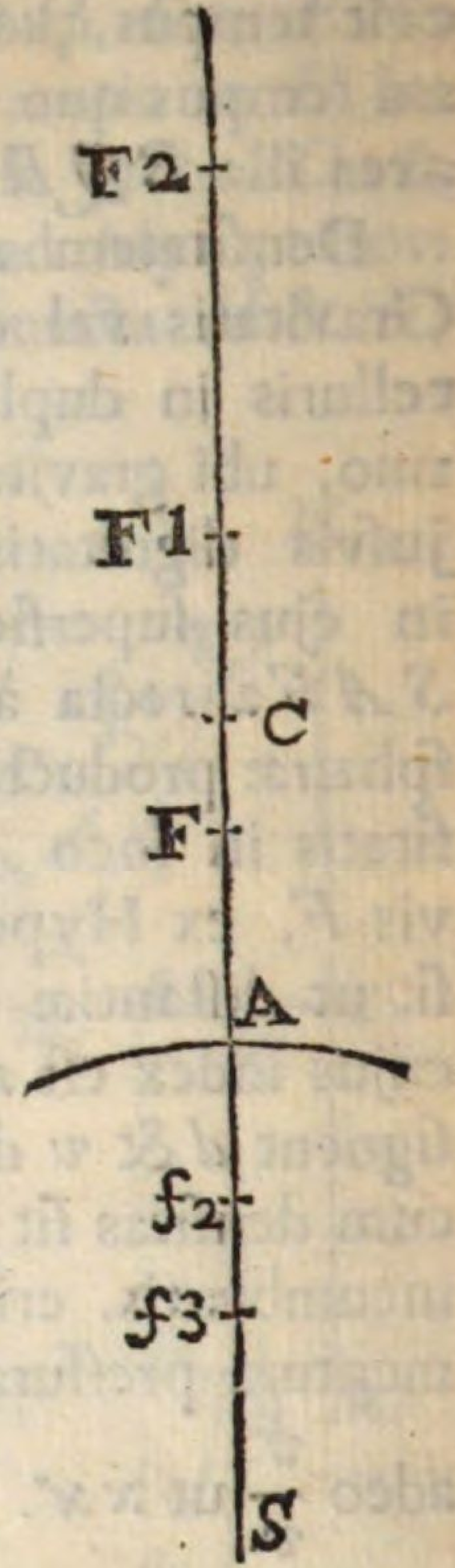


Erit itaque, in hoc caſu, $AC\frac{\dot{v}}{v} = \dot{x} = \frac{\dot{x}x^n}{SA^n}$. Quoniam ergo, ubicunque ſumeretur punctum F, erat $\frac{\dot{v}}{v}$ ut $\dot{x}x^n$: erit porro $AC\frac{\dot{v}}{v} = \frac{\dot{x}x^n}{SA^n}$, ubicunque ſumatur punctum F.

Jam

PARS PRIMA.

Jam ſi gravitatio ſit reciproce ut diſtantia à centro, ſive ut $\frac{1}{x}$ vel x^{-1}; erit $n = -1$, atque inde $AC\frac{\dot{v}}{v} = SA\frac{\dot{x}}{x}$; unde ſi Fluentes ſtatuantur æquales, menſura rationis inter denſitates d & v ad Modulum AC, æquabitur menſuræ rationis inter diſtantias SF & SA ad Modulum SA.

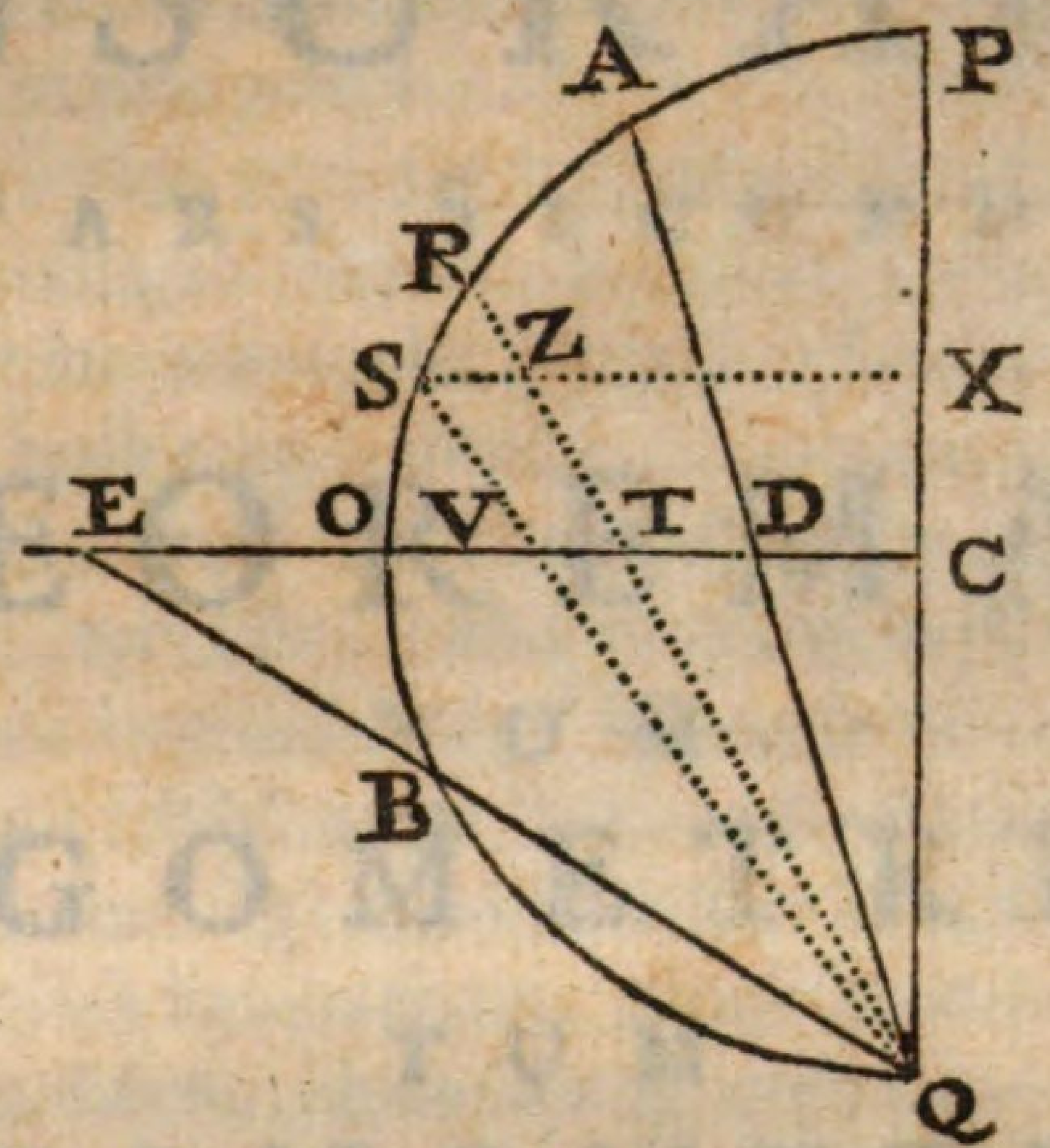


Si gravitationis ſit alia quævis Lex: quoniam eſt $AC\frac{\dot{v}}{v} = \frac{\dot{x}x^n}{SA^n}$; ſi Fluentes ſtatuantur æquales, erit $\frac{1}{n+1}$ in $\frac{SF^{n+1}}{SA^n} - SA$ menſura rationis inter denſitates d & v ad Modulum AC. Itaque ſi ſumantur in progreſſione Geometrica termini creſcentes SA, SF, $SF1$, $SF2$, &c: decreſcentes SF, SA, $Sf2$, $Sf3$, &c: menſura rationis inter denſitates Aeris in A & F ad Modulum AC, erit $\frac{1}{2}Af3$, ſi gravitatio ſit reciproce in triplicata ratione diſtantiæ; erit $Af2$, ſi gravitatio ſit reciproce in duplicata ratione diſtantiæ; erit AF, ſi gravitatio uniformis ſtatuatur; erit $\frac{1}{2}AF1$, ſi gravitatio ſit ut diſtantia; erit $\frac{1}{3}AF2$, ſi gravitatio ſit in duplicata ratione diſtantiæ. Et ſic proceditur in infinitum.

Denique ut plenius conſtet, Syntheticas etiam demonſtrationes ex elementis præmiſſis levi negotio concinnari poſſe; ſufficiet unicum inſuper addidiſſe Exemplum, tædet utique plura jam proferre. Repetatur itaque diviſio illa Nautica Meridianæ quam ſupra attigimus, & videamus etiam abſque ope Curvæ cujuſpiam Logometricæ, annon ſimplicior aliquanto ſit futura demonſtratio ad modum ſequentem. Sit $PXCQ$ Telluris axis, CO ſemidiameter Æquatoris, $PAOBQ$ Meridianus; & invenienda ſit in planiſphærio Nautico magnitudo cujuſvis arcus AB. Ad arcus illius terminos A & B ducantur ab alterutro Polorum P vel Q rectæ QA, QB, ſemidiametro CO occurrentes in D & E: Dico magnitudinem Nauticam arcus AB æqualem eſſe menſuræ rationis inter EC & DC ad Modulum OC.

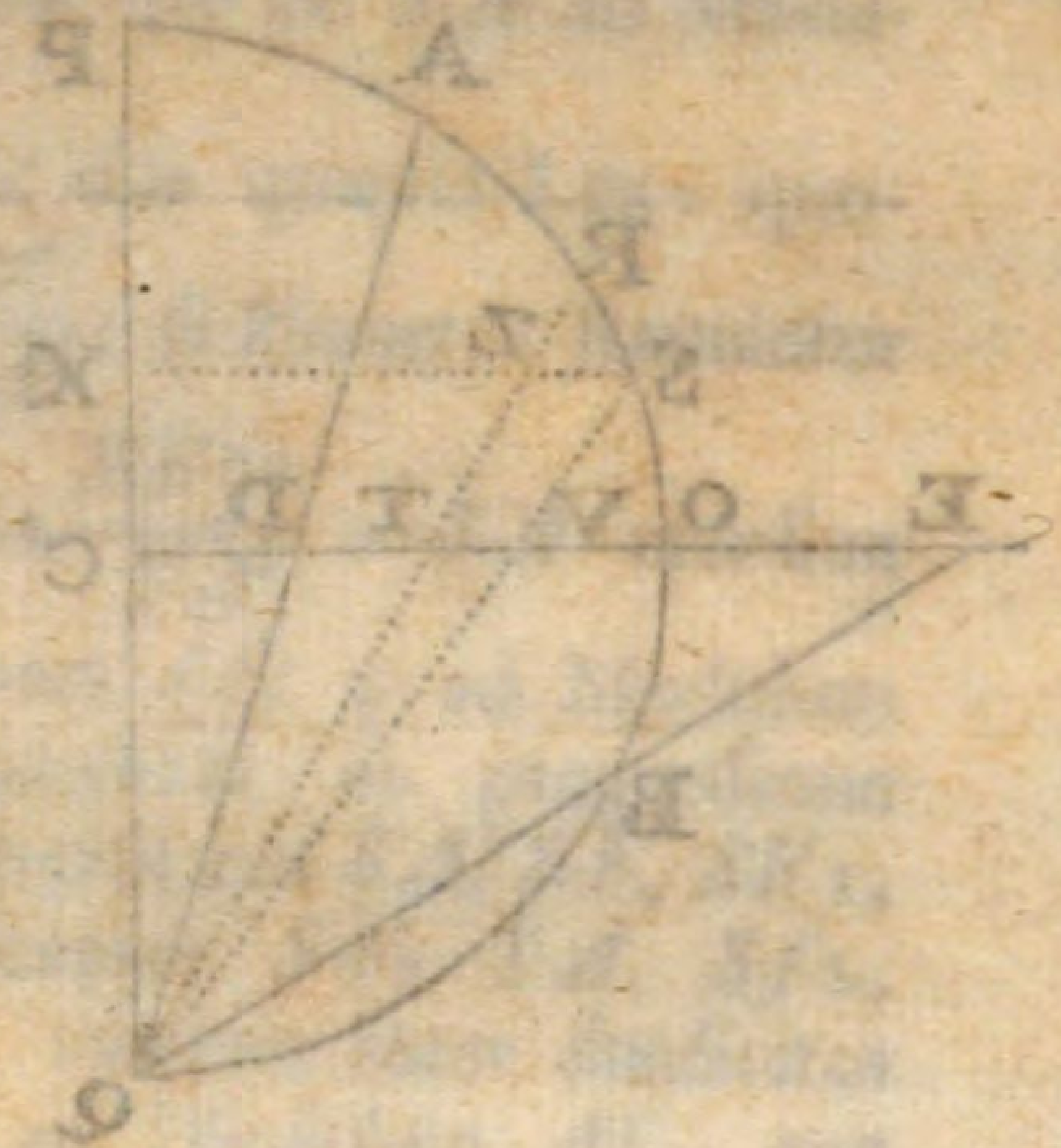
Nam

Nam divisus intelligatur arcus AB in particulas quam minimas RS, & jungantur QR, QS quæ secent CO in T & V; & demisso in axem perpendiculo SX quod rectæ QR occurrat in Z, erit lineola SZ æqualis particulæ RS. Itaque magnitudo Nautica nascentis arcus RS, erit ad Sphæræ semidiametrum OC, ut arcus ille RS sive PARS PRIMA.



lineola SZ ad SX, hoc est, ut VT ad VC. Unde (per Corol. 2. Prop. 1.) magnitudo illa Nautica æquatur mensuræ rationis inter VC & TC ad Modulum OC: & similes utrobique summas colligendo, magnitudo Nautica totius arcus AB æquabitur mensuræ totius rationis inter EC & DC ad eundem Modulum OC.

Nam divisus intelligatur arcus *AB* in particulas quam minimas *RS*, & jungantur *QR*, *QS* quæ secent *CO* in *T* & *V*; & demisso in axem perpendiculo *SX* quod rectæ *QR* occurrat in *Z*, erit lineola *SZ* æqualis particulæ *RS*. Itaque magnitudo Nautica nascentis arcus *RS*, erit ad Sphæræ semidiametrum *OC*, ut arcus ille *RS* sive



lineola *SZ* ad *SX*, hoc est, ut *VT* ad *VC*. Unde (per Corol. 2. Prop. 1.) magnitudo illa Nautica æquatur mensuræ rationis inter *VC* & *TC* ad Modulum *OC*: & similes utrobique summas colligendo, magnitudo Nautica totius arcus *AB* æquabitur mensuræ totius rationis inter *EC* & *DC* ad eundem Modulum *OC*.

ROGERI COTES
HARMONIA MENSURARUM.

PARS SECUNDA.

THEOREMATA
TUM
LOGOMETRICA
TUM
TRIGONOMETRICA,
QUÆ DATARUM
FLUXIONUM FLUENTES
EXHIBENT PER
MENSURAS.

ADJICIUNTUR
THEOREMATA
DE
CONTINUATIONE
FORMARUM ET FLUENTIUM
AD INFINITUM.

AUCTORIS PRÆFATIO.

Ecce tibi, Lector benevole, Theoremata illa tum Logometrica tum Trigonometrica, quæ aliquot abhinc annis, dum ſcilicet elementa traderem Logometriæ, ad uſum parata dixi me aſſervare. Hinc autem intelliges qua arte Conſtructiones illæ per Menſuras rationum atque angulorum inventæ fuerint, & quanta porro facilitate conſimiles alias innumeras invenire licebit. Quin & mirabilis illa Menſurarum utriuſque generis Harmonia clarius inde perſpicienda dabitur, quæ ſane tanta eſt, ut unica ſolum utrique communis adhibenda ſit notatio, ſive rationis ſive anguli quantitas deſignanda propoſito inſerviat. Hujus vero notationis ea eſt indoles ut manifeſto & facillimo indicio ſtatim prodat Menſuræ genus quodnam ſit. Ipſa vero Menſura qualiſcunque fuerit eundem ſemper Modulum habet ſibimet ipſi homogeneum.

De Modulo menſuræ rationis, antehac, ni fallor, prolixe ſatis egimus: reſtant pauca quædam prælibanda de Modulo menſuræ angularis priuſquam ad ipſa Theoremata perveniamus. Anguli dimenſio maxime uſitata fit per arcum circuli, cujus centrum eſt ipſum angulare punctum, & cujus magnitudo ea eſt quæ concluſa inter anguli crura continetur. Hæc vero Menſura necdum ſatis eſt definita, niſi inſuper conſtiterit qualis vel quantus fuerit Circulus cujus arcus adhibetur: etenim quo major eſt iſte circulus eo major erit arcus concluſus, & quo minor circulus eo minor arcus; unde facile apparet ex arcus magnitudine ſola neutiquam definiri poſſe magnitudinem anguli. Itaque præter magnitudinem arcus oportet etiam ex aliquo Modulo circuli magnitudinem determinare. Id autem pluribus modis fieri poteſt, aſſumendo nempe pro Modulo illo vel ipſam circumferentiam integram, vel partem ejus aliquam datam, vel adſcriptam quamvis rectam inſigniorem, quæ, aucta vel diminuta circumferentia in eadem ratione ſimul augetur vel diminuitur. Hujuſmodi rectæ ſunt ipſa circuli diameter, vel radius, vel etiam latus Polygoni cujuſvis regularis inſcripti vel circumſcripti. Vides quanta quærentibus ſtatim fit oblata Modulorum multitudo, ut jam pene nos obruat eligendi copia. Verum bono animo eſto, dabitur enim exitus non infelix nec difficilis, modo naturam ipſam ſequamur ducem. Utique hæc optima Magiſtra indicio non obſcuro ſignificat eligendum eſſe Radium. At

quo

quo indicio, interrogabis; eo nimirum, quod cum rationis menſura mutetur in arcum circuli, menſuram ſcilicet anguli, eodem tempore ſemper mutatur illius Modulus in ejuſdem circuli radium. Igitur in ſequentibus quotieſcunque ſumitur arcus circuli pro anguli menſura, toties ſemper ejuſdem circuli radius ſumendus eſt pro Modulo.

Verum ut Rationes ita quoque Anguli recipiunt menſuras diverſorum generum: quis enim negabit angulum æque commode metiri circularis areæ ſegmentum interclusum, atque peripheriæ circularis interclusum arcum? quamvis illa menſura ſit plana ſuperficies, hæc autem ſit linea latitudinis expers. Quid porro dubitemus quantitates adhibere cujuſcunque generis, quarum magnitudines magnitudinibus angulorum ſint analogæ; quod unicum ſcilicet requiritur ut fiant menſuræ. In unoquoque vero Syſtemate diverſus habebitur Modulus: erit autem iſte Modulus ad menſuram cujuſvis anguli dati, ut Radius circuli ad ejuſdem arcum qui angulum eundem datum metiretur. Atque hæc de Modulo conſtituendo dixiſſe ſufficiat.

Pergamus nunc ad Theoremata, quæ Fluxionum datarum Fluentes exhibent per Menſuras. In his omnibus Fluxionibus unica eſt variabilis & indeterminata quantitas z, ejuſque Fluxio $\dot{z}$, reliquæ notæ deſignant quantitates conſtantes ac prorſus immutabiles: quantitas η eſt index poteſtatis ipſius z ubi ſcribitur vel legitur poſt z. Quantitates R, S, T (quarum valor ſemper apponitur) deſignant vel rationem vel angulum per cujus menſuram exhibenda eſt Fluxionis quantitas Fluens. Nempe ſi R ſit radix quadratica quantitatis affirmativæ, deſignabunt rationem, quæ ratio ſemper erit ea quam habet $R+T$ ad S: ſin R ſit radix quadratica quantitatis negativæ, deſignabunt angulum, qui angulus ſemper erit is cujus tangens & ſecans ſunt ad Radium ut T & S ad R, ubi quantitas illa negativa mutato ſigno converſa fuerit in poſitivam. Ceterum ut ordo ſervetur Fluxiones ad formas quaſdam revocantur. Theorematum quoque Continuatio ad infinitum docetur poſt ipſa Theoremata.

FORMA

THEOREMATA

θ	FORMA I. $\dfrac{dz z^{\theta\eta-1}}{e+fz^\eta}$.
&c.	
5.	$\dfrac{dz^{4\eta}}{4\eta f} - \dfrac{dez^{3\eta}}{3\eta ff} + \dfrac{deez^{2\eta}}{2\eta f^3} - \dfrac{de^3z^\eta}{\eta f^4} + \dfrac{de^4}{\eta f^5}\,\Big\|\,\dfrac{e+fz^\eta}{e}$.
4.	$\dfrac{dz^{3\eta}}{3\eta f} - \dfrac{dez^{2\eta}}{2\eta ff} + \dfrac{deez^\eta}{\eta f^3} - \dfrac{de^3}{\eta f^4}\,\Big\|\,\dfrac{e+fz^\eta}{e}$.
3.	$\dfrac{dz^{2\eta}}{2\eta f} - \dfrac{dez^\eta}{\eta ff} + \dfrac{dee}{\eta f^3}\,\Big\|\,\dfrac{e+fz^\eta}{e}$.
2.	$\dfrac{dz^\eta}{\eta f} - \dfrac{de}{\eta ff}\,\Big\|\,\dfrac{e+fz^\eta}{e}$.
1.	$\dfrac{d}{\eta f}\,\Big\|\,\dfrac{e+fz^\eta}{e}$.
0.	$\dfrac{-d}{\eta e}\,\Big\|\,\dfrac{e+fz^\eta}{fz^\eta}$.
−1.	$\dfrac{-d}{\eta ez^\eta} + \dfrac{df}{\eta ee}\,\Big\|\,\dfrac{e+fz^\eta}{fz^\eta}$.
−2.	$\dfrac{-d}{2\eta ez^{2\eta}} + \dfrac{df}{\eta eez^\eta} - \dfrac{dff}{\eta e^3}\,\Big\|\,\dfrac{e+fz^\eta}{fz^\eta}$.
−3.	$\dfrac{-d}{3\eta ez^{3\eta}} + \dfrac{df}{2\eta eez^{2\eta}} - \dfrac{dff}{\eta e^3z^\eta} + \dfrac{df^3}{\eta e^4}\,\Big\|\,\dfrac{e+fz^\eta}{fz^\eta}$.
−4.	$\dfrac{-d}{4\eta ez^{4\eta}} + \dfrac{df}{3\eta eez^{3\eta}} - \dfrac{dff}{2\eta e^3z^{2\eta}} + \dfrac{df^3}{\eta e^4z^\eta} - \dfrac{df^4}{\eta e^5}\,\Big\|\,\dfrac{e+fz^\eta}{fz^\eta}$.
&c.	

θ	FORMA II. $\dfrac{d\dot{z}z^{\theta\eta+\frac{1}{2}\eta-1}}{e+fz^{\eta}}$
&c.	
3.	$\frac{2dz^{\frac{5}{2}\eta}}{5\eta f}-\frac{2dez^{\frac{3}{2}\eta}}{3\eta ff}+\frac{2deez^{\frac{1}{2}\eta}}{\eta f^{3}}-\frac{2ee}{\eta f^{3}}dR\,\Big\vert\,\frac{R+T}{S}.$
2.	$\frac{2dz^{\frac{3}{2}\eta}}{3\eta f}-\frac{2dez^{\frac{1}{2}\eta}}{\eta ff}+\frac{2e}{\eta ff}dR\,\Big\vert\,\frac{R+T}{S}.$
1.	$\frac{2dz^{\frac{1}{2}\eta}}{\eta f}-\frac{2}{\eta f}dR\,\Big\vert\,\frac{R+T}{S}.$
0.	$\frac{2}{\eta e}dR\,\Big\vert\,\frac{R+T}{S}.$
−1.	$\frac{-2d}{\eta ez^{\frac{1}{2}\eta}}+\frac{2}{\eta e}d\mathrm{r}\,\Big\vert\,\frac{\mathrm{r}+\mathrm{t}}{\mathrm{s}}.$
−2.	$\frac{-2d}{3\eta ez^{\frac{3}{2}\eta}}+\frac{2df}{\eta eez^{\frac{1}{2}\eta}}-\frac{2f}{\eta ee}d\mathrm{r}\,\Big\vert\,\frac{\mathrm{r}+\mathrm{t}}{\mathrm{s}}.$
−3.	$\frac{-2d}{5\eta ez^{\frac{5}{2}\eta}}+\frac{2df}{3\eta eez^{\frac{3}{2}\eta}}-\,-\frac{2dff}{\eta e^{3}z^{\frac{1}{2}\eta}}+\frac{2ff}{\eta e^{3}}d\mathrm{r}\,\Big\vert\,\frac{\mathrm{r}+\mathrm{t}}{\mathrm{s}}.$
&c.	

PARS SECUNDA.

$$R=\sqrt{\frac{-e}{f}},\quad T=z^{\frac{1}{2}\eta},\quad S=\sqrt{\frac{e+fz^{\eta}}{f}}.$$

$$\mathrm{r}=\sqrt{\frac{-f}{e}},\quad \mathrm{t}=\frac{1}{z^{\frac{1}{2}\eta}},\quad \mathrm{s}=\sqrt{\frac{e+fz^{\eta}}{ez^{\eta}}}.$$

THEOREMATA

θ	FORMA III. $dz z^{\theta\eta-1}\sqrt{e+fz^{\eta}}$
&c.	
4.	$\frac{-96e^4+48e^3fz^{\eta}-36eeffz^{2\eta}+30ef^3z^{3\eta}+210f^4z^{4\eta}}{945\eta f^4}dP.$
3.	$\frac{16e^3-8eefz^{\eta}+6effz^{2\eta}+30f^3z^{3\eta}}{105\eta f^3}dP.$
2.	$\frac{-4ee+2efz^{\eta}+6ffz^{2\eta}}{15\eta ff}dP.$
1.	$\frac{2e+2fz^{\eta}}{3\eta f}dP.$
0.	$\frac{2}{\eta}dP-\frac{2}{\eta}dR \,\Big\vert\, \frac{R+T}{S}.$
−1.	$\frac{-1}{\eta z^{\eta}}dP-\frac{f}{\eta e}dR \,\Big\vert\, \frac{R+T}{S}.$
−2.	$\frac{-2e-fz^{\eta}}{4\eta ez^{2\eta}}dP+\frac{ff}{4\eta ee}dR \,\Big\vert\, \frac{R+T}{S}.$
−3.	$\frac{-8ee-2efz^{\eta}+3ffz^{2\eta}}{24\eta eez^{3\eta}}dP-\frac{f^3}{8\eta e^3}dR \,\Big\vert\, \frac{R+T}{S}.$
&c.	

$P=\sqrt{e+fz^{\eta}}.\quad R=\sqrt{e},\quad T=\sqrt{e+fz^{\eta}},\quad S=\sqrt{fz^{\eta}}.$

θ	FORMA IV. $d\dot{z}z^{\theta\eta+\frac{1}{2}\eta-1}\sqrt{e+fz^{\eta}}$
&c.	
3.	$\frac{15e^{3}z_{\eta}-10eefz^{2}_{\eta}+8effz^{3\eta}+48f^{3}z^{4\eta}}{192\eta f^{3}}d\mathrm{P}-\frac{5e^{4}}{64\eta f^{4}}d\mathrm{R}\,\Big\|\,\frac{\mathrm{R+T}}{\mathrm{S}}$
2.	$\frac{-3eez^{\eta}+2efz^{2\eta}+8ffz^{3\eta}}{24\eta ff}d\mathrm{P}+\frac{e^{3}}{8\eta f^{3}}d\mathrm{R}\,\Big\|\,\frac{\mathrm{R+T}}{\mathrm{S}}.$
1.	$\frac{ez^{\eta}+2fz^{2\eta}}{4\eta f}d\mathrm{P}-\frac{ee}{4\eta ff}d\mathrm{R}\,\Big\|\,\frac{\mathrm{R+T}}{\mathrm{S}}.$
0.	$\frac{z^{\eta}}{\eta}d\mathrm{P}+\frac{e}{\eta f}d\mathrm{R}\,\Big\|\,\frac{\mathrm{R+T}}{\mathrm{S}}.$
−1.	$\frac{-2}{\eta}d\mathrm{P}+\frac{2}{\eta}d\mathrm{R}\,\Big\|\,\frac{\mathrm{R+T}}{\mathrm{S}}.$
−2.	$\frac{-2e-2fz^{\eta}}{3\eta ez^{\eta}}d\mathrm{P}.$
−3.	$\frac{-6ee-2efz^{\eta}+4ffz^{2\eta}}{15\eta eez^{2\eta}}d\mathrm{P}.$
−4.	$\frac{-30e^{3}-6eefz^{\eta}+8effz^{2\eta}-16f^{3}z^{3\eta}}{105\eta e^{3}z^{3\eta}}d\mathrm{P}.$
&c.	

PARS SECUNDA

$$\mathrm{P}=\sqrt{\frac{e+fz^{\eta}}{z^{\eta}}}.\quad \mathrm{R}=\sqrt{f},\quad \mathrm{T}=\sqrt{\frac{e+fz^{\eta}}{z^{\eta}}},\quad \mathrm{S}=\sqrt{\frac{e}{z^{\eta}}}.$$

THEOREMATA

θ	FORMA V. $\dfrac{d\dot{z}z^{\theta\eta-1}}{\sqrt{e+fz^{\eta}}}$.
&c.	
4.	$\dfrac{-96e^{3}+48eefz^{\eta}-36effz^{2\eta}+30f^{3}z^{3\eta}}{105\eta f^{4}}dP.$
3.	$\dfrac{16ee-8efz^{\eta}+6ffz^{2\eta}}{15\eta f^{3}}dP.$
2.	$\dfrac{-4e+2fz^{\eta}}{3\eta ff}dP.$
1.	$\dfrac{2}{\eta f}dP.$
0.	$\dfrac{-2}{\eta e}dR \,\Big\vert\, \dfrac{R+T}{S}.$
-1.	$\dfrac{-1}{\eta e z^{\eta}}dP+\dfrac{f}{\eta ee}dR \,\Big\vert\, \dfrac{R+T}{S}.$
-2.	$\dfrac{-2e+3fz^{\eta}}{4\eta eez^{2\eta}}dP-\dfrac{3ff}{4\eta e}dR \,\Big\vert\, \dfrac{R+T}{S}.$
-3.	$\dfrac{-8ee+10efz^{\eta}-15ffz^{2\eta}}{24\eta e^{3}z^{3\eta}}dP+\dfrac{5f^{3}}{8\eta e^{4}}dR \,\Big\vert\, \dfrac{R+T}{S}.$
&c.	

$P=\sqrt{e+fz^{\eta}}.\quad R=\sqrt{e},\quad T=\sqrt{e+fz^{\eta}},\quad S=\sqrt{fz^{\eta}}.$

PARS SECUNDA.

θ	FORMA VI. $\dfrac{dz\,z^{\theta\eta+\frac{1}{2}\eta-1}}{\sqrt{e+fz^{\eta}}}$
&c.	
3.	$\dfrac{15eez^{\eta}-10efz^{2\eta}+8ffz^{3\eta}}{24\eta f^{3}}dP-\dfrac{5e^{3}}{8\eta f^{4}}dR\Big\vert\dfrac{R+T}{S}.$
2.	$\dfrac{-3ez^{\eta}+2fz^{2\eta}}{4\eta ff}dP+\dfrac{3ee}{4\eta f^{3}}dR\Big\vert\dfrac{R+T}{S}.$
1.	$\dfrac{z^{\eta}}{\eta f}dP-\dfrac{e}{\eta ff}dR\Big\vert\dfrac{R+T}{S}.$
0.	$\dfrac{2}{\eta f}dR\Big\vert\dfrac{R+T}{S}.$
−1.	$\dfrac{-2}{\eta e}dP.$
−2.	$\dfrac{-2e+4fz^{\eta}}{3\eta eez^{\eta}}dP.$
−3.	$\dfrac{-6ee+8efz^{\eta}-16ffz^{2\eta}}{15\eta e^{3}z^{2\eta}}dP.$
−4.	$\dfrac{-30e^{3}+36eefz^{\eta}-48effz^{2\eta}+96f^{3}z^{3\eta}}{105\eta e^{4}z^{3\eta}}dP.$
&c.	

$$P=\sqrt{\frac{e+fz^{\eta}}{z^{\eta}}}.\quad R=\sqrt{f},\quad T=\sqrt{\frac{e+fz^{\eta}}{z^{\eta}}},\quad S=\sqrt{\frac{e}{z^{\eta}}}.$$

Theoremata

θ	FORMA VII. $\dfrac{dz\,z^{\theta n-1}\sqrt{e+fz^n}}{g+hz^n}$.
&c.	
3.	$\dfrac{\begin{matrix}-4eehh\\ -10efgh-10ffghz^n\\ +30ffgg+2efhhz^n+6ffhhz^{2n}\end{matrix}}{15\eta ffh^3}dP-\dfrac{2gg}{\eta h^3}dR\left\|\dfrac{R+T}{S}\right.$.
2.	$\dfrac{\begin{matrix}+2eh\\ -6fg+2fhz^n\end{matrix}}{3\eta fhh}dP+\dfrac{2g}{\eta hh}dR\left\|\dfrac{R+T}{S}\right.$.
1.	$\dfrac{2}{\eta h}dP-\dfrac{2}{\eta h}dR\left\|\dfrac{R+T}{S}\right.$.
0.	$\dfrac{2}{\eta g}dR\left\|\dfrac{R+T}{S}\right.-\dfrac{2}{\eta g}d\mathrm{r}\left\|\dfrac{\mathrm{r}+\mathrm{t}}{\mathrm{s}}\right.$.
−1.	$\dfrac{-1}{\eta gz^n}dP-\dfrac{2h}{\eta gg}dR\left\|\dfrac{R+T}{S}\right.+\dfrac{2eh-fg}{\eta egg}d\mathrm{r}\left\|\dfrac{\mathrm{r}+\mathrm{t}}{\mathrm{s}}\right.$.
−2.	$\dfrac{\begin{matrix}+4ehz^n\\ -2eg-fgz^n\end{matrix}}{4\eta eggz^{2n}}dP+\dfrac{2hh}{\eta g^3}dR\left\|\dfrac{R+T}{S}\right.+\dfrac{\begin{matrix}-8eehh\\ +4efgh\\ +ffgg\end{matrix}}{4\eta eeg^3}d\mathrm{r}\left\|\dfrac{\mathrm{r}+\mathrm{t}}{\mathrm{s}}\right.$.
&c.	

$P=\sqrt{e+fz^n}$. $R=\sqrt{\dfrac{eh-fg}{h}}$, $T=\sqrt{e+fz^n}$, $S=\sqrt{\dfrac{f\,\overline{g+hz^n}}{h}}$.

$\mathrm{r}=\sqrt{e}$, $\mathrm{t}=\sqrt{e+fz^n}$, $\mathrm{s}=\sqrt{fz^n}$.

Pars Secunda

θ	FORMA VIII. $\dfrac{d\dot{z}z^{\theta\eta+\frac{1}{2}\eta-1}\sqrt{e+fz^\eta}}{g+hz^\eta}$.
&c.	
2.	$\dfrac{\begin{matrix}-4fgz^\eta\\+ehz^\eta+2fhz^{2\eta}\end{matrix}}{4\eta fhh}dP-\dfrac{2gg}{\eta h^3}dR\,\Big\|\dfrac{R+T}{S}+\dfrac{\begin{matrix}+8ffgg\\-4efgh\\-eehh\end{matrix}}{4\eta ffh^3}d\mathrm{r}\,\Big\|\dfrac{\mathrm{r}+\mathrm{t}}{\mathrm{s}}$.
1.	$\dfrac{z^\eta}{\eta h}dP+\dfrac{2g}{\eta hh}dR\,\Big\|\dfrac{R+T}{S}+\dfrac{eh-2fg}{\eta fhh}d\mathrm{r}\,\Big\|\dfrac{\mathrm{r}+\mathrm{t}}{\mathrm{s}}$.
0.	$\dfrac{-2}{\eta h}dR\,\Big\|\dfrac{R+T}{S}+\dfrac{2}{\eta h}d\mathrm{r}\,\Big\|\dfrac{\mathrm{r}+\mathrm{t}}{\mathrm{s}}$.
−1.	$\dfrac{-2}{\eta g}dP+\dfrac{2}{\eta g}dR\,\Big\|\dfrac{R+T}{S}$.
−2.	$\dfrac{\begin{matrix}-2fgz^\eta\\-2eg+6ehz^\eta\end{matrix}}{3\eta eggz^\eta}dP-\dfrac{2h}{\eta gg}dR\,\Big\|\dfrac{R+T}{S}$.
−3.	$\dfrac{\begin{matrix}+4ffggz^{2\eta}\\-2efggz^\eta+10efghz^{2\eta}\\-6eegg+10eeghz^\eta-30eehhz^{2\eta}\end{matrix}}{15\eta eeg^3z^{2\eta}}dP+\dfrac{2hh}{\eta g^3}dR\,\Big\|\dfrac{R+T}{S}$.
&c.	

$$P=\sqrt{\frac{e+fz^\eta}{z^\eta}}.\quad R=\sqrt{\frac{fg-eh}{g}},\quad T=\sqrt{\frac{e+fz^\eta}{z^\eta}},\quad S=\sqrt{\frac{e,\overline{g+hz^\eta}}{gz^\eta}}.$$

$$\mathrm{r}=\sqrt{f},\quad \mathrm{t}=\sqrt{\frac{e+fz^\eta}{z^\eta}},\quad \mathrm{s}=\sqrt{\frac{e}{z^\eta}}.$$

THEOREMATA

θ	FORMA IX. $\dfrac{dzz^{\theta\eta-1}}{\overline{g+hz^\eta}\sqrt{e+fz^\eta}}$
&c.	
4.	$\dfrac{\begin{array}{l}+16eehh\\ +20efgh-8efhhz^\eta\\ +30ffgg-10ffghz^\eta+6ffhhz^{2\eta}\end{array}}{15\eta f^3h^3}dP+\dfrac{2g^3}{\eta h^3,\overline{eh-fg}}dR\left\|\dfrac{R+T}{S}\right.$
3.	$\dfrac{\begin{array}{l}-4eh\\ -6fg+2fhz^\eta\end{array}}{3\eta ffhh}dP-\dfrac{2gg}{\eta hh,\overline{eh-fg}}dR\left\|\dfrac{R+T}{S}\right.$.
2.	$\dfrac{2}{\eta fh}dP+\dfrac{2g}{\eta h,\overline{eh-fg}}dR\left\|\dfrac{R+T}{S}\right.$.
1.	$\dfrac{-2}{\eta,\overline{eh-fg}}dR\left\|\dfrac{R+T}{S}\right.$.
0.	$\dfrac{+2h}{\eta g,\overline{eh-fg}}dR\left\|\dfrac{R+T}{S}\right.-\dfrac{2}{\eta eg}dr\left\|\dfrac{r+t}{s}\right.$.
−1.	$\dfrac{-1}{\eta egz^\eta}dP-\dfrac{2hh}{\eta gg,\overline{eh-fg}}dR\left\|\dfrac{R+T}{S}\right.+\dfrac{2eh+fg}{\eta eegg}dr\left\|\dfrac{r+t}{s}\right.$.
&c.	

$P=\sqrt{e+fz^\eta}$. $R=\sqrt{\dfrac{eh-fg}{h}}$, $T=\sqrt{e+fz^\eta}$, $S=\sqrt{\dfrac{f,\overline{g+hz^\eta}}{h}}$.

$r=\sqrt{e}$, $t=\sqrt{e+fz^\eta}$, $s=\sqrt{fz^\eta}$.

PARS SECUNDA.

θ	FORMA X. $\dfrac{d\dot{z}z^{\theta\eta+\frac{1}{2}\eta-1}}{\overline{g+hz^\eta}\sqrt{e+fz^\eta}}$.
&c.	
2.	$\dfrac{z^\eta}{\eta fh}dP+\dfrac{2gg}{\eta hh,\overline{fg-eh}}dR\left\|\dfrac{R+T}{S}\right.+\dfrac{-2fg-eh}{\eta ffhh}dr\left\|\dfrac{r+t}{s}\right.$.
1.	$\dfrac{-2g}{\eta h,\overline{fg-eh}}dR\left\|\dfrac{R+T}{S}\right.+\dfrac{2}{\eta fh}dr\left\|\dfrac{r+t}{s}\right.$.
0.	$\dfrac{+2}{\eta,\overline{fg-eh}}dR\left\|\dfrac{R+T}{S}\right.$.
$-$1.	$\dfrac{-2}{\eta eg}dP-\dfrac{2h}{\eta g,\overline{fg-eh}}dR\left\|\dfrac{R+T}{S}\right.$.
$-$2.	$\dfrac{\begin{matrix}+4fgz^\eta\\-2eg+6ehz^\eta\end{matrix}}{3\eta eeggz^\eta}dP+\dfrac{2hh}{\eta gg,\overline{fg-eh}}dR\left\|\dfrac{R+T}{S}\right.$.
$-$3.	$\dfrac{\begin{matrix}-16ffggz^{2\eta}\\+8efggz^\eta-20efghz^{2\eta}\\-6eegg+10eeghz^\eta-30eehhz^{2\eta}\end{matrix}}{15\eta e^3g^3z^{2\eta}}dP-\dfrac{2h^3}{\eta g^3,\overline{fg-eh}}dR\left\|\dfrac{R+T}{S}\right.$
&c.	

$$P=\sqrt{\frac{e+fz^\eta}{z^\eta}}.\quad R=\sqrt{\frac{fg-eh}{g}},\quad T=\sqrt{\frac{e+fz^\eta}{z^\eta}},\quad S=\sqrt{\frac{e,\overline{g+hz^\eta}}{gz^\eta}}.$$

$$r=\sqrt{f},\quad t=\sqrt{\frac{e+fz^\eta}{z^\eta}},\quad s=\sqrt{\frac{e}{z^\eta}}.$$

Theoremata

θ	FORMA XI. $d\dot{z}z^{\theta\eta-1}\sqrt{\dfrac{e+fz^\eta}{g+hz^\eta}}$.
&c.	
3.	$\dfrac{-3eehh-4efgh+2efhhz^\eta+15ffgg-10ffghz^\eta+8ffhhz^{2\eta}}{24\eta ffh^3}dPQ+\dfrac{+3eehh+6efgh+15ffgg \text{ in } \overline{eh-fg}}{24\eta f^3h^3}dR$
2.	$\dfrac{+eh-3fg+2fhz^\eta}{4\eta fhh}dPQ+\dfrac{-eh-3fg \text{ in } \overline{eh-fg}}{4\eta ffhh}dR\left\vert\dfrac{R+T}{S}\right.$.
1.	$\dfrac{1}{\eta h}dPQ+\dfrac{eh-fg}{\eta fh}dR\left\vert\dfrac{R+T}{S}\right.$.
0.	$\dfrac{2}{\eta}dR\left\vert\dfrac{R+T}{S}\right.-\dfrac{2}{\eta}dr\left\vert\dfrac{r+t}{s}\right.$.
−1.	$\dfrac{-1}{\eta gz^\eta}dPQ-\dfrac{+fg-eh}{\eta eg}dr\left\vert\dfrac{r+t}{s}\right.$.
−2.	$\dfrac{-fgz^\eta-2eg+3ehz^\eta}{4\eta eggz^{2\eta}}dPQ+\dfrac{+fg+3eh \text{ in } \overline{fg-eh}}{4\eta eegg}dr\left\vert\dfrac{r+t}{s}\right.$.
−3.	$\dfrac{+3ffggz^{2\eta}-2efggz^\eta+4efghz^{2\eta}-8eegg+10eeghz^\eta-15eehhz^{2\eta}}{24\eta eeg^3z^{3\eta}}dPQ-\dfrac{+3ffgg+6efgh+15eehh \text{ in } \overline{fg-eh}}{24\eta e^3g^3}dr$
&c.	

$P=\sqrt{e+fz^\eta}$. $R=\sqrt{\dfrac{f}{h}}$, $T=\sqrt{\dfrac{e+fz^\eta}{g+hz^\eta}}$, $S=\sqrt{\dfrac{eh-fg}{h,\overline{g+hz^\eta}}}$.

$Q=\sqrt{g+hz^\eta}$. $r=\sqrt{\dfrac{e}{g}}$, $t=\sqrt{\dfrac{e+fz^\eta}{g+hz^\eta}}$, $s=\sqrt{\dfrac{fgz^\eta-ehz^\eta}{g,\overline{g+hz^\eta}}}$.

PARS SECUNDA

θ	FORMA XII. $\frac{dzz^{\theta n-1}}{k+lz^n}\sqrt{\frac{e+fz^n}{g+hz^n}}$.
&c.	
3.	$\frac{\begin{matrix}-4fhk\\-3fgl\\+ehl+2fhlz^n\end{matrix}}{4\eta fhhll}dPQ-\frac{2kk}{\eta l^3}dR\Big\| + \frac{\begin{matrix}-eehhll+8ffhhkk\\-2efghll+4ffghkl\\+3ffggll-4efhhkl\end{matrix}}{4\eta ffhhl^3}dr\Big\|$
2.	$\frac{1}{\eta hl}dPQ+\frac{2k}{\eta ll}dR\Big\|\frac{R+T}{S}+\frac{ehl-fgl-2fhk}{\eta fhll}dr\Big\|\frac{r+t}{s}$.
1.	$\frac{-2}{\eta l}dR\Big\|\frac{R+T}{S}+\frac{2}{\eta l}dr\Big\|\frac{r+t}{s}$.
0.	$\frac{2}{\eta k}dR\Big\|\frac{R+T}{S}-\frac{2}{\eta k}d\varrho\Big\|\frac{\varrho+\tau}{\sigma}$.
−1.	$\frac{-1}{\eta gkz^n}dPQ-\frac{2l}{\eta kk}dR\Big\|\frac{R+T}{S}+\frac{ehk-fgk+2egl}{\eta egkk}d\varrho\Big\|\frac{\varrho+\tau}{\sigma}$.
−2.	$\frac{\begin{matrix}-fgkz^n\\+3ehkz^n\\-2egk+4eglz^n\end{matrix}}{4\eta eggkkz^{2n}}dPQ+\frac{2ll}{\eta k^3}dR\Big\| + \frac{\begin{matrix}-3eehhkk+4efggkl\\+2efghkk-4eeghkl\\+ffggkk-8eeggll\end{matrix}}{4\eta eeggk^3}d\varrho\Big\|$
&c.	

$P=\sqrt{e+fz^n}$. $R=\sqrt{\frac{el-fk}{gl-hk}}$, $T=\sqrt{\frac{e+fz^n}{g+hz^n}}$, $S=\sqrt{\frac{fg-eh,\,k+lz^n}{gl-hk,\,g+hz^n}}$

$Q=\sqrt{g+hz^n}$. $r=\sqrt{\frac{f}{h}}$, $t=\sqrt{\frac{e+fz^n}{g+hz^n}}$, $s=\sqrt{\frac{eh-fg}{g,\,g+hz^n}}$

$\varrho=\sqrt{\frac{e}{g}}$, $\tau=\sqrt{\frac{e+fz^n}{g+hz^n}}$, $\sigma=\sqrt{\frac{fg-eh,\,z^n}{g,\,g+hz^n}}$

Theoremata

θ	FORMA XIII. $\dfrac{dz z^{\theta\eta-1}}{e+fz^{\eta}+gz^{2\eta}}$
&c.	
4.	$\frac{dz^{2\eta}}{2\eta g} - \frac{dfz^{\eta}}{\eta gg} + \frac{ff-eg}{2\eta g^{3}} d\mathrm{N} + \frac{f^{3}-3efg}{2\eta g^{3}\mathrm{q}} d\mathrm{R} \vert \frac{\mathrm{R}+\mathrm{T}}{\mathrm{S}}.$
3.	$\frac{dz^{\eta}}{\eta g} - \frac{df}{2\eta gg}\mathrm{N} + \frac{2eg-ff}{2\eta gg\mathrm{q}} d\mathrm{R} \vert \frac{\mathrm{R}+\mathrm{T}}{\mathrm{S}}.$
2.	$\frac{d}{2\eta g}\mathrm{N} + \frac{f}{2\eta g\mathrm{q}} d\mathrm{R} \vert \frac{\mathrm{R}+\mathrm{T}}{\mathrm{S}}.$
1.	$\frac{-1}{\eta\mathrm{q}} d\mathrm{R} \vert \frac{\mathrm{R}+\mathrm{T}}{\mathrm{S}}.$
0.	$\frac{-d}{2\eta e}\mathrm{n} + \frac{f}{2\eta e\mathrm{q}} d\mathrm{R} \vert \frac{\mathrm{R}+\mathrm{T}}{\mathrm{S}}.$
−1.	$\frac{-d}{\eta e z^{\eta}} + \frac{df}{2\eta ee}\mathrm{n} + \frac{2eg-ff}{2\eta ee\mathrm{q}} d\mathrm{R} \vert \frac{\mathrm{R}+\mathrm{T}}{\mathrm{S}}.$
−2.	$\frac{-d}{2\eta e z^{2\eta}} + \frac{df}{\eta ee z^{\eta}} + \frac{eg-ff}{2\eta e^{3}} d\mathrm{n} + \frac{f^{3}-3efg}{2\eta e^{3}\mathrm{q}} d\mathrm{R} \vert \frac{\mathrm{R}+\mathrm{T}}{\mathrm{S}}.$
&c.	

$\mathrm{q} = \tfrac{1}{4}ff - eg.$ $\mathrm{N} = 1 \vert \frac{e+fz^{\eta}+gz^{2\eta}}{e}.$ $\mathrm{n} = 1 \vert \frac{e+fz^{\eta}+gz^{2\eta}}{gz^{2\eta}}.$

$\mathrm{R} = \sqrt{\mathrm{q}}.$ $\mathrm{T} = \tfrac{1}{2}f + gz^{\eta}.$ $\mathrm{S} = \sqrt{g,\overline{e+fz^{\eta}+gz^{2\eta}}}.$

Si ponantur $\mathrm{r} = \sqrt{\mathrm{q}}$, $\mathrm{t} = \frac{e+\frac{1}{2}fz^{\eta}}{z^{\eta}}$, $\mathrm{s} = \frac{\sqrt{e,\overline{e+fz^{\eta}+gz^{2\eta}}}}{z^{\eta}}$,

adhiberi poteſt $-\mathrm{r} \vert \frac{\mathrm{r}+\mathrm{t}}{\mathrm{s}}$ pro $+\mathrm{R} \vert \frac{\mathrm{R}+\mathrm{T}}{\mathrm{S}}.$

PARS SECUNDA.

θ	FORMA XIV. $\frac{d\dot{z}z^{\theta\eta-1}}{\overline{k+lz^{\eta}},\overline{e+fz^{\eta}+gz^{2\eta}}}$.
&c.	
4.	$\frac{dz^{\eta}}{\eta gl}+\frac{-fl-gk}{2\eta ggll}d\mathrm{N}+\frac{dk^{3}}{2\eta ll\mathrm{p}}\mathrm{M}+\frac{\substack{-effl+f^{3}k\\2eegl-3efgk}}{2\eta gg\mathrm{pq}}d\mathrm{R}\Big\vert\frac{\mathrm{R}+\mathrm{T}}{\mathrm{S}}$
3.	$\frac{d}{2\eta gl}\mathrm{N}-\frac{dkk}{2\eta l\mathrm{p}}\mathrm{M}+\frac{efl-ffk+2egk}{2\eta g\mathrm{pq}}d\mathrm{R}\Big\vert\frac{\mathrm{R}+\mathrm{T}}{\mathrm{S}}$.
2.	$\frac{dk}{2\eta\mathrm{p}}\mathrm{M}+\frac{fk-2el}{2\eta\mathrm{pq}}d\mathrm{R}\Big\vert\frac{\mathrm{R}+\mathrm{T}}{\mathrm{S}}$.
1.	$\frac{-dl}{2\eta\mathrm{p}}\mathrm{M}+\frac{fl-2gk}{2\eta\mathrm{pq}}d\mathrm{R}\Big\vert\frac{\mathrm{R}+\mathrm{T}}{\mathrm{S}}$.
0.	$\frac{-d}{2\eta ek}\mathrm{n}+\frac{dll}{2\eta k\mathrm{p}}\mathrm{M}+\frac{fgk-ffl+2egl}{2\eta e\mathrm{pq}}d\mathrm{R}\Big\vert\frac{\mathrm{R}+\mathrm{T}}{\mathrm{S}}$.
−1.	$\frac{-d}{\eta ekz^{\eta}}+\frac{fk+el}{2\eta eekk}d\mathrm{n}-\frac{dl^{3}}{2\eta kk\mathrm{p}}\mathrm{M}+\frac{\substack{-ffgk+f^{3}l\\2eggk-3efgl}}{2\eta ee\mathrm{pq}}d\mathrm{R}\Big\vert\frac{\mathrm{R}+\mathrm{T}}{\mathrm{S}}$.
&c.	

$$\mathrm{p}=ell-fkl+gkk,\quad \mathrm{q}=\tfrac{1}{4}ff-eg.$$

$$\mathrm{N}=1\Big\vert\frac{e+fz^{\eta}+gz^{2\eta}}{e},\quad \mathrm{n}=1\Big\vert\frac{e+fz^{\eta}+gz^{2\eta}}{gz^{2\eta}},\quad \mathrm{M}=1\Big\vert\frac{e+fz^{\eta}+gz^{2\eta}}{\overline{k+lz^{\eta}}\vert^{2}}.$$

$$\mathrm{R}=\sqrt{\mathrm{q}},\quad \mathrm{T}=\tfrac{1}{2}f+gz^{\eta},\quad \mathrm{S}=\sqrt{g,\overline{e+fz^{\eta}+gz^{2\eta}}}.$$

Theoremata

θ	FORMA XV. $d\dot{z}z^{\theta\eta-1}\sqrt{e+fz^{\eta}+gz^{2\eta}}$.
&c.	
3.	$\frac{\begin{matrix}+15f^{3}+24eggz^{\eta}\\ -52efg-10ffgz^{\eta}+8fggz^{2\eta}+48g^{3}z^{3\eta}\end{matrix}}{192\eta g^{3}}dP+\frac{\begin{matrix}-16eegg\\ +24effg-5f^{4}\end{matrix}}{128\eta g^{4}}dR\,\vert$
2.	$\frac{\begin{matrix}+8eg\\ -3ff+2fgz^{\eta}+8ggz^{2\eta}\end{matrix}}{24\eta gg}dP+\frac{f^{3}-4efg}{16\eta g^{3}}dR\,\vert\frac{R+T}{S}$
1.	$\frac{f+2gz^{\eta}}{4\eta g}dP+\frac{4eg-ff}{8\eta gg}dR\,\vert\frac{R+T}{S}$.
0.	$\frac{1}{\eta}dP+\frac{f}{2\eta g}dR\,\vert\frac{R+T}{S}-\frac{1}{\eta}dr\,\vert\frac{r+t}{s}$.
−1.	$\frac{-1}{\eta z^{\eta}}dP-\frac{f}{2\eta e}dr\,\vert\frac{r+t}{s}+\frac{1}{\eta}dR\,\vert\frac{R+T}{S}$.
−2.	$\frac{-2e-fz^{\eta}}{4\eta ez^{2\eta}}dP+\frac{ff-4eg}{8\eta ee}dr\,\vert\frac{r+t}{s}$.
−3.	$\frac{\begin{matrix}-8egz^{2\eta}\\ -8ee-2efz^{\eta}+3ffz^{2\eta}\end{matrix}}{24\eta eez^{3\eta}}dP+\frac{4efg-f^{3}}{16\eta e^{3}}dr\,\vert\frac{r+t}{s}$.
−4.	$\frac{\begin{matrix}-24eegz^{2\eta}-15f^{3}z^{3\eta}\\ -48e^{3}-8eefz^{\eta}+10effz^{2\eta}+52efgz^{3\eta}\end{matrix}}{192\eta e^{3}z^{4\eta}}dP+\frac{\begin{matrix}-24effg\\ +16eegg+5f^{4}\end{matrix}}{128\eta e^{4}}dr\,\vert$
&c.	

Quantitates P, R, T, S, r, t, s eosdem habent valores ac in Forma subsequente.

PARS SECUNDA.

θ	FORMA XVI. $\dfrac{dz z^{\theta n-1}}{\sqrt{e+fz^n+gz^{2n}}}$
&c.	
4.	$\dfrac{+15ff-16eg-10fgz^n+8ggz^{2n}}{24\eta g^3}dP+\dfrac{12efg-5f^3}{16\eta g^4}dR\Big\|\dfrac{R+T}{S}.$
3.	$\dfrac{-3f+2gz^n}{4\eta gg}dP+\dfrac{3ff-4eg}{8\eta g^3}dR\Big\|\dfrac{R+T}{S}.$
2.	$\dfrac{1}{\eta g}dP-\dfrac{f}{2\eta gg}dR\Big\|\dfrac{R+T}{S}.$
1.	$\dfrac{1}{\eta g}dR\Big\|\dfrac{R+T}{S}.$
0.	$\dfrac{-1}{\eta e}dr\Big\|\dfrac{r+t}{s}.$
−1.	$\dfrac{-1}{\eta ez^n}dP+\dfrac{f}{2\eta ee}dr\Big\|\dfrac{r+t}{s}.$
−2.	$\dfrac{-2e+3fz^n}{4\eta eez^{2n}}dP+\dfrac{4eg-3ff}{8\eta e^3}dr\Big\|\dfrac{r+t}{s}.$
−3.	$\dfrac{-15ffz^{2n}-8ee+10efz^n+16egz^{2n}}{24\eta e^3z^{3n}}dP+\dfrac{5f^3-12efg}{16\eta e^4}dr\Big\|\dfrac{r+t}{s}.$
&c.	

$$P=\sqrt{e+fz^n+gz^{2n}}.\quad R=\sqrt{g},\quad T=\frac{\frac{1}{2}f+gz^n}{\sqrt{e+fz^n+gz^{2n}}},\quad S=\sqrt{\frac{\frac{1}{4}ff-eg}{e+fz^n+gz^{2n}}}$$

$$r=\sqrt{e},\quad t=\frac{e+\frac{1}{2}fz^n}{\sqrt{e+fz^n+gz^{2n}}},\quad s=z^n\sqrt{\frac{\frac{1}{4}ff-eg}{e+fz^n+gz^{2n}}}$$

θ	FORMA XVII. $\dfrac{dz\,z^{\theta n-1}\sqrt{e+fz^{n}+gz^{2n}}}{k+lz^{n}}$.
&c.	
3.	$\frac{-6fgkl+24ggkk-3ffll-12ggklz^{n}+8egll+2fgllz^{n}+8ggllz^{2n}}{24\eta ggl^{3}}dP-\frac{kk}{\eta l^{4}}dR\Big\vert+\frac{+2ffgkll-8eggkll+f^{3}l^{3}+8fggkkl-4efgl^{3}-16g^{3}k^{3}}{16\eta g^{3}l^{4}}dr\Big\vert$
2.	$\frac{+fl-4gk+2glz^{n}}{4\eta gll}dP+\frac{k}{\eta l^{3}}dR\Big\vert\frac{R+T}{S}+\frac{-ffll-4fgkl+4egll+8ggkk}{8\eta ggl^{3}}dr\Big\vert\frac{r+t}{s}.$
1.	$\frac{1}{\eta l}dP-\frac{1}{\eta ll}dR\Big\vert\frac{R+T}{S}+\frac{fl-2gk}{2\eta gll}dr\Big\vert\frac{r+t}{s}.$
0.	$\frac{1}{\eta kl}dR\Big\vert\frac{R+T}{S}+\frac{1}{\eta l}dr\Big\vert\frac{r+t}{s}-\frac{1}{\eta k}d\varrho\Big\vert\frac{\varrho+\tau}{\sigma}.$
−1.	$\frac{-1}{\eta kz^{n}}dP-\frac{1}{\eta kk}dR\Big\vert\frac{R+T}{S}+\frac{2el-fk}{2\eta ekk}d\varrho\Big\vert\frac{\varrho+\tau}{\sigma}.$
−2.	$\frac{+4elz^{n}-2ek-fkz^{n}}{4\eta ekkz^{2n}}dP+\frac{l}{\eta k^{3}}dR\Big\vert\frac{R+T}{S}+\frac{-4egkk+4efkl+ffkk-8eell}{8\eta eek^{3}}d\varrho\Big\vert\frac{\varrho+\tau}{\sigma}.$
−3.	$\frac{+3ffkkz^{2n}+6efklz^{2n}-2efkkz^{n}-8egkkz^{2n}-8eekk+12eeklz^{n}-24eellz^{2n}}{24\eta eek^{3}z^{3n}}dP-\frac{ll}{\eta k^{4}}dR\Big\vert+\frac{+16e^{3}l^{3}-8eefkll-f^{3}k^{3}-2effkkl+4efgk^{3}+8eegkkl}{16\eta e^{3}k^{4}}d\varrho\Big\vert$
&c.	

Quantitates P, p; R, T, S; r, t, s; ϱ, τ, σ, eosdem habent valores ac in Forma subsequente.

θ	FORMA XVIII. $\dfrac{\dot{d}z\, z^{\theta n-1}}{\overline{k+lz^n}, \sqrt{e+fz^n+gz^{2n}}}$.
&c.	
4.	$\dfrac{-3fl-4gk+2glz^n}{4ngg ll}dP+\dfrac{k^3}{nl^3p}dR\left\|\dfrac{R+T}{S}\right.+\dfrac{-4egll+8ggkk+3ffll+4fgkl}{8ng^3l^3}dr\left\|\dfrac{r+t}{s}\right.$
3.	$\dfrac{1}{ngl}dP-\dfrac{kk}{nllp}dR\left\|\dfrac{R+T}{S}\right.+\dfrac{-fl-2gk}{2nggll}dr\left\|\dfrac{r+t}{s}\right.$.
2.	$\dfrac{k}{nlp}dR\left\|\dfrac{R+T}{S}\right.+\dfrac{1}{ngl}dr\left\|\dfrac{r+t}{s}\right.$.
1.	$\dfrac{-1}{np}dR\left\|\dfrac{R+T}{S}\right.$.
0.	$\dfrac{l}{nkp}dR\left\|\dfrac{R+T}{S}\right.-\dfrac{1}{nek}d\varrho\left\|\dfrac{\varrho+\tau}{\sigma}\right.$.
−1.	$\dfrac{-1}{nekz^n}dP-\dfrac{ll}{nkkp}dR\left\|\dfrac{R+T}{S}\right.+\dfrac{fk+2el}{2neekk}d\varrho\left\|\dfrac{\varrho+\tau}{\sigma}\right.$.
−2.	$\dfrac{+4elz^n-2ek+3fkz^n}{4neekkz^{2n}}dP+\dfrac{l^3}{nk^3p}dR\left\|\dfrac{R+T}{S}\right.+\dfrac{+4egkk-8eell-3ffkk-4efkl}{8ne^3k^3}d\varrho\left\|\dfrac{\varrho+\tau}{\sigma}\right.$
&c.	

$P=\sqrt{e+fz^n+gz^{2n}}$. $\quad p=ell-fkl+gkk$,

$R=\sqrt{p}$, $\quad T=\dfrac{l,\overline{e+\frac{1}{2}fz^n}-k,\overline{\frac{1}{2}f+gz^n}}{\sqrt{e+fz^n+gz^{2n}}}$, $\quad S=\overline{k+lz^n}\sqrt{\dfrac{\frac{1}{4}ff-eg}{e+fz^n+gz^{2n}}}$.

$r=\sqrt{g}$, $\quad t=\dfrac{\frac{1}{2}f+gz^n}{\sqrt{e+fz^n+gz^{2n}}}$, $\quad s=\sqrt{\dfrac{\frac{1}{4}ff-eg}{e+fz^n+gz^{2n}}}$.

$\varrho=\sqrt{e}$, $\quad \tau=\dfrac{e+\frac{1}{2}fz^n}{\sqrt{e+fz^n+gz^{2n}}}$, $\quad \sigma=z^n\sqrt{\dfrac{\frac{1}{4}ff-eg}{e+fz^n+gz^{2n}}}$.

FORMA	Adhibere licet vel eos valores quantitatum R, T, S, &c. qui singulis Formis apponuntur vel hos qui sequuntur, prout commodum videbitur.
II.	$R = \sqrt{\frac{-e}{f}}$. $T = \frac{-e}{fz^{\frac{1}{2}n}}$. $S = \frac{\sqrt{e,\overline{e+fz^n}}}{fz^{\frac{1}{2}n}}$. $r = \sqrt{\frac{-f}{e}}$. $t = \frac{-fz^{\frac{1}{2}n}}{e}$. $s = \frac{\sqrt{f,\overline{e+fz^n}}}{e}$.
III. V.	$R = \sqrt{e}$. $T = \frac{e}{\sqrt{e+fz^n}}$. $S = \sqrt{\frac{-efz^n}{e+fz^n}}$.
IV. VI.	$R = \sqrt{f}$. $T = \frac{fz^{\frac{1}{2}n}}{\sqrt{e+fz^n}}$. $S = \sqrt{\frac{-ef}{e+fz^n}}$.
VII. IX.	$R = \sqrt{\frac{eh-fg}{h}}$. $T = \frac{eh-fg}{h\sqrt{e+fz^n}}$. $S = \frac{1}{h}\sqrt{\frac{f,\overline{fg-eh},\overline{g+hz^n}}{e+fz^n}}$ $r = \sqrt{e}$. $t = \frac{e}{\sqrt{e+fz^n}}$. $s = \sqrt{\frac{-efz^n}{e+fz^n}}$.
VIII. X.	$R = \sqrt{\frac{fg-eh}{g}}$. $T = \frac{\overline{fg-eh},z^{\frac{1}{2}n}}{g\sqrt{e+fz^n}}$. $S = \frac{1}{g}\sqrt{\frac{e,\overline{eh-fg},\overline{g+hz^n}}{e+fz^n}}$ $r = \sqrt{f}$. $t = \frac{fz^{\frac{1}{2}n}}{\sqrt{e+fz^n}}$. $s = \sqrt{\frac{-ef}{e+fz^n}}$.
XI.	$R = \sqrt{\frac{f}{h}}$. $T = \frac{f}{h}\sqrt{\frac{g+hz^n}{e+fz^n}}$. $S = \frac{1}{h}\sqrt{\frac{f,\overline{fg-eh}}{e+fz^n}}$. $r = \sqrt{\frac{e}{g}}$. $t = \frac{e}{g}\sqrt{\frac{g+hz^n}{e+fz^n}}$. $s = \frac{1}{g}\sqrt{\frac{\overline{eh-fg},ez^n}{e+fz^n}}$.

FORMA	
XII.	$R = \sqrt{\frac{el-fk}{gl-hk}}$. $T = \frac{el-fk}{gl-hk}\sqrt{\frac{g+hz^n}{e+fz^n}}$. $S = \frac{1}{gl-hk}\sqrt{\frac{\overline{eh-fg},\,\overline{el-fk},\,\overline{k+lz^n}}{e+fz^n}}$
	$r = \sqrt{\frac{f}{h}}$. $t = \frac{f}{h}\sqrt{\frac{g+hz^n}{e+fz^n}}$. $s = \frac{1}{h}\sqrt{\frac{f,\,\overline{fg-eh}}{e+fz^n}}$.
	$\varrho = \sqrt{\frac{e}{g}}$. $\tau = \frac{e}{g}\sqrt{\frac{g+hz^n}{e+fz^n}}$. $\sigma = \frac{1}{g}\sqrt{\frac{\overline{eh-fg},\,ez^n}{e+fz^n}}$.
XIII. XIV.	$R = \sqrt{q}$. $T = \frac{q}{\frac{1}{2}f+gz^n}$. $S = \frac{\sqrt{-gq,\,\overline{e+fz^n+gz^{2n}}}}{\frac{1}{2}f+gz^n}$.
XV. XVI.	$R = \sqrt{g}$. $T = \frac{g\sqrt{e+fz^n+gz^{2n}}}{\frac{1}{2}f+gz^n}$. $S = \frac{\sqrt{g,\,\overline{eg-\frac{1}{4}ff}}}{\frac{1}{2}f+gz^n}$.
	$r = \sqrt{e}$, $t = \frac{e\sqrt{e+fz^n+gz^{2n}}}{e+\frac{1}{2}fz^n}$. $s = \frac{z^n\sqrt{e,\,\overline{eg-\frac{1}{4}ff}}}{e+\frac{1}{2}fz^n}$.
XVII. XVIII.	$R = \sqrt{p}$, $T = \frac{p\sqrt{e+fz^n+gz^{2n}}}{l,\,\overline{e+\frac{1}{2}fz^n}-k,\,\overline{\frac{1}{2}f+gz^n}}$, $S = \frac{\overline{k+lz^n}\sqrt{-pq}}{l,\,\overline{e+\frac{1}{2}fz^n}-k,\,\overline{\frac{1}{2}f+gz^n}}$
	$r = \sqrt{g}$, $t = \frac{g\sqrt{e+fz^n+gz^{2n}}}{\frac{1}{2}f+gz^n}$, $s = \frac{\sqrt{g,\,\overline{eg-\frac{1}{4}ff}}}{\frac{1}{2}f+gz^n}$.
	$\varrho = \sqrt{e}$, $\tau = \frac{e\sqrt{e+fz^n+gz^{2n}}}{e+\frac{1}{2}fz^n}$, $\sigma = \frac{z^n\sqrt{e,\,\overline{eg-\frac{1}{4}ff}}}{e+\frac{1}{2}fz^n}$.

DE

CONTINUATIONE

FORMARUM ET FLUENTIUM

AD INFINITUM.

THEOREMA I.

Positis

$Z = e + fz^n \qquad \dot{A} = d\dot{z}z^{\theta n - 1} Z^{\omega - 1} \qquad \dot{F} = d\dot{z}z^{\theta n - 1} Z^{\omega}$

$\dot{B} = d\dot{z}z^{\theta n + n - 1} Z^{\omega - 1}$

Erit

$$\tfrac{1}{n} dz^{\theta n} Z^{\omega} = \theta e A + \overline{\theta + \omega}, fB$$

Erit

$$F = eA + fB$$

Atque inde

$$A = \frac{-\tfrac{1}{n} dz^{\theta n} Z^{\omega} + \overline{\theta + \omega}, F}{\omega e}. \qquad B = \frac{+\tfrac{1}{n} dz^{\theta n} Z^{\omega} - \theta F}{\omega f}.$$

THEOREMA II.

Positis

$Z = e + fz^n \qquad \dot{A} = d\dot{z}z^{\theta n - 1} Z^{\omega - 1} Y^{\psi - 1}$

$Y = g + hz^n \qquad \dot{B} = d\dot{z}z^{\theta n + n - 1} Z^{\omega - 1} Y^{\psi - 1}$

$\dot{C} = d\dot{z}z^{\theta n + 2n - 1} Z^{\omega - 1} Y^{\psi - 1}$

$\dot{F} = d\dot{z}z^{\theta n - 1} Z^{\omega} Y^{\psi - 1} \qquad \dot{I} = d\dot{z}z^{\theta n - 1} Z^{\omega - 1} Y^{\psi}$

$\dot{G} = d\dot{z}z^{\theta n + n - 1} Z^{\omega} Y^{\psi - 1} \qquad \dot{K} = d\dot{z}z^{\theta n + n - 1} Z^{\omega - 1} Y^{\psi}$

Erit

Erit

$$\tfrac{1}{n} d z^{\theta n} Z^{\omega} Y^{\psi} = \theta e g A + \overline{\overline{\theta+\omega}, fg + \overline{\theta+\psi}, eh}, B + \overline{\theta+\omega+\psi}, fhC$$

Erunt

$F = eA + fB \qquad I = gA + hB$

$G = eB + fC \qquad K = gB + hC$

Atque inde

$$A = \frac{+f, \tfrac{1}{n} d z^{\theta n} Z^{\omega} Y^{\psi} + \overline{\omega eh - \overline{\theta-\omega}, fg}, F - \overline{\theta-\omega-\psi}, fhG}{\omega e, \overline{eh-fg}} = \frac{+h, \tfrac{1}{n} d z^{\theta n} Z^{\omega} Y^{\psi} + \overline{\psi fg - \overline{\theta-\psi}, eh}, I - \overline{\theta-\omega-\psi}, fhK}{\psi g, \overline{fg-eh}}.$$

$$B = \frac{-\tfrac{1}{n} d z^{\theta n} Z^{\omega} Y^{\psi} + \theta g F + \overline{\theta+\omega+\psi}, hG}{\omega, \overline{eh-fg}} = \frac{-\tfrac{1}{n} d z^{\theta n} Z^{\omega} Y^{\psi} + \theta e I + \overline{\theta+\omega+\psi}, fK}{\psi, \overline{fg-eh}}.$$

$$C = \frac{+e, \tfrac{1}{n} d z^{\theta n} Z^{\omega} Y^{\psi} - \theta e g F + \overline{\overline{-\theta-\psi}, eh - \omega fg}, G}{\omega f, \overline{eh-fg}} = \frac{+g, \tfrac{1}{n} d z^{\theta n} Z^{\omega} Y^{\psi} - \theta e g I + \overline{\overline{-\theta-\omega}, fg - \psi eh}, K}{\psi h, \overline{fg-eh}}$$

THEOREMA III.

Positis

$$Z = e + fz^{\eta} + gz^{2\eta}$$

$$\dot{A} = d\dot{z}z^{\overline{\theta\eta - 1}}\, Z^{\overline{\omega - 1}} \qquad \dot{F} = d\dot{z}z^{\overline{\theta\eta - 1}}\, Z^{\omega}$$

$$\dot{B} = d\dot{z}z^{\overline{\theta\eta + \eta - 1}}\, Z^{\overline{\omega - 1}} \qquad \dot{G} = d\dot{z}z^{\overline{\theta\eta + \eta - 1}}\, Z^{\omega}$$

$$\dot{C} = d\dot{z}z^{\overline{\theta\eta + 2\eta - 1}}\, Z^{\overline{\omega - 1}}$$

$$\dot{D} = d\dot{z}z^{\overline{\theta\eta + 3\eta - 1}}\, Z^{\overline{\omega - 1}}$$

Erunt

$$\tfrac{1}{\eta}dz^{\theta\eta}Z^{\omega} = \theta eA + \overline{\theta + \omega},fB + \overline{\theta + 2\omega},gC.$$

$$\tfrac{1}{\eta}dz^{\theta\eta + \eta}Z^{\omega} = \overline{\theta + 1},eB + \overline{\theta + 1 + \omega},fC + \overline{\theta + 1 + 2\omega},gD.$$

Erunt

$$F = eA + fB + gC$$

$$G = eB + fC + gD.$$

Atque inde

$$A = \frac{+\overline{ff - 2eg},\tfrac{1}{\eta}dz^{\theta\eta}Z^{\omega} + fg,\tfrac{1}{\eta}dz^{\theta\eta + \eta}Z^{\omega} - \overline{\overline{\theta - \omega},ff + \overline{2\theta + 4\omega},eg},F - \overline{\theta - 1 - 2\omega},fgG}{\omega e,\overline{4eg - ff}}. \qquad B = \frac{-f,\tfrac{1}{\eta}dz^{\theta\eta}Z^{\omega} - 2g,\tfrac{1}{\eta}dz^{\theta\eta + \eta}Z^{\omega} + \theta fF + \overline{2\theta + 2 + 4\omega},gG}{\omega,\overline{4eg - ff}}.$$

$$C = \frac{+2e,\tfrac{1}{\eta}dz^{\theta\eta}Z^{\omega} + f,\tfrac{1}{\eta}dz^{\theta\eta + \eta}Z^{\omega} - 2\theta eF - \overline{\theta - 1 - 2\omega},fG}{\omega,\overline{4eg - ff}} \qquad D = \frac{-ef,\tfrac{1}{\eta}dz^{\theta\eta}Z^{\omega} + \overline{2eg - ff},\tfrac{1}{\eta}dz^{\theta\eta + \eta}Z^{\omega} + \theta efF + \overline{\overline{\theta + 1 + \omega},ff - \overline{2\theta - 2},eg},G}{\omega g,\overline{4eg - ff}}$$

THEOREMA IV.

Positis

$$Z = e + fz^{\eta} \qquad \dot{A} = d\dot{z}z^{\theta\eta - 1}Z^{\omega - 1}Y^{\psi - 1}X^{\chi - 1}$$
$$Y = g + hz^{\eta} \qquad \dot{B} = d\dot{z}z^{\theta\eta + \eta - 1}Z^{\omega - 1}Y^{\psi - 1}X^{\chi - 1}$$
$$X = k + lz^{\eta} \qquad \dot{C} = d\dot{z}z^{\theta\eta + 2\eta - 1}Z^{\omega - 1}Y^{\psi - 1}X^{\chi - 1}$$
$$\dot{D} = d\dot{z}z^{\theta\eta + 3\eta - 1}Z^{\omega - 1}Y^{\psi - 1}X^{\chi - 1}$$

$$\dot{F} = d\dot{z}z^{\theta\eta - 1}Z^{\omega}Y^{\psi - 1}X^{\chi - 1} \qquad \dot{I} = d\dot{z}z^{\theta\eta - 1}Z^{\omega - 1}Y^{\psi}X^{\chi - 1}$$
$$\dot{G} = d\dot{z}z^{\theta\eta + \eta - 1}Z^{\omega}Y^{\psi - 1}X^{\chi - 1} \qquad \dot{K} = d\dot{z}z^{\theta\eta + \eta - 1}Z^{\omega - 1}Y^{\psi}X^{\chi - 1}$$
$$\dot{H} = d\dot{z}z^{\theta\eta + 2\eta - 1}Z^{\omega}Y^{\psi - 1}X^{\chi - 1} \qquad \dot{L} = d\dot{z}z^{\theta\eta + 2\eta - 1}Z^{\omega - 1}Y^{\psi}X^{\chi - 1}$$

$$\dot{M} = d\dot{z}z^{\theta\eta - 1}Z^{\omega - 1}Y^{\psi - 1}X^{\chi}$$
$$\dot{N} = d\dot{z}z^{\theta\eta + \eta - 1}Z^{\omega - 1}Y^{\psi - 1}X^{\chi}$$
$$\dot{O} = d\dot{z}z^{\theta\eta + 2\eta - 1}Z^{\omega - 1}Y^{\psi - 1}X^{\chi}$$

Erit

$$\tfrac{1}{\eta}dz^{\theta\eta}Z^{\omega}Y^{\psi}X^{\chi} = \theta egk\,A$$
$$+\overline{\theta+\omega},fgk + \overline{\theta+\psi},ehk + \overline{\theta+\chi},egl,\,B$$
$$+\overline{\theta+\omega+\psi},fhk + \overline{\theta+\omega+\chi},fgl + \overline{\theta+\psi+\chi},ehl,\,C$$
$$+\overline{\theta+\omega+\psi+\chi},fhl\,D$$

Erunt

$$F = eA + fB. \qquad I = gA + hB. \qquad M = kA + lB.$$
$$G = eB + fC. \qquad K = gB + hC. \qquad N = kB + lC.$$
$$H = eC + fD. \qquad L = gC + hD. \qquad O = kC + lD.$$

Atque inde

$$A = \frac{\begin{array}{l} +ff, \tfrac{1}{n} d z^{\theta n} Z^{\omega} Y^{\psi} X^{\chi} \\ -\overline{\overline{\theta-\omega}, ffgk + \omega e, \overline{fhk+fgl-ehl}}, F \\ -\overline{\overline{\theta-\omega-\psi}, ffhk - \overline{\theta-\omega-\chi}, ffgl + \omega efhl}, G \\ -\overline{\theta-\omega-\psi-\chi}, ffhl H \end{array}}{\omega e, \overline{eh-fg}, \overline{fk-el}}$$

$$B = \frac{\begin{array}{l} -f, \tfrac{1}{n} d z^{\theta n} Z^{\omega} Y^{\psi} X^{\chi} \\ +\theta fgk F \\ \overline{-\omega ehl + \overline{\theta+\omega+\psi}, fhk + \overline{\theta+\omega+\chi}, fgl}, G \\ +\overline{\theta+\omega+\psi+\chi}, fhl H \end{array}}{\omega, \overline{eh-fg}, \overline{fk-el}}$$

$$C = \frac{\begin{array}{l} +e, \tfrac{1}{n} d z^{\theta n} Z^{\omega} Y^{\psi} X^{\chi} \\ -\theta egk F \\ \overline{-\omega fgk - \overline{\theta-\psi}, ehk - \overline{\theta-\chi}, egl}, G \\ -\overline{\theta-\omega-\psi-\chi}, ehl H \end{array}}{\omega, \overline{eh-fg}, \overline{fk-el}}$$

$$D = \frac{\begin{array}{l} -ee, \tfrac{1}{n} d z^{\theta n} Z^{\omega} Y^{\psi} X^{\chi} \\ +\theta eegk F \\ \overline{+\omega efgk + \overline{\theta+\psi}, eehk + \overline{\theta+\chi}, eegl}, G \\ \overline{+\omega f, \overline{ehk+egl-fgk} + \overline{\theta+\psi+\chi}, eehl}, H \end{array}}{\omega f, \overline{eh-fg}, \overline{fk-el}}$$

Vel

$$A = \frac{\begin{array}{l} +hh, \tfrac{1}{n} d z^{\theta n} Z^{\omega} Y^{\psi} X^{\chi} \\ -\overline{\theta-\psi}, ehhk + \psi g, \overline{fhk+ehl-fgl}, I \\ -\overline{\theta-\omega-\psi}, fhhk - \overline{\theta-\psi-\chi}, ehhl + \psi fghl, K \\ -\overline{\theta-\omega-\psi-\chi}, fhhl L \end{array}}{\psi g, \overline{fg-eh}, \overline{hk-gl}}$$

$$B = \frac{\begin{array}{l} -h, \tfrac{1}{n} d z^{\theta n} Z^{\omega} Y^{\psi} X^{\chi} \\ +\theta ehk I \\ -\psi fgl + \overline{\theta+\omega+\psi}, fhk + \overline{\theta+\psi+\chi}, ehl, K \\ +\overline{\theta+\omega+\psi+\chi}, fhl L \end{array}}{\psi, \overline{fg-eh}, \overline{hk-gl}}$$

$$C = \frac{\begin{array}{l} +g, \tfrac{1}{n} d z^{\theta n} Z^{\omega} Y^{\psi} X^{\chi} \\ -\theta egk I \\ -\psi ehk - \overline{\theta-\omega}, fgk - \overline{\theta-\chi}, egl, K \\ -\overline{\theta-\omega-\psi-\chi}, fgl L \end{array}}{\psi, \overline{fg-eh}, \overline{hk-gl}}$$

$$D = \frac{\begin{array}{l} -gg, \tfrac{1}{n} d z^{\theta n} Z^{\omega} Y^{\psi} X^{\chi} \\ +\theta eggk I \\ +\psi eghk + \overline{\theta+\omega}, fggk + \overline{\theta+\chi}, eggl, K \\ +\psi h, \overline{fgk+egl-ehk} + \overline{\theta+\omega+\chi}, fggl, L \end{array}}{\psi h, \overline{fg-eh}, \overline{hk-gl}}$$

Vel

Vel

$$A = \frac{\begin{array}{l} +ll, \tfrac{1}{n} d z^{\theta n} Z^{\omega} Y^{\psi} X^{\chi} \\ \overline{-\overline{\theta-\chi}, egll + \chi k, \overline{ehl+fgl-fhk}}, M \\ \overline{-\overline{\theta-\psi-\chi}, ehll - \overline{\theta-\omega-\chi}, fgll + \chi fhkl}, N \\ -\overline{\theta-\omega-\psi-\chi}, fhllO \end{array}}{\chi k, \overline{hk-gl}, \overline{el-fk}}$$

$$B = \frac{\begin{array}{l} -l, \tfrac{1}{n} d z^{\theta n} Z^{\omega} Y^{\psi} X^{\chi} \\ +\theta eglM \\ \overline{-\chi fhk + \overline{\theta+\psi+\chi}, ehl + \overline{\theta+\omega+\chi}, fgl}, N \\ +\overline{\theta+\omega+\psi+\chi}, fhlO \end{array}}{\chi, \overline{hk-gl}, \overline{el-fk}}$$

$$C = \frac{\begin{array}{l} +k, \tfrac{1}{n} d z^{\theta n} Z^{\omega} Y^{\psi} X^{\chi} \\ -\theta egkM \\ \overline{-\chi egl - \overline{\theta-\psi}, ehk - \overline{\theta-\omega}, fgk}, N \\ -\overline{\theta-\omega-\psi-\chi}, fhkO \end{array}}{\chi, \overline{hk-gl}, \overline{el-fk}}$$

$$D = \frac{\begin{array}{l} -kk, \tfrac{1}{n} d z^{\theta n} Z^{\omega} Y^{\psi} X^{\chi} \\ +\theta egkkM \\ \overline{+\chi egkl + \overline{\theta+\psi}, ehkk + \overline{\theta+\omega}, fgkk}, N \\ \overline{+\chi l, \overline{ehk+fgk-egl} + \overline{\theta+\omega+\psi}, fhkk}, O \end{array}}{\chi l, \overline{hk-gl}, \overline{el-fk}}$$

THEO-

THEOREMA V.

Positis

$Z = e + fz^{n} + gz^{2n}$

$Y = k + lz^{n}$

$\dot{A} = d\dot{z}z^{\theta n - 1} Z^{\omega - 1} Y^{\psi - 1}$

$\dot{B} = d\dot{z}z^{\theta n + n - 1} Z^{\omega - 1} Y^{\psi - 1}$

$\dot{C} = d\dot{z}z^{\theta n + 2n - 1} Z^{\omega - 1} Y^{\psi - 1}$

$\dot{D} = d\dot{z}z^{\theta n + 3n - 1} Z^{\omega - 1} Y^{\psi - 1}$

$\dot{E} = d\dot{z}z^{\theta n + 4n - 1} Z^{\omega - 1} Y^{\psi - 1}$

$\dot{F} = d\dot{z}z^{\theta n - 1} Z^{\omega} Y^{\psi - 1}$

$\dot{G} = d\dot{z}z^{\theta n + n - 1} Z^{\omega} Y^{\psi - 1}$

$\dot{H} = d\dot{z}z^{\theta n + 2n - 1} Z^{\omega} Y^{\psi - 1}$

$\dot{I} = d\dot{z}z^{\theta n - 1} Z^{\omega - 1} Y^{\psi}$

$\dot{K} = d\dot{z}z^{\theta n + n - 1} Z^{\omega - 1} Y^{\psi}$

$\dot{L} = d\dot{z}z^{\theta n + 2n - 1} Z^{\omega - 1} Y^{\psi}$

Erunt

$$\begin{aligned}\tfrac{1}{n} d z^{\theta n} Z^{\omega} Y^{\psi} = {} & \theta e k A \\ & + \overline{\overline{\theta + \omega},fk + \overline{\theta + \psi},el},B \\ & + \overline{\theta \overline{+ 2\omega},gk + \theta \overline{+ \omega + \psi},fl},C \\ & + \theta \overline{+ 2\omega + \psi},gl D.\end{aligned}$$

$$\begin{aligned}\tfrac{1}{n} d z^{\theta n + n} Z^{\omega} Y^{\psi} = {} & \overline{\theta + 1},ekB \\ & + \overline{\overline{\theta + 1 + \omega},fk + \overline{\theta + 1 + \psi},el},C \\ & + \overline{\overline{\theta + 1 + 2\omega},gk + \overline{\theta + 1 + \omega + \psi},fl},D \\ & + \overline{\theta + 1 + 2\omega + \psi},gl E\end{aligned}$$

Erunt

$F = eA + fB + gC$ $\qquad$ $I = kA + lB.$

$G = eB + fC + gD$ $\qquad$ $K = kB + lC.$

$H = eC + fD + gE$ $\qquad$ $L = kC + lD.$

Atque inde

$$
A = \frac{
\begin{array}{l}
+\overline{3efgl-2eggk+ffgk-f^{3}l},\tfrac{1}{n}dz^{\theta n}Z^{\omega}Y^{\psi} \\
+\overline{2eggl+fggk-ffgl},\tfrac{1}{n}dz^{\theta n+n}Z^{\omega}Y^{\psi} \\
+\overline{\overline{2\theta+4\omega},eggkk-\overline{3\theta-4\omega},efgkl+\overline{\theta+\omega},f^{3}kl-ffgkk+\omega,\overline{4eegll-effll}},F \\
+\overline{\overline{\theta+\omega+\psi},f^{3}ll-\overline{3\theta-4\omega-3\psi},efgll+\overline{1+\omega-\psi},ffgkl-\overline{2+2\psi},eggkl-\overline{\theta-1-2\omega},fggkk},G \\
+\overline{\theta+1+2\omega+\psi},\overline{ffgll-2eggll-fggkl},H
\end{array}
}{\omega e,\overline{4eg-ff},\overline{ell-fkl+gkk}}
$$

$$
B = \frac{
\begin{array}{l}
+\overline{ffl-fgk-2egl},\tfrac{1}{n}dz^{\theta n}Z^{\omega}Y^{\psi} \\
+\overline{fgl-2ggk},\tfrac{1}{n}dz^{\theta n+n}Z^{\omega}Y^{\psi} \\
+\theta,\overline{2egkl+fgkk-ffkl},F \\
+\overline{\overline{2\theta+4\omega+2\psi},egll-\overline{\theta-\omega-\psi},ffll-\overline{1-2\omega+\psi},fgkl+\overline{2\theta+2+4\omega},ggkk},G \\
+\overline{\theta+1+2\omega+\psi},\overline{2ggkl-fgll},H
\end{array}
}{\omega,\overline{4eg-ff},\overline{ell-fkl+gkk}}
$$

$$
C = \frac{
\begin{array}{l}
+\overline{2egk - efl}, \tfrac{1}{n} dz^{\theta n} Z^{\omega} Y^{\psi} \\
+\overline{fgk - 2egl}, \tfrac{1}{n} dz^{\theta n + n} Z^{\omega} Y^{\psi} \\
+\theta, \overline{efkl - 2egkk}, F \\
+\overline{\overline{\theta + \psi}, efll + \overline{2 - 2\psi}, egkl + \omega ffkl - \overline{\theta - 1 - 2\omega}, fgkk}, G \\
+\overline{\theta + 1 + 2\omega + \psi}, \overline{2egll - fgkl}, H
\end{array}
}{
\omega, \overline{4eg - ff}, \overline{ell - fkl + gkk}
}
$$

$$
D = \frac{
\begin{array}{l}
+\overline{2eel - efk}, \tfrac{1}{n} dz^{\theta n} Z^{\omega} Y^{\psi} \\
+\overline{2egk - ffk + efl}, \tfrac{1}{n} dz^{\theta n + n} Z^{\omega} Y^{\psi} \\
+\theta, \overline{efkk - 2eekl}, F \\
+\overline{\overline{\theta + 1 + \omega}, ffkk - \overline{2\theta - 2}, egkk - \overline{1 - 2\omega + \psi}, efkl - \overline{2\theta - 2\psi}, eell}, G \\
+\overline{\theta + 1 + 2\omega + \psi}, \overline{ffkl - 2egkl - efll}, H
\end{array}
}{
\omega, \overline{4eg - ff}, \overline{ell - fkl + gkk}
}
$$

$$
E = \frac{\begin{array}{l}
+\overline{effk - eefl - 2eegk}.\ \tfrac{1}{n}dz^{\theta n} Z^{\omega} Y^{\psi} \\
+\overline{2eegl - 3efgk - effl + f^3 k},\ \tfrac{1}{n}dz^{\theta n + n} Z^{\omega} Y^{\psi} \\
+\theta,\ \overline{2eegkk + eefkl - effkk},\ F \\
+\overline{\theta + \psi},\ eefll + \overline{1 + \omega + \psi},\ effkl - \overline{2 + 2\psi},\ eegkl + \overline{3\theta + 3 + 2\omega},\ efgkk - \overline{\theta - 1 - \omega},\ f^3 kk,\ G \\
+4\omega eggkk - \omega ffgkk + \overline{3\theta + 3 + 2\omega + 3\psi},\ efgkl - \overline{\theta - 1 - \omega - \psi}.\ f^3 kl + \overline{\theta + 1 + \omega + \psi},\ effll - \overline{2\theta - 2 - 2\psi},\ eegll,\ H
\end{array}}{\omega g,\ \overline{4eg - ff},\ \overline{ell - fk + gkk}}
$$

Vel

$$
A = \frac{\begin{array}{l}
-ll,\ \tfrac{1}{n}dz^{\theta n} Z^{\omega} Y^{\psi} \\
+\overline{\theta + \psi},\ \overline{ell - \psi fkl + \psi gkk},\ I \\
+\overline{\theta + \omega + \psi},\ \overline{fll - \psi gkl},\ K \\
+\overline{\theta + 2\omega + \psi},\ gll\,L
\end{array}}{\psi k,\ \overline{ell - fkl + gkk}}
\qquad
B = \frac{\begin{array}{l}
+l,\ \tfrac{1}{n}dz^{\theta n} Z^{\omega} Y^{\psi} \\
-\theta el\,I \\
-\overline{\theta - \omega - \psi},\ \overline{fl + \psi gk},\ K \\
-\overline{\theta - 2\omega - \psi},\ gl\,L
\end{array}}{\psi,\ \overline{ell - fkl + gkk}}
$$

$$
C = \frac{\begin{array}{l}
-k,\ \tfrac{1}{n}dz^{\theta n} Z^{\omega} Y^{\psi} \\
+\theta ek\,I \\
+\overline{\theta + \omega},\ \overline{fk - \theta el},\ K \\
+\overline{\theta + 2\omega + \psi},\ gk\,L
\end{array}}{\psi,\ \overline{ell - fkl + gkk}}
\qquad
D = \frac{\begin{array}{l}
+kk,\ \tfrac{1}{n}dz^{\theta n} Z^{\omega} Y^{\psi} \\
-\theta ekk\,I \\
-\overline{\theta - \omega},\ \overline{fkk - \psi ekl},\ K \\
-\overline{\theta - 2\omega},\ \overline{gkk - \psi fkl + \psi ell},\ L
\end{array}}{\psi l,\ \overline{ell - fkl + gkk}}
$$

ROGERI COTES
HARMONIA MENSURARUM.

PARS TERTIA.

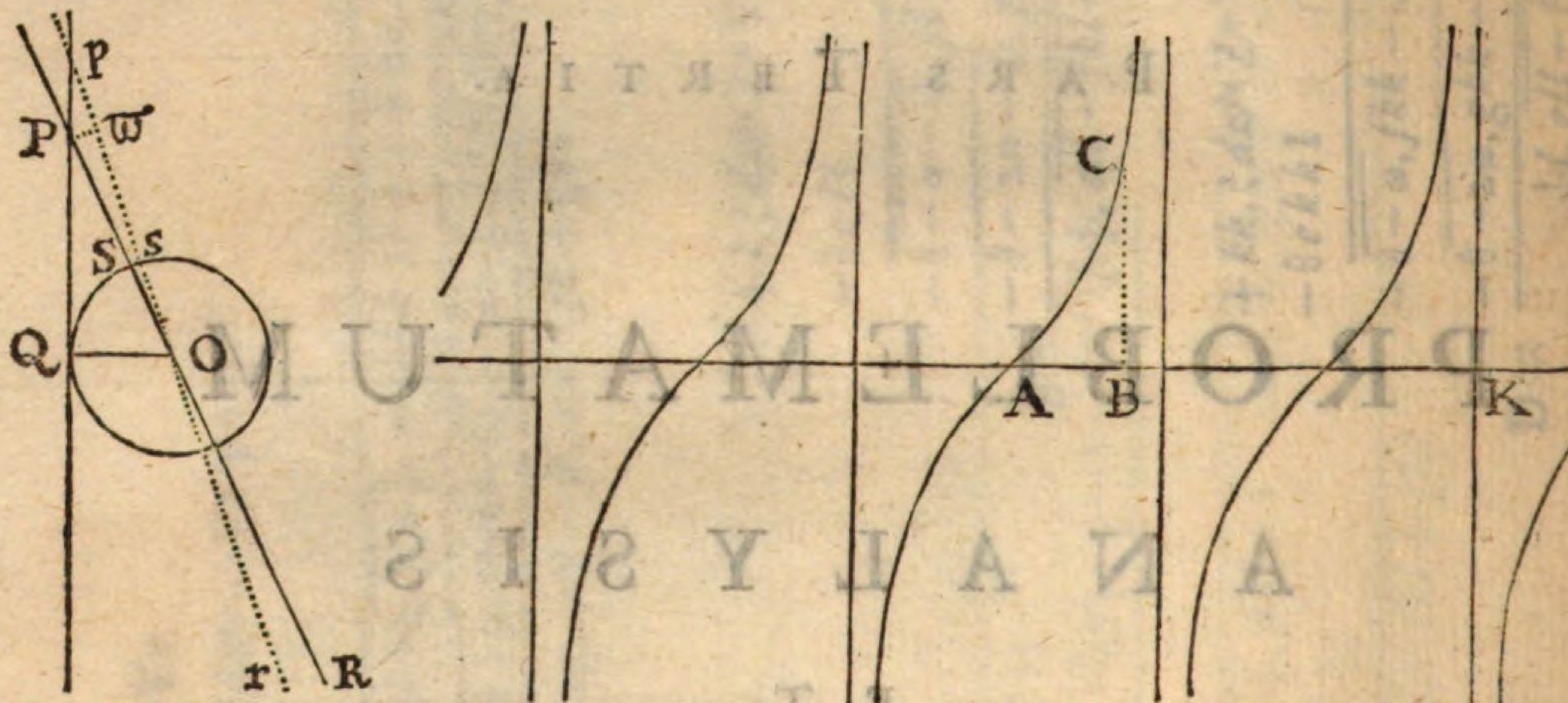
PROBLEMATUM
ANALYSIS
ET
CONSTRUCTIO
PER
FORMAS
PRÆCEDENTES.

PROBLE-MATA

PROBLEMA I.

Quadrare Figuram Tangentium.

CIRCULUM OQS deſcriptum O centro tangat indefinita recta PQ in puncto Q, ſecet vero tum circulum in S tum tangentem in P indefinita recta POR circa centrum volubilis. Deinde ſi in Baſi infinita AK detur punctum A, atque arcui QS ponatur ſemper æqualis AB recta, huic autem ad terminum ejus B inſiſtat Ordinata rectangula BC Tangenti QP æqualis: generabit hæc ordinata ductu ejus in baſem AK Figuram illam Tangentium quam quadrare volumus.



Vocentur itaque circuli radius OQ, r; arcus QS vel abſciſſa AB, x; tangens QP vel ordinata BC, y: & provoluta quam minimum $PSOR$ recta, punctiſque adeo P, S in ſitum novum p, s delatis, linearum x & y fluxiones $\dot{x}$ & $\dot{y}$ erunt ad invicem ut Ss ad Pp. Demiſſo autem perpendiculo $P\varpi$ in rectam Op, erit Ss ad Pp ut Ss ad $P\varpi$ & $P\varpi$ ad Pp conjunctim, hoc eſt (ob ſimilia triangula SOs & $PO\varpi$, ϖPp & QOP) ut OS ſive OQ ad OP & OQ ad OP conjunctim, id eſt ut OQq ad OPq. Unde $\dot{x}$ ad $\dot{y}$ ut rr ad $rr+yy$, ſive $\dot{x} = \dfrac{rr\dot{y}}{rr+yy}$ atque ita Fluxio Areæ ABC, nempe $\dot{x}y$, erit $\dfrac{rr\dot{y}y}{rr+yy}$.

Hæc

Hæc autem Fluxio ad FORMAM I referenda est. Nam pro z, η, θ, d, e, f scriptis respective y, 2, 1, rr, rr, 1, erit $\frac{d\dot{z}z^{\theta\eta-1}}{e+fz^{\eta}}$ PARS TERTIA.

$= \frac{rr\dot{y}y}{rr+yy}$, cujus Fluens (existente $\theta = 1$) erit $\frac{d}{\eta f}\left|\frac{e+fz^{\eta}}{e}\right. =$

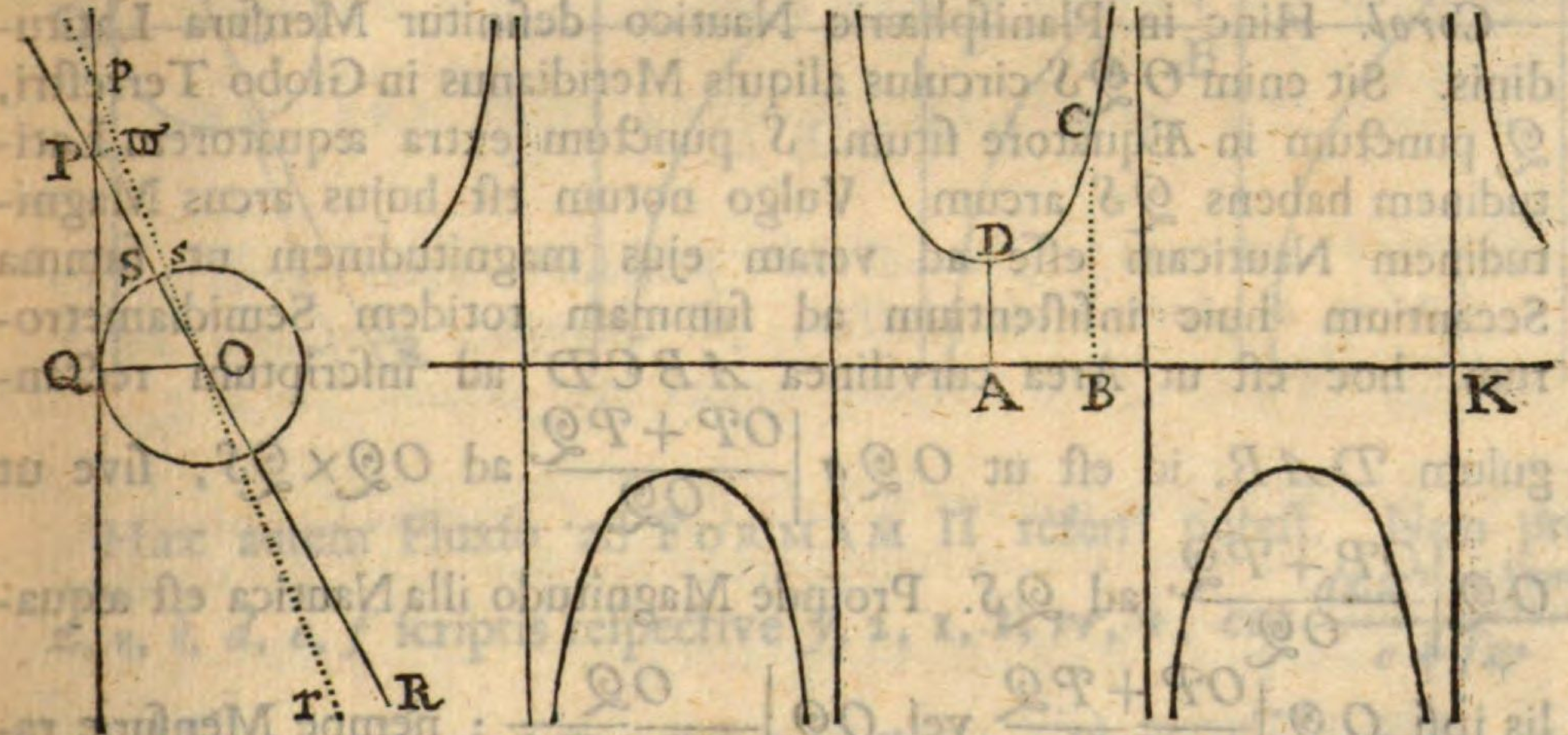
$\frac{1}{2}rr\left|\frac{rr+yy}{rr}\right. = \frac{1}{2}OQq\left|\frac{OPq}{OQq}\right. = OQq\left|\frac{OP}{OQ}\right.$ $= OQ^2 \log(\sec QS)$

Igitur Area istæc ABC, quæ basem habet AB æqualem arcui QS, æqualis est Mensuræ rationis inter Secantem ejusdem arcus atque Radium ad quadratum Radii pro Modulo.

PROBLEMA II.

Quadrare Figuram Secantium.

RELIQUIS manentibus quæ posita sunt in Problemate primo, sit nunc ordinata BC non Tangenti QP sed Secanti OP æqualis, & mutabitur Figura Tangentium in Figuram Secantium, cujus Aream $ABCD$ ordinatis AD, BC interjectam oportet invenire.



Itaque si vocentur OQ sive AD, r; AB sive QS, x; BC sive OP, y; erit $\dot{x}$ ad $\dot{y}$ ut Ss ad ϖp, hoc est ut Ss ad $P\varpi$ & $P\varpi$ ad ϖp conjunctim, sive ut OS vel OQ ad OP & OQ ad QP conjunctim, id est ut OQq ad $OP \times QP$, vel rr ad $y\sqrt{yy-rr}$.

Hinc

PROBLEMATA

Hinc vero $\dot{x} = \frac{rr\dot{y}}{y\sqrt{yy-rr}}$ atque adeo $\dot{x}y$ five Fluxio Areæ *ABCD* erit $\frac{rr\dot{y}}{\sqrt{yy-rr}}$.

Hæc autem Fluxio ad FORMAM VI referri poteſt. Nam pro z, η, θ, d, e, f ſcriptis reſpective $y, 2, 0, rr, -rr, 1,$ erit $\frac{d\dot{z}z^{\theta\eta+\frac{1}{2}\eta-1}}{\sqrt{e+fz^{\eta}}} = \frac{rr\dot{y}}{\sqrt{yy-rr}}$. Itaque cum in hac Forma ſit $R = \sqrt{f} = 1$, * $T = \frac{fz^{\frac{1}{2}\eta}}{\sqrt{e+fz^{\eta}}} = \frac{y}{\sqrt{yy-rr}}$, $S = \sqrt{\frac{-ef}{e+fz^{\eta}}} = \frac{r}{\sqrt{yy-rr}}$; erunt R, T, S ad invicem ut $\sqrt{yy-rr}, y, r$; hoc eſt ut *QP*, *OP*, *OQ*: atque inde Fluens (exiſtente $\theta = 0$) erit $\frac{\eta f}{2} dR \left| \frac{R+T}{S} \right. = OQq \left| \frac{OP+PQ}{OQ} \right.$.

Igitur Area iſtæc *ABCD*, quæ baſem habet *AB* æqualem arcui *QS*, æqualis eſt Menſuræ rationis inter ſummam Secantis & Tangentis ejuſdem arcus atque Radium ad quadratum Radii pro Modulo.

Corol. Hinc in Planiſphærio Nautico definitur Menſura Latitudinis. Sit enim *OQS* circulus aliquis Meridianus in Globo Terreſtri, *Q* punctum in Æquatore ſitum, *S* punctum extra æquatorem Latitudinem habens *QS* arcum. Vulgo notum eſt hujus arcus Magnitudinem Nauticam eſſe ad veram ejus magnitudinem ut ſumma Secantium huic inſiſtentium ad ſummam totidem Semidiametrorum, hoc eſt ut Area curvilinea *ABCD* ad inſcriptum rectangulum *DAB*, id eſt ut $OQq \left| \frac{OP+PQ}{OQ} \right.$ ad $OQ \times QS$, ſive ut $OQ \left| \frac{OP+PQ}{OQ} \right.$ ad *QS*. Proinde Magnitudo illa Nautica eſt æqualis ipſi $OQ \left| \frac{OP+PQ}{OQ} \right.$ vel $OQ \left| \frac{OQ}{OP-PQ} \right.$; nempe Menſuræ rationis inter Radium & Tangentem ſemiſſis complementi Latitudinis ad Semidiametrum Globi pro Modulo.

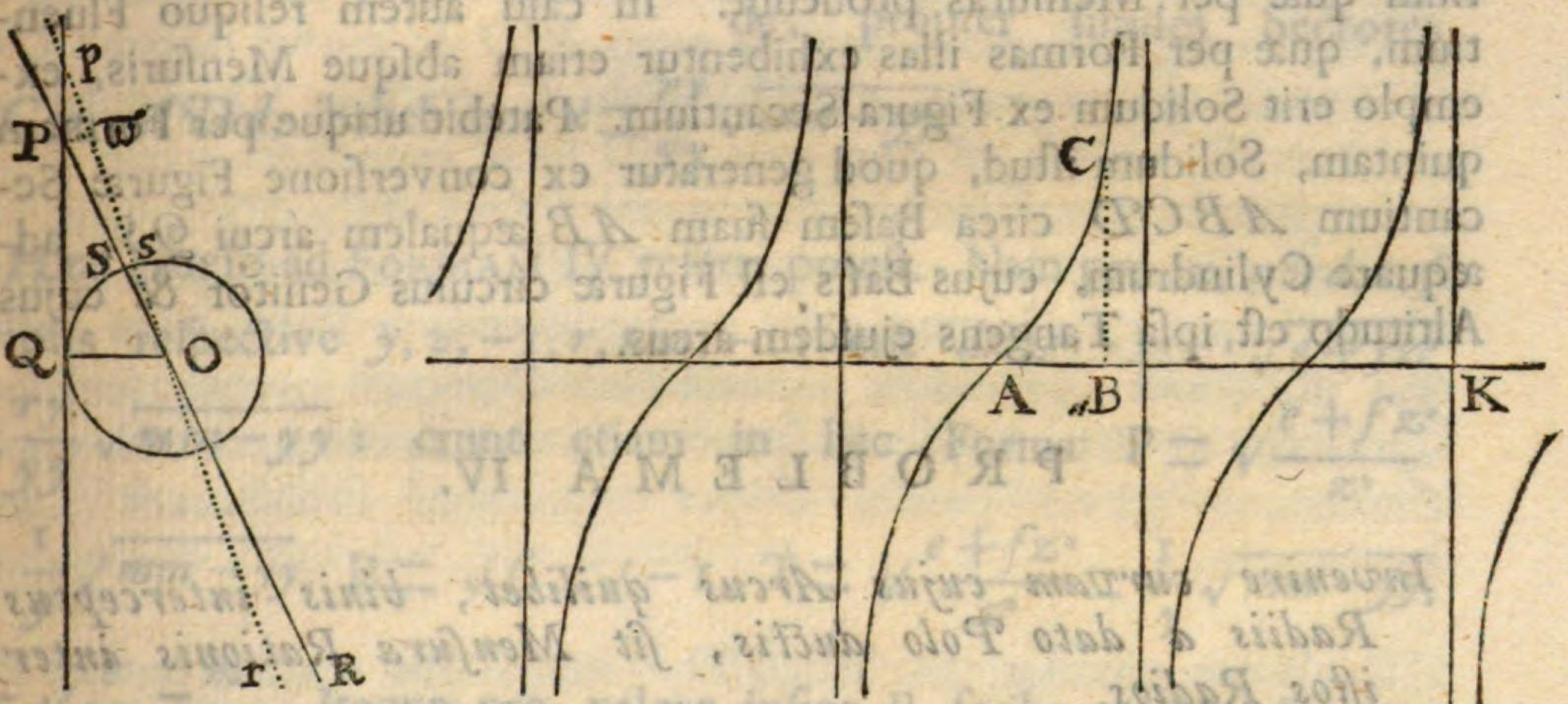
* *Vide pag.* 64.

PRO-

PROBLEMA III.

Solido ex Figuræ Tangentium converſione circa Baſem ſuam genito, æqualem Cylindrum invenire.

MANENTIBUS adhuc quæ poſita ſunt in Problemate primo, convertatur Figura ABC circa Baſem ſuam AB & Solidum inde genitum erit ut Fluens quantitatis $\dot{x}yy$ ſive $\frac{rr\dot{y}y^2}{rr+yy}$; erat enim ſupra $\dot{x} = \frac{rr\dot{y}}{rr+yy}$. Inde vero ſi Figuræ circulus Genitor OQS ponatur Baſis Cylindri quæſiti, Altitudo ejus æqualis erit Fluenti quantitatis $\frac{\dot{y}y^2}{rr+yy}$.



Hæc autem Fluxio ad FORMAM II referri poteſt. Nam pro z, η, θ, d, e, f ſcriptis reſpective y, 2, 1, 1, rr, 1; erit $\frac{dzz^{\theta\eta+\frac{1}{2}\eta-1}}{e+fz^{\eta}} = \frac{\dot{y}y^2}{rr+yy}$: erunt etiam in hac Forma $R = \sqrt{\frac{-e}{f}} = \sqrt{-rr}$, $T = z^{\frac{1}{2}\eta} = y$, $S = \sqrt{\frac{e+fz^{\eta}}{f}} = \sqrt{rr+yy}$. At vero propter ſignum negativum quantitatis $\sqrt{-rr}$, pro valores ipſius R ſcribo $\sqrt{+rr}$.

PROBLEMATA

Ex quo Fluxionis propositæ quantitas Fluens exhibenda est per Mensuram Anguli, cujus Radius, Tangens & Secans sunt ad invicem ut R, T, S, id est ut $r, y, \sqrt{rr+yy}$, vel OQ, QP, OP. Itaque Fluens hæcce (existente $\theta = 1$) erit $\frac{2dz^{\frac{1}{2}n}}{nf} - \frac{2d}{nf} R\left(\frac{R.T}{S}\right) = y - r(R:T:S::OQ:QP:OP) = QP - OQ$ (Ang. QOP) hoc est $= QP - QS$.

Proinde Solidum istud, quod generatur ex conversione Figuræ Tangentium *ABC* circa Basem suam *AB* æqualem arcui *QS*, æquale est Cylindro cujus Basis est Figuræ circulus Genitor & cujus Altitudo est excessus Tangentis ejusdem arcus supra Arcum ipsum.

Scholium.

Hactenus illustravimus usum FORMARUM per omnes casus Fluentium quæ per Mensuras prodeunt. In casu autem reliquo Fluentium, quæ per Formas illas exhibentur etiam absque Mensuris, exemplo erit Solidum ex Figura Secantium. Patebit utique per Formam quintam, Solidum istud, quod generatur ex conversione Figuræ Secantium *ABCD* circa Basem suam *AB* æqualem arcui *QS*, adæquare Cylindrum, cujus Basis est Figuræ circulus Genitor & cujus Altitudo est ipsa Tangens ejusdem arcus.

PROBLEMA IV.

Invenire curvam cujus Arcus quilibet, binis interceptus Radiis a dato Polo ductis, sit Mensura Rationis inter istos Radios.

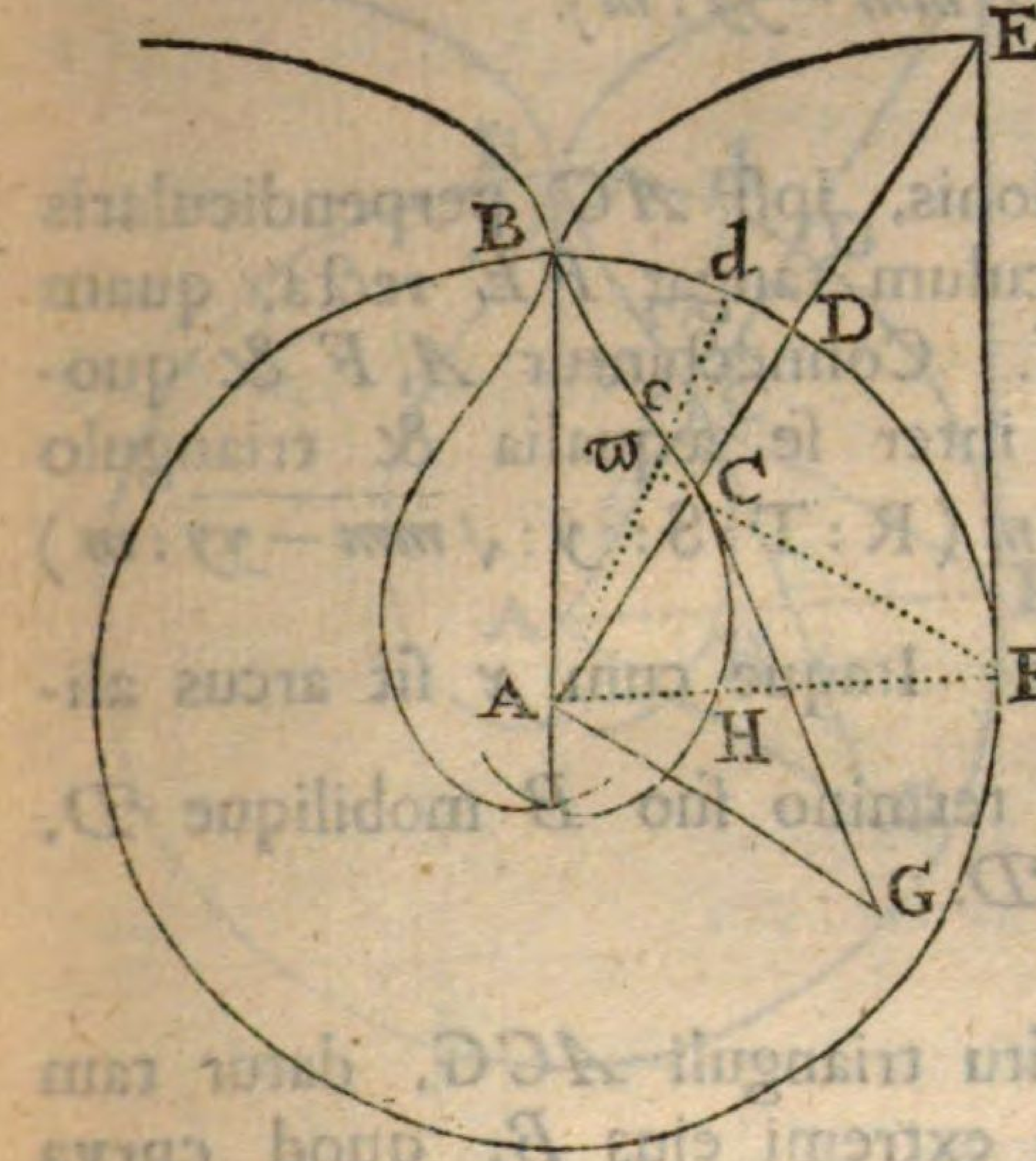
ESTO *AC* radius quilibet a dato Polo *A* eductus: oportet invenire curvam *BCH* cujus arcus quilibet *CH* sit mensura rationis inter *AC* & *AH*.

Si detur positione *CG* recta, quæ tangat curvam in puncto *C*, & fiat angulus *CAG* rectus: tangens illa *CG* erit Mensurarum Modulus, per Cor. 2 Prop. I LOGOMETRIÆ, adeoque ejusdem ubique magnitudinis. Nam provoluto quam minimum radio *AC* in situm novum *Ac* & in hanc demisso perpendiculo $C\varpi$: propter

similia

ſimilia triangula CAG, $c\varpi C$, tangens illa CG erit ad arcum quam minimum cC, menſuram ſcilicet rationis inter Ac & AC (ex natur a Problematis), ut terminus AC ad terminorum differentiam ϖc.

PARS TERTIA.



Centro A & intervallo quovis dato AD deſcribatur circulus BDF, qui Radiis AC, Ac ſi opus productis occurrat in D & d: & ſi vocentur CG, m; AC, y; ϖc, $\dot{y}$; AD, r; Dd, $\dot{x}$; erit $AG = \sqrt{mm-yy}$ adeoque $C\varpi = \frac{\dot{y}}{y}\sqrt{mm-yy}$. Atque inde, propter ſimiles Sectores $AC\varpi$, ADd, habebimus $\dot{x} = \frac{r\dot{y}}{yy}\sqrt{mm-yy}$.

Hæc Fluxio ad FORMAM IV referri poteſt. Nam pro z, η, θ, d, e, f ſcriptis reſpective $y, 2, -1, r, mm, -1$; erit $d\dot{z}z^{\theta\eta+\frac{1}{2}\eta-1}\sqrt{e+fz^{\eta}} = \frac{r\dot{y}}{yy}\sqrt{mm-yy}$: erunt etiam in hac Forma $P = \sqrt{\frac{e+fz^{\eta}}{z^{\eta}}} = \frac{1}{y}\sqrt{mm-yy}$, $R = \sqrt{f} = \sqrt{-1}$, $T = \sqrt{\frac{e+fz^{\eta}}{z^{\eta}}} = \frac{1}{y}\sqrt{mm-yy}$, $S = \sqrt{\frac{e}{z^{\eta}}} = \frac{m}{y}$. Itaque pro valore ipſius R ſcribo $\sqrt{+1}$. Ex quo habebimus Angulum menſurandum, cujus R, T, S erunt ad invicem ut y, $\sqrt{mm-yy}$, m. Adeoque Fluens (exiſtente $\theta = -1$) erit $\frac{-2}{\eta}dP + \frac{2}{\eta}dR\left(\frac{R.T}{S}\right) = \frac{-r}{y}\sqrt{mm-yy} + r(R:T:S::y:\sqrt{mm-yy}:m) = x$. Ubi cum quantitas x fluendo diminuitur dum y augetur, & viciſſim; æquatio illa correcta erit $x = +\frac{r}{y}\sqrt{mm-yy} - r(R:T:S::y:\sqrt{mm-yy}:m)$. Circuli radius r pro arbitrio ſumi poteſt.

PROBLE-MATA potest. Ponatur itaque $r = m$ & æquatio paullo simplicior evadet

$$x = \frac{m}{y}\sqrt{mm - yy} - m\,(\mathrm{R:T:S} :: y : \sqrt{mm - yy} : m).$$

Ad constructionem hujus æquationis, ipsi *AC* perpendicularis erigatur *AF* & ad punctum *F* circulum tangat *FE* recta, quam radius *ACD* productus secet in *E*. Connectantur *A*, *F* & quoniam triangula *CAG*, *ACF* sunt inter se æqualia & triangulo *AFE* similia, erit arcus $FD = m\,(\mathrm{R:T:S} :: y : \sqrt{mm - yy} : m)$ & ejus tangens $FE = \frac{m}{y}\sqrt{mm - yy}$. Itaque cum x sit arcus aliquis *BD* qui intercipitur immobili termino suo *B* mobilique *D*, habebimus tandem $BD = FE - FD$.

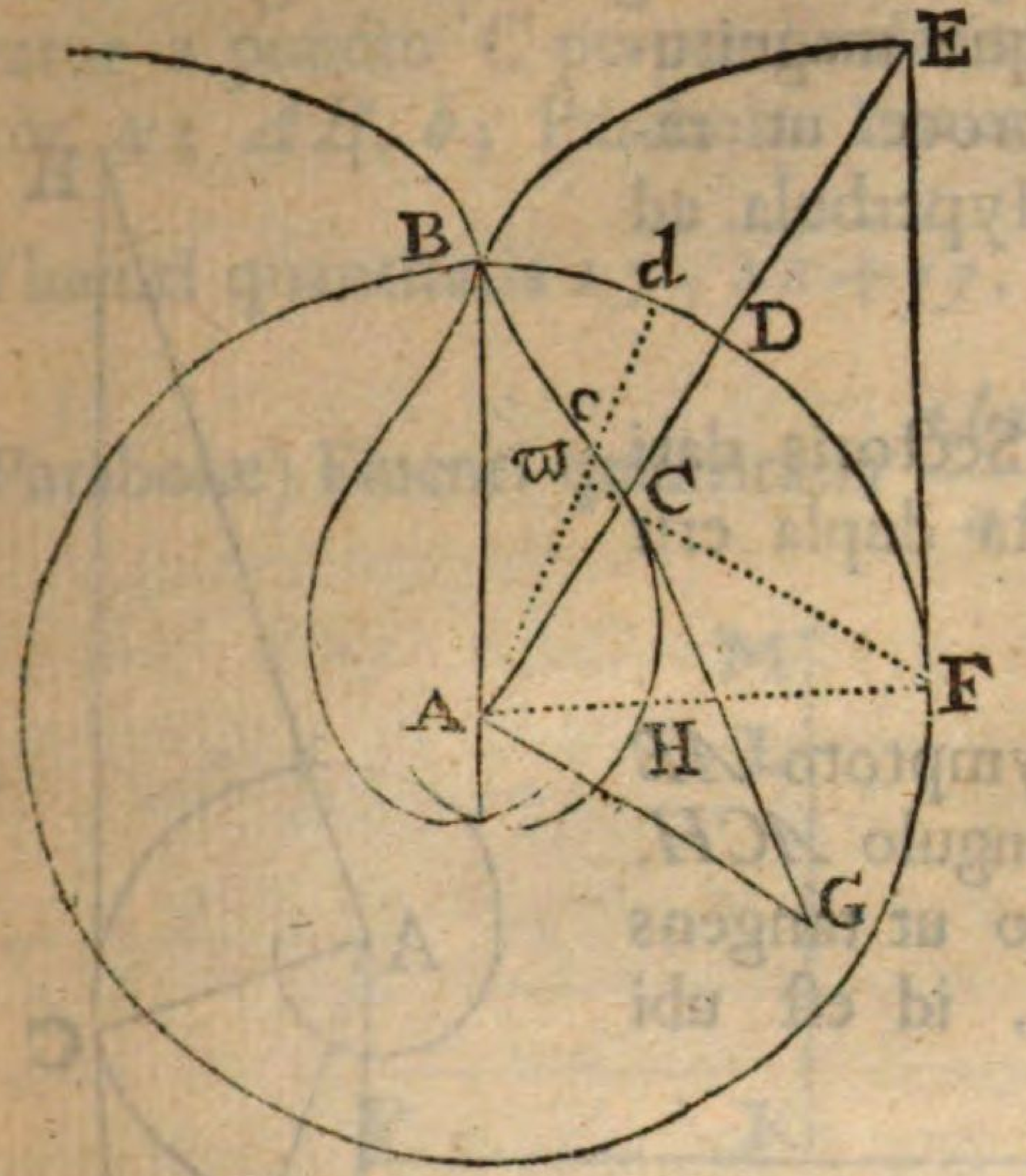
Ex hac æquatione & ex dato situ trianguli *ACG*, datur tam magnitudo arcus *BD* quam positio extremi ejus *B*, quod curva quæsita attinget si modo jaceant arcus *BD* & curvæ tangens *CG* ad diversas partes radii *ACD*. Nam si punctum *D* accedat ad punctum *B*, necesse est cum arcu *BD* simul evanescere arcum *FD* & tangentem ejus *FE*, ideoque æquales fieri *AC*, *AD* atque ita punctum *C* coincidere cum punctis *D*, *B*. Perveniet igitur curva ad punctum illud *B*, atqui non perget ulterius a Polo *A* ne quantitas illa $\sqrt{mm - yy}$ evadat impossibilis. Denique invento semel ut supra, vel ex dato imprimis, puncto *B*, deinde dabuntur alia quotlibetcunque curvæ puncta *C* per sequentem Problematis Constructionem.

Evolvatur circulus quilibet *BDF* incipiendo a dato puncto *B*, ita ut *FE* recta sit semper æqualis arcui *FB* & sit *BE* curva hujusmodi Evolutione descripta. Hanc secet in *E* productus quilibet radius *AD*, in quo ipsis *AE*, *AD* sumatur tertia proportionalis *AC*: erit punctum *C* ad Spiralem *BCH* quæ est Complicata * *Tractrix*. Nempe hujus Tangens *CG* erit ubique æqualis Radio *AB*, & hujus Arcus quilibet *CH* erit Mensura Rationis inter *AC* & *AH* ad Modulum *AB*.

* De Simplici *Tractrice* vide N. 345. Phil. Tran. Reg. Soc. Lond. & Mem. Ann. 1715. Acad. Reg. Sci. Par.

Notan-

PARS TERTIA.



Notandum eſt, ſi tangens ad *C* ſit parallela Radio *AB*, erit ea magnitudo anguli *BAC* ſive *ACG* ut tangens ejus ſit arcui ejus duplo æqualis, adeoq; inter *66* & *67* gradus. Quod ſi deſideretur hujus Spiralis Area *BCA*, hæc ſtatim prodibit per eandem Formam quartam, &, connexis *C*, *F*, æqualis erit Areæ circulari dimidiatæ *CDF*.

Scholium.

Non abſimili ratione complures curvæ Logometricæ facillime inveniri poſſunt, ex data ſcilicet Menſurarum relatione ad curvas inveniendas. Hujus generis alteram hic adjungam Spiralem, quam *Litui* Figuram appello propter formæ ſimilitudinem. Hæc vero confeſtim ſeſe obviam dabit quærenti curvam cujus Area, binis intercepta Radiis a dato Polo eductis, ſit Menſura rationis inter iſtos radios; ejuſque proprietates erunt hæ quæ ſequuntur.

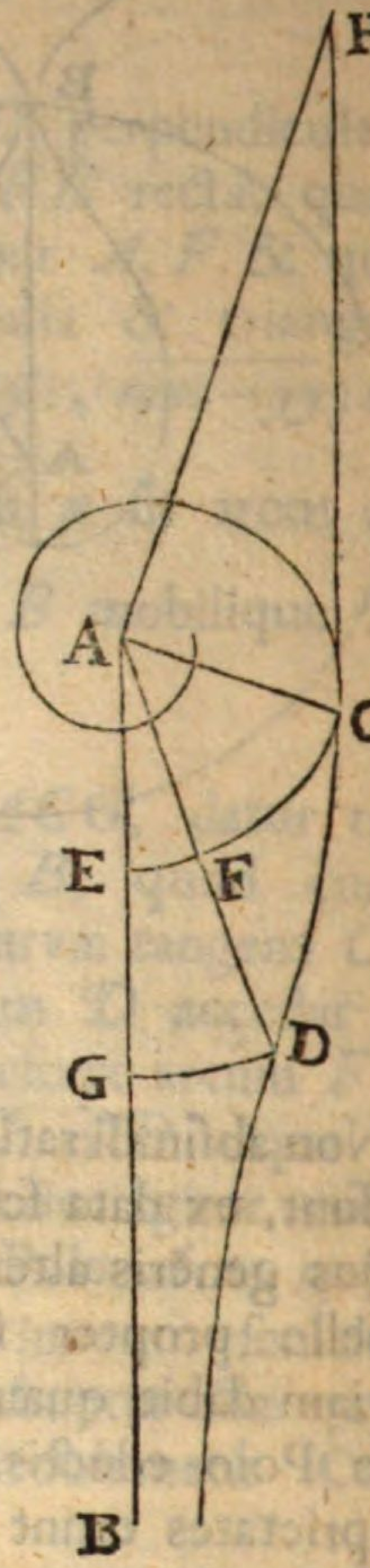
Eſto *A* Polus; *AB* Radius primus & Aſymptotos; *AC*, *AD* bini quilibet radii Spiralem ſecantes in *C* & *D*. His intervallis & centro *A* deſcribantur arcus *CE*, *DG* aſymptoto occurrentes in *E* & *G*, arcus autem *CE* radium *AD* ſecet in *F*.

Erunt anguli *BAD* reciproce ut Quadrata Radiorum *AD*: atque inde $EF \times ADq = EC \times ACq$.

Sectores *AEC*, *AGD* erunt ubique æquales: & arcus *DG* ubique ut radius *AD* reciproce.

PROBLEMATA

Triangulum ACH inter radium AC, tangentem CH & subtangentem AH erit ejusdem ubique magnitudinis: & subtangens AH reciproce ut radius AC. Quemadmodum in Hyperbola ad Asymptoton.



Idem triangulum datum ACH Sectoris dati AEC duplum erit: & AH recta dupla erit Arcus EC.

Tangens CH parallela erit Asymptoto AB ubi angulus BAC fuerit æqualis angulo ACH, nempe ubi ea est anguli magnitudo ut tangens ejus sit arcui ejus duplo æqualis, id est ubi angulus est inter *66* & *67* gradus.

Denique quod præcipuum est. Area CAD est Mensura Rationis inter AD & AC ad Modulum qui sit Triangulo ACH vel duplicato Sectori ADG æqualis. Et quoniam istæc area CAD est ubique æqualis areæ $CDGE$; erit hæc Area Mensura Rationis ejusdem inter AG & AE vel inter arcus EC & GD ad Modulum supradictum. Qua proprietate refert Hyperbolam ad Asymptoton. Quæ Hyperbola ita se habet ad hanc * Figuram ut Parabola ad Spiralem *Archimedis* utque Logistica ad Spiralem Reciprocam.

PROBLEMA V.

Superficiei genitæ ex arcus Parabolici conversione, circa rectam quamvis axi Parabolæ perpendicularem, æqualem Circulum invenire.

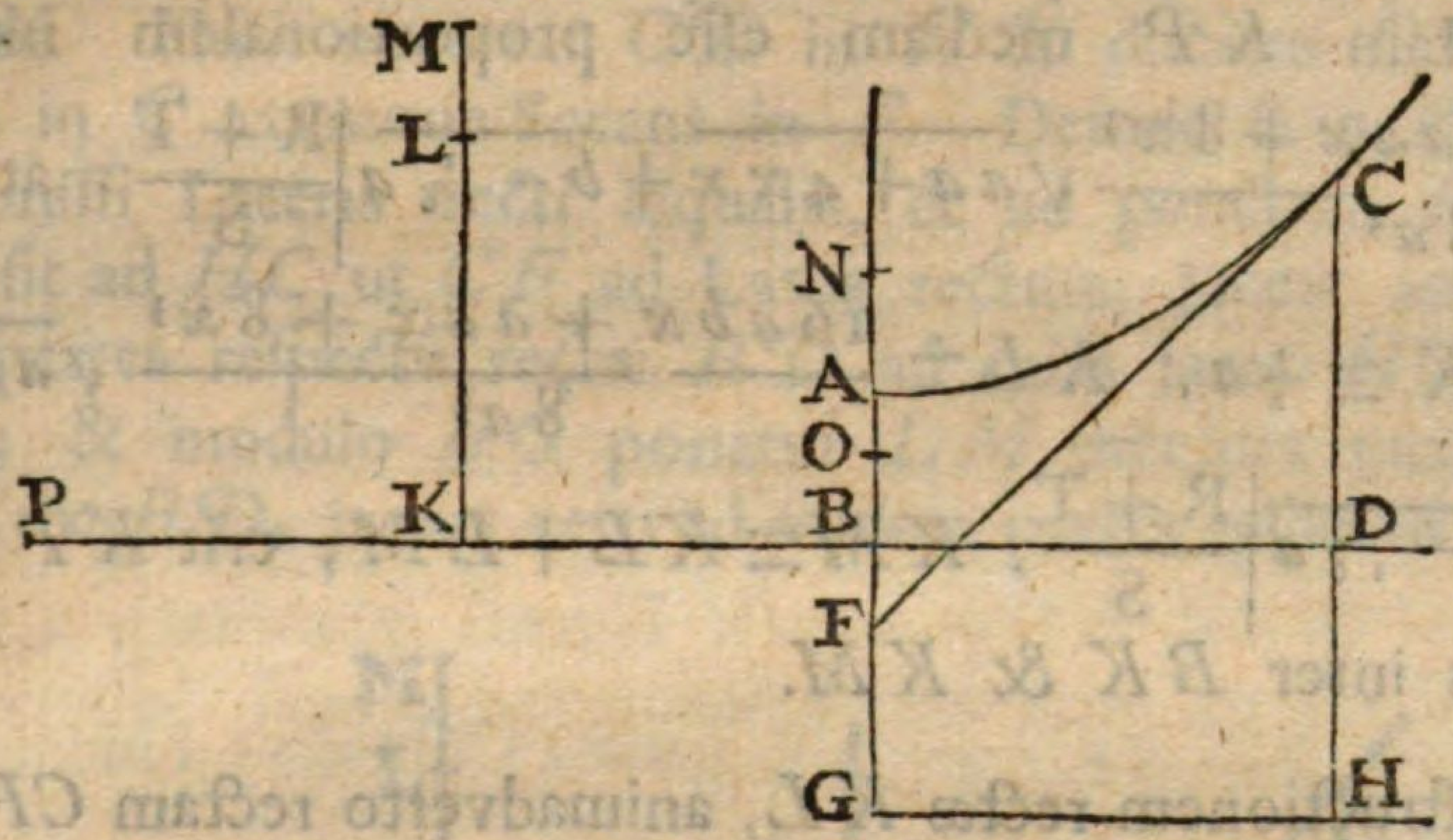
SIT AB axis Parabolæ, A vertex, AC arcus oblatus, BD recta ad axem normalis circa quam convertitur arcus AC: oportet invenire KP radium circuli, cujus area superficiem ex ista conversione genitam adæquet.

* Cl. *Varignonius* de Spiralibus Mem. An. 1704. Acad. Reg. Sci. Par. vocat hanc Figuram Spiralem Hyperbolicam concentricam, quæ ibi prodit ponendo $m = 2$.

Esto

Eſto igitur extra Parabolam poſita recta BD, & in eam demittatur a puncto C perpendicularis CD, quæ vocetur y; BD vocetur x; AB, b; Parabolæ Latus rectum, a: & erit KPq æquale Pars Tertia.

Fluenti quantitatis $2y\sqrt{\dot{x}\dot{x}+\dot{y}\dot{y}}$, vel (propter $y = b + \frac{xx}{a}$ ex natura Parabolæ) Fluenti quantitatis $\frac{2b\dot{x}}{a}\sqrt{aa+4xx} + \frac{2\dot{x}x^2}{aa}\sqrt{aa+4xx}$.



Hujus quantitatis pars utraque pertinet ad Formam IV, poſito ſcilicet $\theta = 0$ pro parte priore, & $\theta = 1$ pro parte poſteriore.

Nam pro z, η, d, e, f ſcriptis reſpective x, 2, $\frac{2b}{a}$, aa, 4; fiet $d\dot{z}z^{\frac{1}{2}\eta-1}\sqrt{e+fz^\eta} = \frac{2b\dot{x}}{a}\sqrt{aa+4xx}$. Hujus autem Fluens $\frac{z^\eta}{\eta}dP + \frac{e}{\eta f}dR \,\Big|\, \frac{R+T}{S}$ evadet $\frac{bx}{a}\sqrt{aa+4xx} + \frac{1}{2}ab \,\Big|\, \frac{R+T}{S}$; pro P & R ſive $\sqrt{\frac{e+fz^\eta}{z^\eta}}$ & $\sqrt{f}$ ſcribendo inſuper $\frac{1}{x}\sqrt{aa+4xx}$ & 2.

Similiter pro z, η, d, e, f, ſcriptis x, 2, $\frac{2}{aa}$, aa, 4; fiet $d\dot{z}z^{\frac{3}{2}\eta-1}\sqrt{e+fz^\eta} = \frac{2\dot{x}x^2}{aa}\sqrt{aa+4xx}$. Hujus autem Fluens $\frac{ez^\eta+2fz^{2\eta}}{4\eta f}dP - \frac{ee}{4\eta ff}dR \,\Big|\, \frac{R+T}{S}$ evadet $\frac{aax+8x^3}{16aa}\sqrt{aa+4xx}$
$-\frac{1}{32}aa$

PROBLEMATA $-\frac{1}{32}aa\Big|\frac{R+T}{S}$. Addantur partium Fluentes modo repertæ, & integra Fluens fiet $\frac{16abx+aax+8x^3}{16aa}\sqrt{aa+4xx}$ $+\overline{\frac{1}{2}ab-\frac{1}{32}aa}\Big|\frac{R+T}{S}=K\mathcal{P}q.$

Jam ut iſtæc Æquatio commodiſſime conſtruatur; animadverto rectam $K\mathcal{P}$ mediam eſſe proportionalem inter $\frac{1}{2}a$ & $\frac{16abx+aax+8x^3}{8a^3}\sqrt{aa+4xx}+\overline{b-\frac{1}{16}a}\Big|\frac{R+T}{S}$: atque adeo ſi fuerit $BK=\frac{1}{2}a$, $KL=\frac{16abx+aax+8x^3}{8a^3}\sqrt{aa+4xx}$, $LM=\overline{b-\frac{1}{16}a}\Big|\frac{R+T}{S}$, $KM=KL+LM$; erit $K\mathcal{P}$ media proportionalis inter BK & KM.

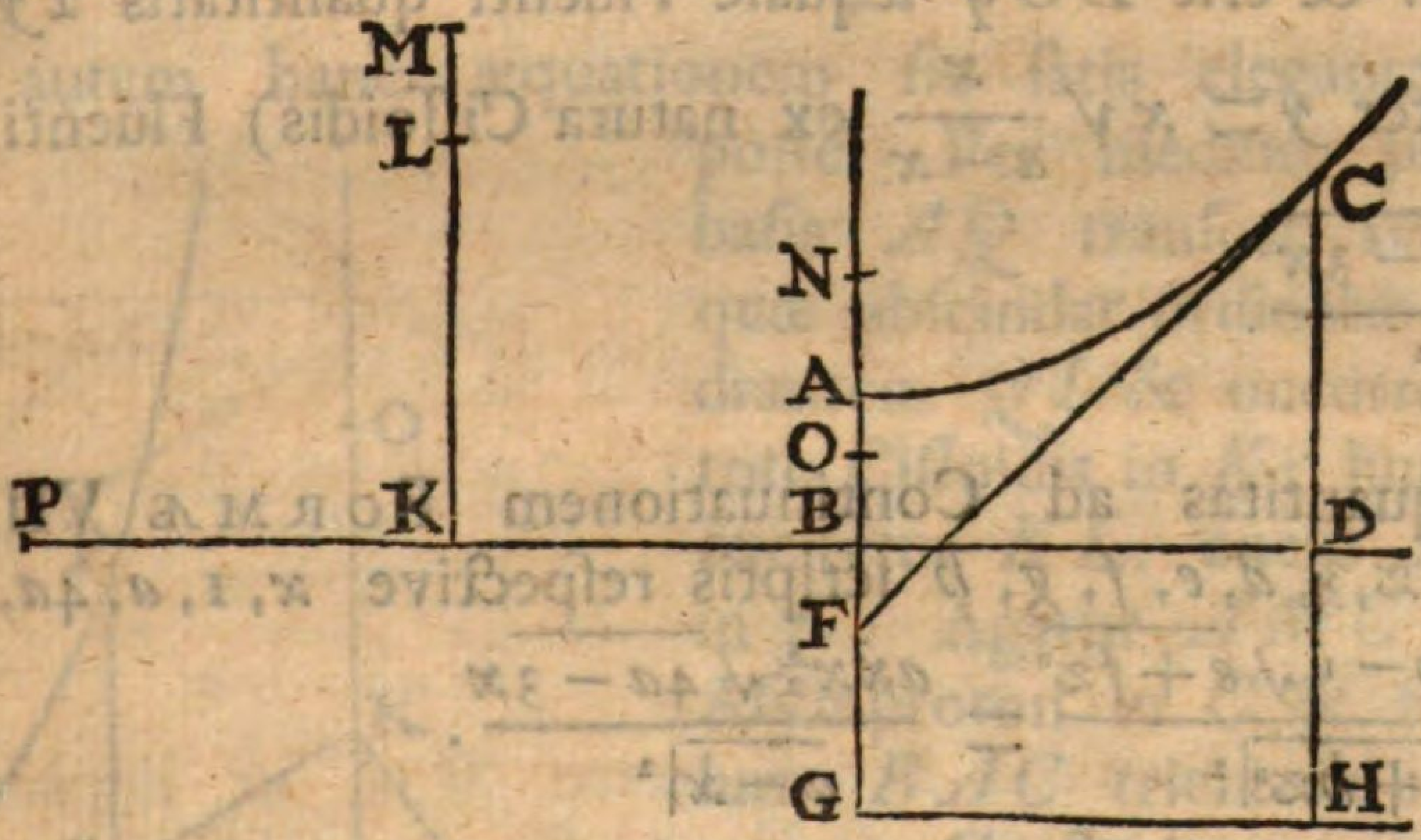
Ad conſtructionem rectæ KL, animadverto rectam CF quæ parabolam tangit in C & axem ejus ſecat in F, æqualem eſſe quantitati $\frac{x}{a}\sqrt{aa+4xx}$. Itaq; $KL=\frac{16ab+aa+8xx}{8aa}\times CF=\frac{2b+\frac{1}{8}a+\frac{xx}{a}}{a}\times CF$ $=\frac{y+b+\frac{1}{8}a}{a}\times CF$. Unde ſi rectæ $C\mathcal{D}$ adjiciatur $\mathcal{D}H=b+\frac{1}{8}a$, erit $HC=y+b+\frac{1}{8}a$, & $KL=\frac{HC\times CF}{a}$: atque adeo KL erit ad HC ut CF ad Latus rectum Parabolæ.

Ad conſtructionem rectæ LM, inquirendum eſt quænam ſit ratio inter R+T & S. Hanc vero eam eſſe animadverto quæ eſt inter $2AF+FC$ & $B\mathcal{D}$. Nam erat ſupra $R=2$, eſt autem in hac Forma $T=\sqrt{\frac{e+fz^n}{z^n}}=\frac{1}{x}\sqrt{aa+4xx}$, $S=\sqrt{\frac{e}{z^n}}=\frac{a}{x}$. Unde R, T, S ſunt ad invicem $2x$, $\sqrt{aa+4xx}$, a, ſive ut $\frac{2xx}{a}$, $\frac{x}{a}\sqrt{aa+4xx}$, x, hoc eſt ut $2AF$, FC, $B\mathcal{D}$. Ergo ſi ponatur $BO=b-\frac{1}{16}a$, erit LM menſura rationis inter $2AF+FC$ & $B\mathcal{D}$ ad Modulum BO.

Sic

Sic itaque Problematis finita est Analysis pro casu illo ubi recta *BD* tota jacet extra Parabolam. Hinc vero deducuntur casus omnes reliqui, mutando tantum signa terminorum ubi opus fuerit, & lineamenta ducendo in plagas contrarias: Unde tandem generalis illa quæ sequitur Constructio emergit.

In axe *AB* sumatur intra Parabolam, *AN* pars octava Lateris recti; extra Parabolam, *AO* pars decima sexta Lateris recti: agatur *GH* parallela rectæ *BD* ad intervallum *BG* quod æquale sit intervallo *BN*. Huic occurrat *CD* in *H* & ducatur *CF* parabolam tangens in *C* axemque Secans in *F*. Deinde si in *BD* capiatur *BK* semissi Lateris recti æqualis, & ad punctum *K* erigatur *KL*, quæ sit ad *HC* ut *CF* ad Latus rectum, jaceat autem *KL* ad easdem partes respectu rectæ *BD* ad quas jacet *HC* respectu rectæ *GH*; & modulo *BO* ponatur *LM* mensura rationis inter $2AF+FC$ & *BD*, jaceat vero ad easdem partes respectu puncti *L*



ad quas jacet *BO* respectu puncti *B*; atque inter *KM* & *KB* ponatur media proportionalis *KP*: Hæc erit radius Circuli cujus area superficiem Parabolicam adæquat; vel erit superficies Parabolica ad Circulum Semidiametro *KB* descriptum ut *KM* ad *KB*.

Notandum tamen, si recta *BD* secet arcum Parabolicum inter *A* & *C*, & in bina Segmenta dividat; in hoc casu circulum radio *KP* descriptum æqualem fore non summæ superficierum a segmentis genitarum, sed earundem differentiæ; & excessus erit penes illius segmenti superficiem quod ad rectæ *BD* partes easdem cadit atque recta *KM*.

Nec indignum erit observatione, si punctum B incidat in punctum O; nempe si recta BD ita sit extra parabolam posita ut ab ejus vertice distet parte decima sexta Lateris recti, constructionem in hoc casu simplicissimam evadere, evanescentibus simul Modulo BO & Mensura LM.

PROBLEMA VI.

Superficiei genitæ ex arcus Cissoidalis conversione circum axem ejus, æqualem Circulum invenire.

SIT A vertex Cissoidis, AB axis ejus vel diameter Circuli Genitoris circa quam rotetur arcus oblatus AP: oportet invenire BO radium Circuli cujus area superficiem ex ista rotatione genitam adæquet.

Ad axem AQB normalis esto PQ, quæ vocetur y; AQ vocetur x, AB, a: & erit BOq æquale Fluenti quantitatis $2y\sqrt{\dot{x}\dot{x}+\dot{y}\dot{y}}$, vel (propter $y = x\sqrt{\frac{x}{a-x}}$ ex natura Cissoidis) Fluenti quantitatis $\frac{a\dot{x}x^{\frac{1}{2}}\sqrt{4a-3x}}{\overline{a-x}|^2}$.

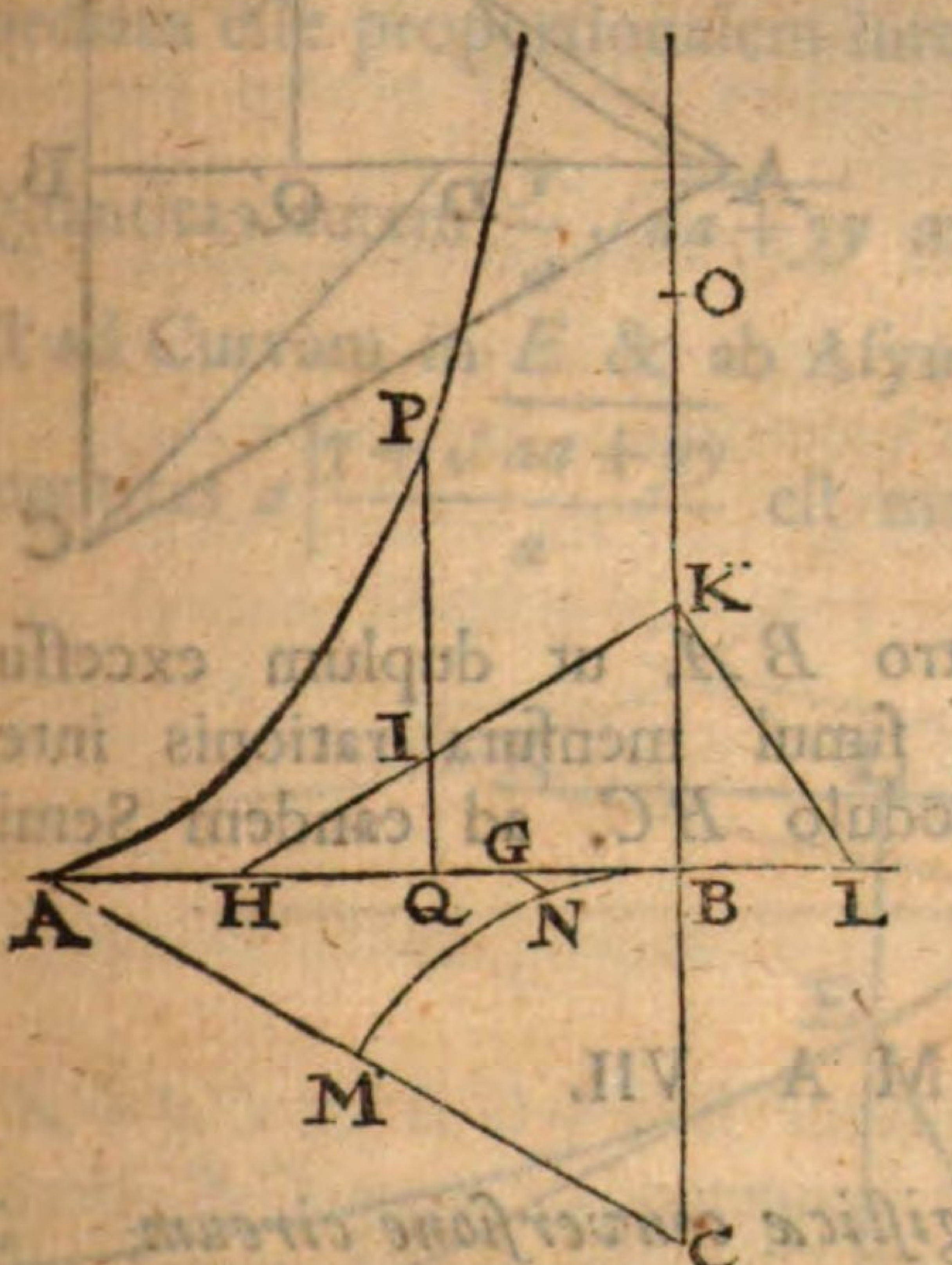
Hæc quantitas ad Continuationem FORMÆ VIII pertinet. Nam pro z, η, d, e, f, g, h scriptis respective $x, 1, a, 4a, -3, a, -1$; fiet $\frac{d\dot{z}z^{\frac{1}{2}\eta-1}\sqrt{e+fz^{\eta}}}{\overline{g+hz^{\eta}}|^2} = \frac{a\dot{x}x^{\frac{1}{2}}\sqrt{4a-3x}}{\overline{a-x}|^2}$.

Jam vero dantur Fluentes quantitatum $\frac{d\dot{z}z^{\frac{1}{2}\eta-1}\sqrt{e+fz^{\eta}}}{g+hz^{\eta}}$ & $\frac{d\dot{z}z^{\frac{1}{2}\eta-1}\sqrt{e+fz^{\eta}}}{g+hz^{\eta}}$; & ex hisce Fluentibus elici poterit Fluens quantitatis $\frac{d\dot{z}z^{\frac{1}{2}\eta-1}\sqrt{e+fz^{\eta}}}{\overline{g+hz^{\eta}}|^2}$ per THEOREMA II DE CONTIN. Utique posito calculo invenio hunc ejus valorem qui sequitur $\frac{2gz^{\eta}+hz^{2\eta}}{\eta hh, \overline{g+hz^{\eta}}} dP$ $+ \frac{3egh-4fgg}{\eta h^3, \overline{eh-fg}} dR \left| \frac{R+T}{S} \right. + \frac{eehh-5efgh+4ffgg}{\eta fh^3, \overline{eh-fg}} dr \left| \frac{r+t}{s} \right.$.

Facta

Facta igitur debita notarum ſubſtitutione, P prodit $\sqrt{\frac{4a-3x}{x}}$; r, t, s prodeunt reſpective * $\sqrt{-3}$, $-3\sqrt{\frac{x}{4a-3x}}$, $\sqrt{\frac{12a}{4a-3x}}$; pro quibus ſcribo $\sqrt{3}$, $3\sqrt{\frac{x}{4a-3x}}$, $\sqrt{\frac{12a}{4a-3x}}$; mutata rationis menſura in menſuram anguli, propter ſignum negativum quantitatis prodeuntis $\sqrt{-3}$; & converſo inſuper hujus menſuræ ſigno in contrarium, propter ſignum negativum quantitatis prodeuntis $-3\sqrt{\frac{x}{4a-3x}}$: quibus poſitis, Fluens quantitatis $\frac{a\dot{x}x^{\frac{1}{2}}\sqrt{4a-3x}}{\overline{a-x}|^{2}}$ prodit $\frac{2aa-ax}{a-x}\sqrt{4ax-3xx}-\frac{8}{3}aa\sqrt{3}\left(\frac{r.t}{s}\right.=BOq$; ubi r, t, s, nempe Radius Tangens & Secans anguli menſurandi, atque adeo Coſinus, Sinus & Radius ſunt ad invicem ut $\sqrt{\frac{4}{3}aa-ax}$, $\sqrt{ax}$, $\sqrt{\frac{4}{3}aa}$.

Invenio autem hanc æquationem ſic ſatis eleganter conſtrui poſſe. Per medium punctum H baſis AQ tranſeat recta HIK quæ abſcindat ordinatæ QP quadrantem QI & occurrat Aſymptoto Ciſſoidis in K: huic normalis excitetur KL, axem AB ſecans in L. Agatur AMC quæ ſecet Aſymptoton in C & faciat angulum BAC trientem recti; centroque C, & intervallo CB deſcribatur circuli arcus BNM; ſumptaque inter AQ & AB media proportionali AG, ducatur GN parallela rectæ AM & abſcindens arcum MN. Deinde ſi inter AB atque octuplum exceſſus quo KL recta ſuperat arcum circularem MN ſtatuatur Media proportionalis BO: hæc erit ſemidiameter Circuli æqualis Ciſſoidali ſuperficiei ex AP arcu convoluto circum axem AB. Inde vero ſuperficies iſthæc eandem habebit



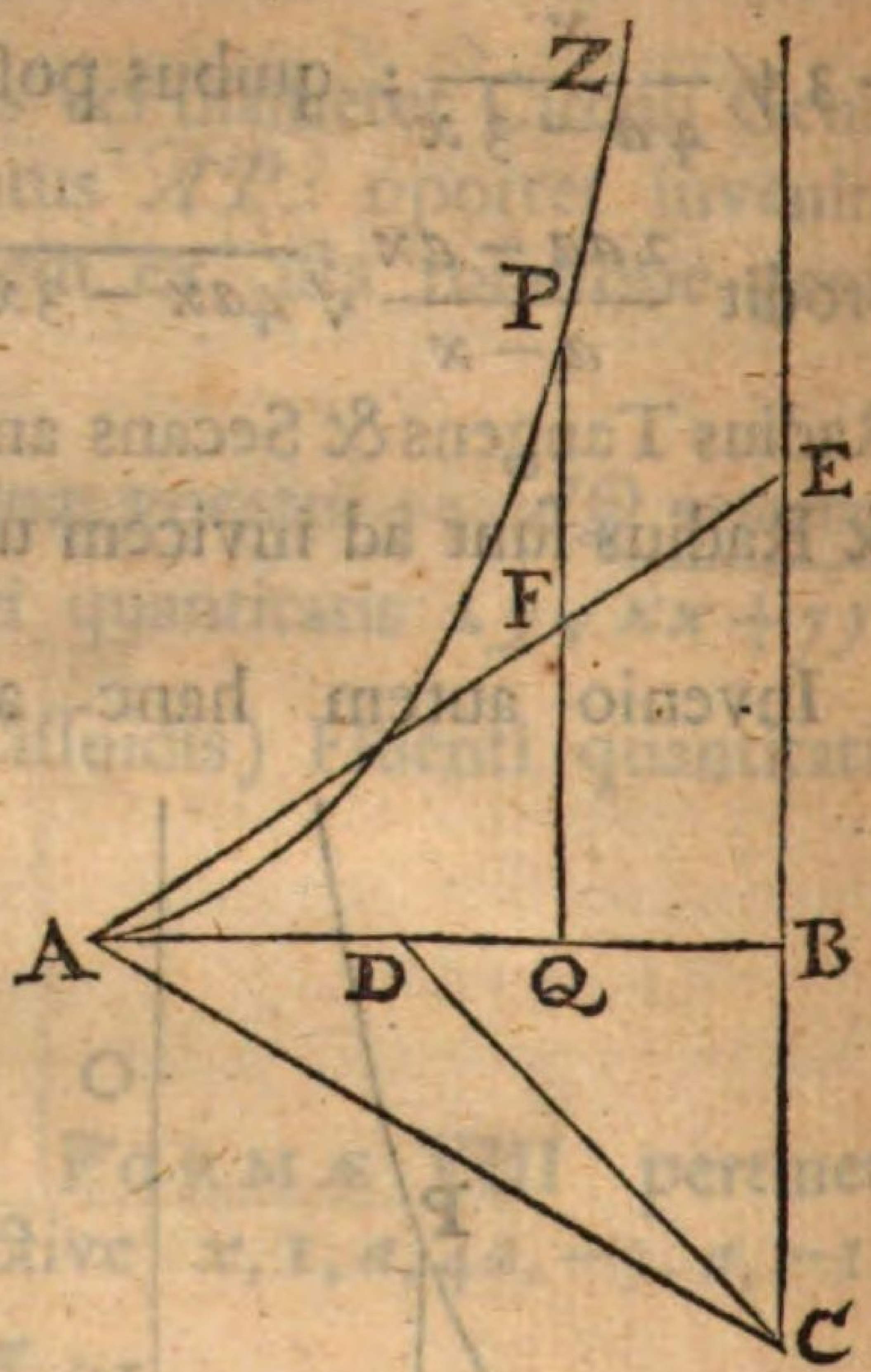
* Vid. Pag. 64.

rationem

 rationem ad Circulum Genitorem Cissoidis, quam habet sedecuplum prædicti excessus ad genitoris semidiametrum $\frac{1}{2}AB$.

Scholium.

Non multum dissimili ratione dimensio invenietur illius superficiei, quæ generatur ab eadem Cissoidis curva circum Asymptoton suam in orbem lata. Reliquis enim manentibus, si per Cissoidis verticem A atque ordinatæ PQ medium punctum F agatur AFE recta, quæ secet Asymptoton BC in E; atque inter BA & BQ capiatur Media proportionalis BD, jungaturque CD: Superficies genita ex arcu infinito PZ, erit ad Circulum Semidiametro BA descriptum, ut duplum EF rectæ & simul mensura rationis inter $BD + DC$ & BC pro Modulo BC, ad semidiametrum illam BA. Hinc autem Superficies ex arcu infinito AZ erit ad Circulum semidiametro BA, ut duplum BA rectæ & simul mensura rationis inter $BA+AC$ & BC pro Modulo BC, ad semidiametrum BA. Inde vero Superficies ex arcu finito AP, erit ad Circulum semidiametro BA, ut duplum excessus rectæ BA supra rectam EF & simul mensura rationis inter $BA+AC$ & $BD+DC$ pro Modulo BC, ad eandem Semidiametrum BA.



PROBLEMA VII.

Superficiei genitæ ex arcus Logisticæ conversione circum Asymptoton suam, æqualem Circulum invenire.

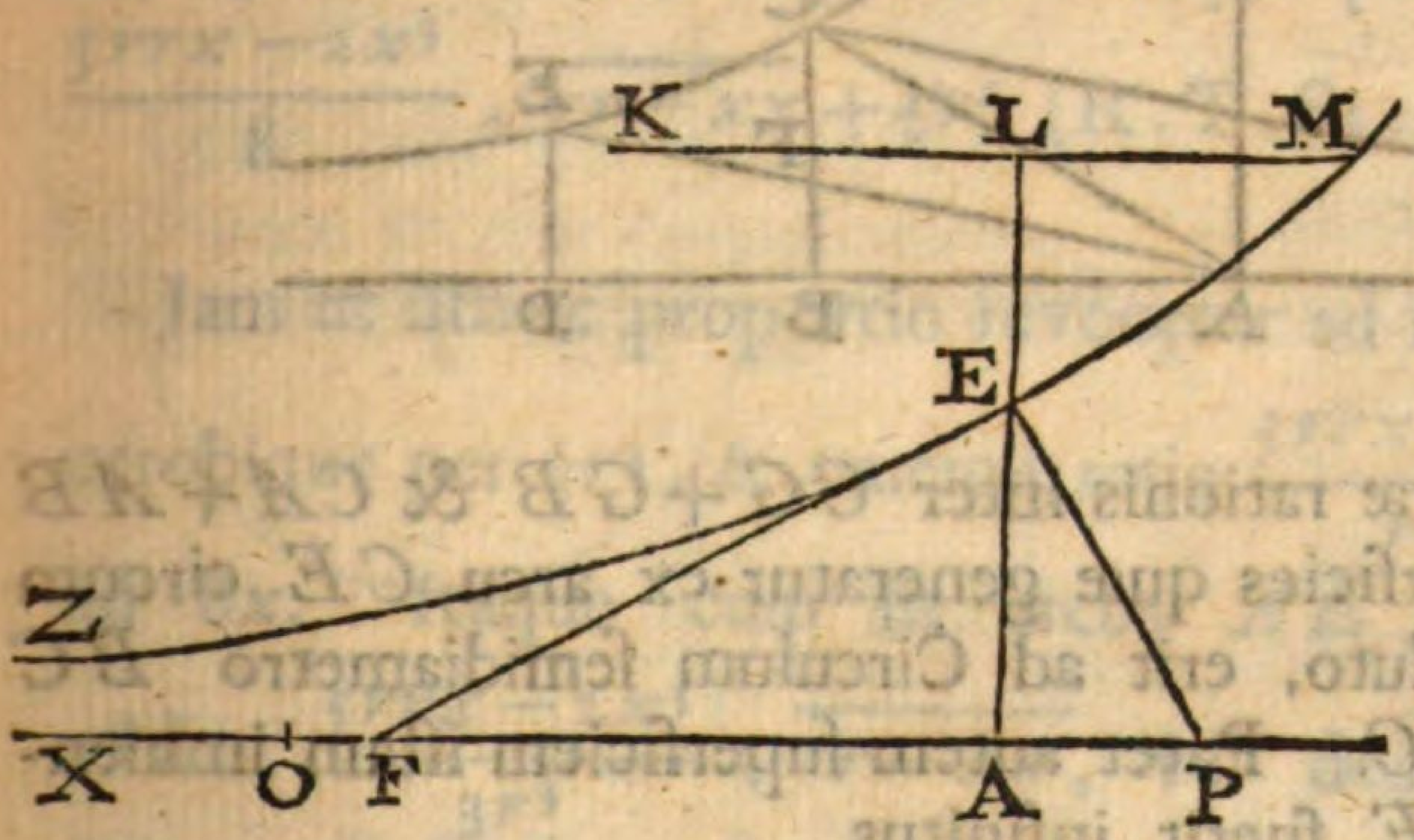
REVOLVATUR Logisticæ arcus infinitus EZ circum Asymptoton suam AX, ad quam perpetuo appropinquet versus partes ZX: & quæratur AO radius Circuli, cujus area sit æqualis superficiei ex isto motu genitæ.

Sit

Sit *EA* ordinata rectangula quæ vocetur y, ejusque Fluxio $\dot{y}$; Fluxio autem Curvæ *ZE* vocetur $\dot{z}$; ductaque *EF* quæ tangat Curvam in *E* secetque Asymptoton in *F*, subtangentis data magnitudo *AF* vocetur a: & erit AOq æquale Fluenti quantitatis $2y\dot{z}$ sive $2\dot{y}\sqrt{aa+yy}$. PARS TERTIA.

Hanc Fluxionem refero ad FORMAM IV: nam pro z, η, θ, d, e, f scriptis respective $y, 2, 0, 2, aa, 1$; fiet $d\dot{z}z^{\theta\eta+\frac{1}{2}\eta-1}\sqrt{e+fz^{\eta}} = 2\dot{y}\sqrt{aa+yy}$. Hujus autem Fluens $\frac{z^{\eta}}{\eta}dP+\frac{e}{\eta f}dR \left| \frac{R+T}{S} \right.$ (pro P, R, T, S sive $\sqrt{\frac{e+fz^{\eta}}{z^{\eta}}}$, $\sqrt{f}$, $\sqrt{\frac{e+fz^{\eta}}{z^{\eta}}}$, $\sqrt{\frac{e}{z^{\eta}}}$ scribendo $\frac{1}{y}\sqrt{aa+yy}$, 1, $\frac{1}{y}\sqrt{aa+yy}$, $\frac{a}{y}$) evadet $y\sqrt{aa+yy}+aa\left|\frac{y+\sqrt{aa+yy}}{a}\right. = AOq$.

Ad Constructionem hujus Æquationis, animadverto rectam *AO* mediam esse proportionalem inter a & $\frac{y}{a}\sqrt{aa+yy}+a\left|\frac{y+\sqrt{aa+yy}}{a}\right.$. Quantitas autem $\frac{y}{a}\sqrt{aa+yy}$ æqualis est *EP* rectæ, quæ normalis est ad Curvam in *E* & ab Asymptoto terminatur in *P*: altera vero quantitas $a\left|\frac{y+\sqrt{aa+yy}}{a}\right.$ est mensura rationis inter *AE*+*EF* & *AF* ad Modulum *AF*, vel (propter similia triangula *AEF*, *APE*) mensura rationis inter *AP*+*PE* & *AE* ad eundem Modulum *AF*.



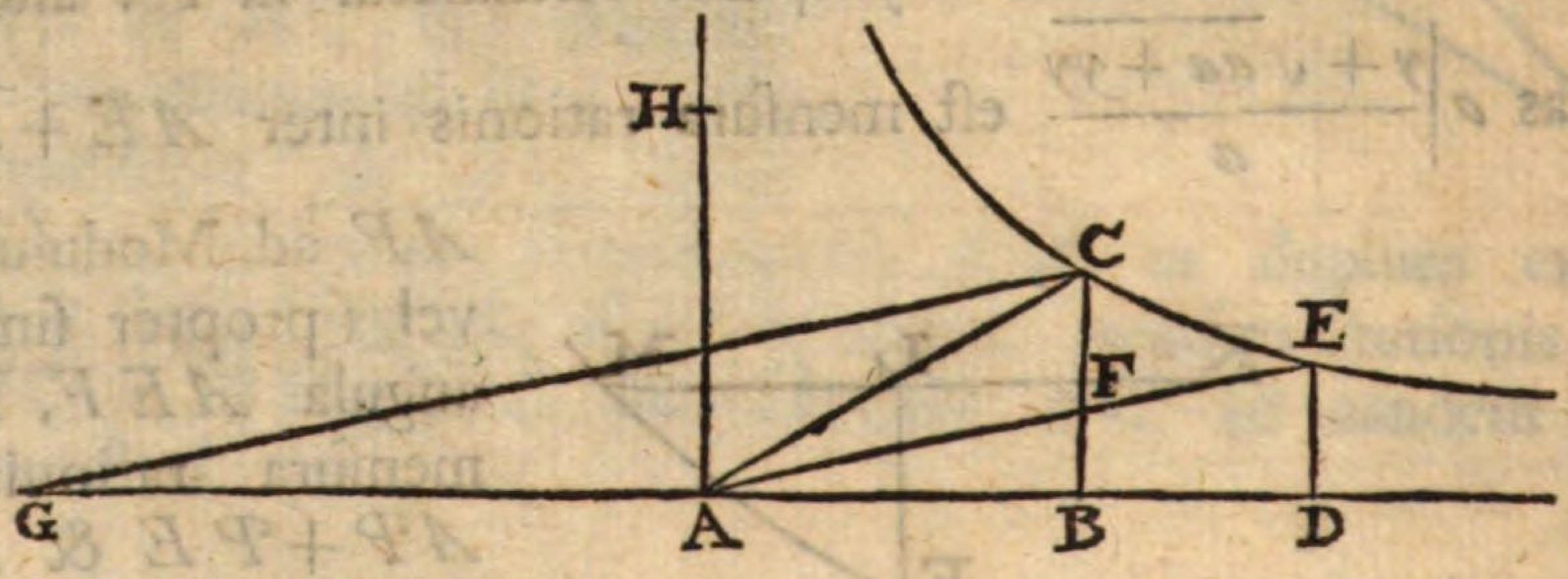
Sic itaque construitur Problema. Fiat *EP* normalis ad Curvam in *E* puncto, & attingat Asymptoton in *P*; producatur ordinata *AE* ultra punctum *E* ad *L*, ut sit *AL* æqualis aggregato linearum *AP*, *PE*; huic ad quas partes jacet curva *EZ*, perpendicularis excitetur *LK* normali *EP* æqualis, quæ

PROBLEMATA producta productam secet curvam in M. Deinde si inter KM & subtangentem AF statuatur media proportionalis AO: hæc erit semidiameter Circuli aream habentis æqualem Superficiei ex arcu infinito EZ. Eritque adeo superficies ista ad Circulum semidiametro AF, ut KM ad AF.

Scholium.

In hoc Problemate & in præcedentis Scholio, Superficiei finitæ ex arcu infinito genitæ dedimus Exempla; qualia sane quam plurima liceret proferre, si foret operæ pretium. Nam ubi area curvilinea infinito Arcui & Asymptoto interjecta finitam habet magnitudinem, ibi semper necesse est & Superficiem genitam ex Arcu illo circum Asymptoton convoluto finitam habere magnitudinem. Et vicissim, si illius magnitudo infinita fuerit, necesse est & hujus magnitudinem infinitam esse: quod Exemplo illustrare lubet.

Sit A centrum Hyperbolæ rectangulæ CE, quæ versus partes $E\mathcal{D}$ perpetuo appropinquet ad Asymptoton suam $GAB\mathcal{D}$; ad puncta C, E sint alteri Asymptoto parallelæ CFB centro propior & $E\mathcal{D}$ ab eodem remotior: jungantur AC, AFE, atque huic parallela ducatur CG: capiatur AH æqualis excessui rectæ AC



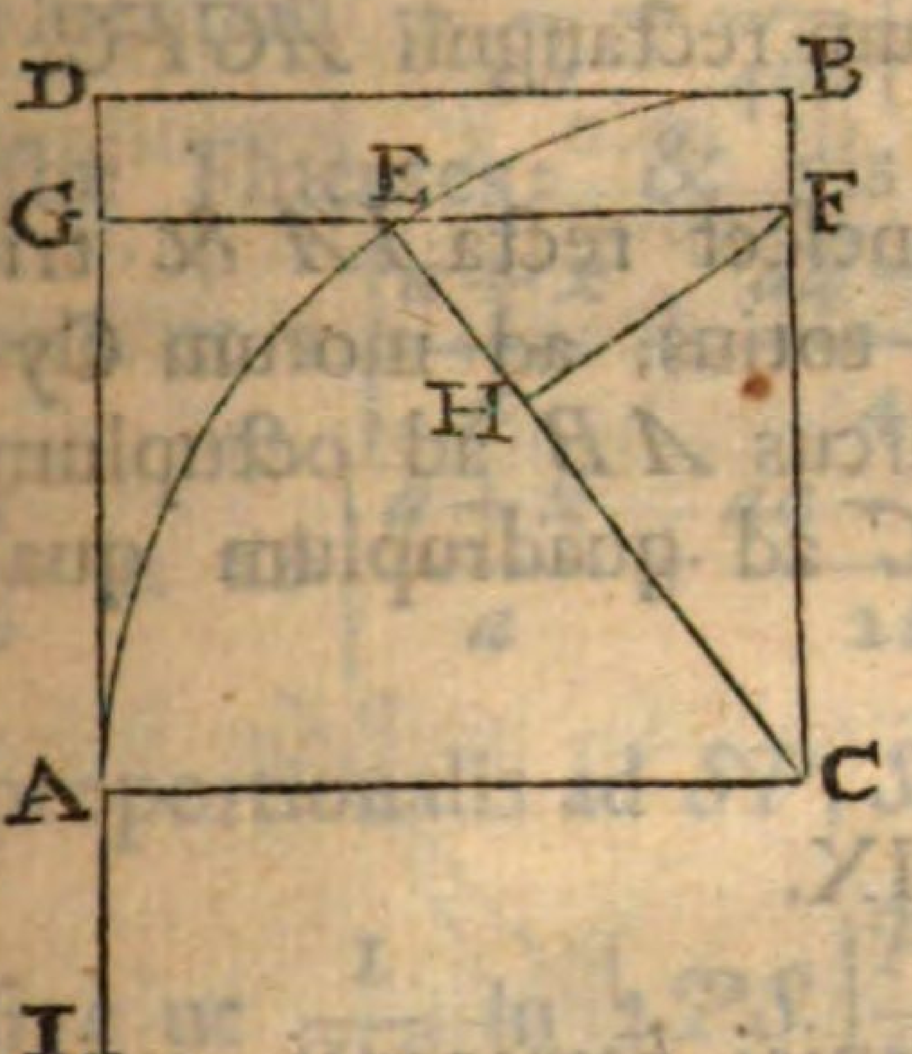
supra AF & simul mensuræ rationis inter $CG+GB$ & $CA+AB$ ad modulum AB, & superficies quæ generatur ex arcu CE circum $B\mathcal{D}$ Asymptoton convoluto, erit ad Circulum semidiametro BC descriptum, ut AH ad BC. Patet autem superficiem istam immensam evadere, ubi arcus CE fuerit infinitus.

PRO-

PROBLEMA VIII.

Invenire motum Sphæræ & circumscripti Cylindri, dum simul revolvuntur circum axem utrique communem.

SIT *AEBC* quadrans Circuli, cujus centrum *C*, semidiameter *CA* vel *CB*: compleatur semidiametri quadratum *ACBD*, hujusque lateribus *AC*, *DB* agatur utcunque parallela *GEF*, quæ reliquis lateribus *AD*, *CB* occurrat in *G*, *F* & arcui quadrantis in *E*. Jam convertantur quadrantis segmentum *AEFC* & quadrati segmentum *AGFC* circum axem *CF*, generentque segmenta Sphæræ & circumscripti Cylindri: oportet horum segmentorum, dum eodem periodico tempore circum axem utrique communem *CF* revolvuntur, proportionem motuum invenire.



Vocentur *CF*, x; *FE*, y; *FG*, r; eritque motus segmenti Sphærici ad motum segmenti Cylindrici, ut Fluens quantitatis $\dot{x}y^3$ ad Fluentem quantitatis $r^3\dot{x}$, hoc est, ut Fluens quantitatis $rr\dot{x}\sqrt{rr-xx}-\dot{x}x^2\sqrt{rr-xx}$ ad Fluentem quantitatis $r^3\dot{x}$, id est, (relata Fluxionis prioris utraque parte ad FORMAM IV) ut $\frac{5rrx-2x^3}{8}\sqrt{rr-xx}+\frac{3}{8}r^4$ (R.T.S::$\sqrt{rr-xx}.x.r$) ad r^3x.

Jam ut isthæc proportio revocetur ad simpliciores terminos, divido utrobique per $\frac{1}{8}r^3$, & emergit ratio $\frac{5rrx-2x^3}{r^3}\sqrt{rr-xx}+3r\,(ACE)$ ad $8x$. Itaque cum sit arcus $AE = r\,(ACE)$, si capiatur $AI = \frac{5rrx-2x^3}{3r^3}\sqrt{rr-xx}$, vel demisso in *CE* perpendiculo *FH*, si capiatur *AI* ad $CE+\frac{2}{3}EH$ ut *FH* ad *CE*: erit motus segmenti Sphærici, ad motum segmenti Cylindrici, ut triplum compositæ lineæ *IAE*, ad octuplum rectæ *CF*.

Si

PROBLEMATA

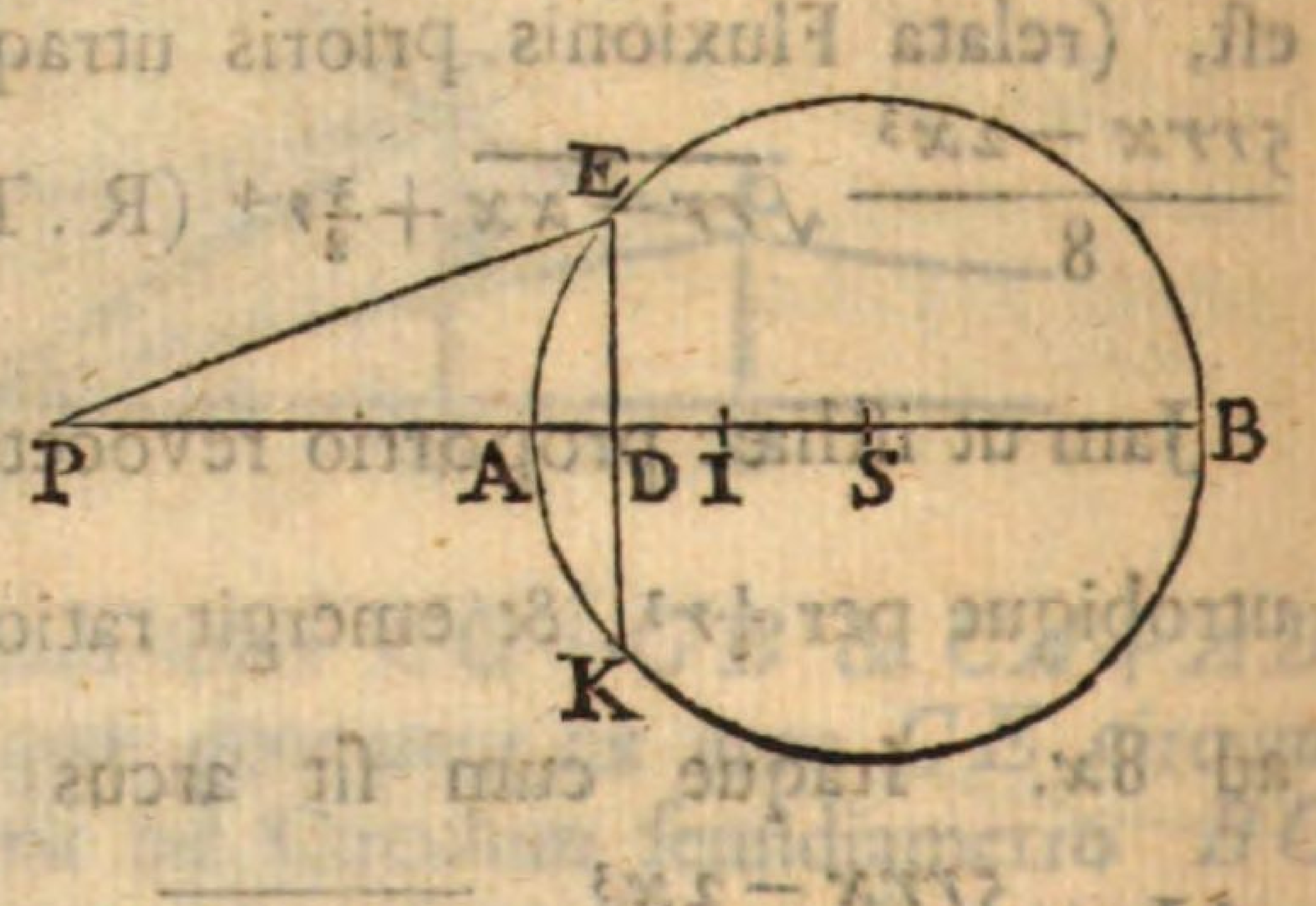
Si fiat utrobique divisio per $\frac{1}{4}rr$, emerget ratio $\frac{5rrx-2x^3}{2rr}\sqrt{rr-xx}$ $+\frac{1}{2}rr\,(ACE)$ ad $4rx$. Itaque cum sit areæ circularis sector $AEC = \frac{1}{2}rr\,(ACE)$, triangulum $ECF = \frac{x}{2}\sqrt{rr-xx}$, triangulum $EFH = \frac{rrx-x^3}{2rr}\sqrt{rr-xx}$: erit motus segmenti Sphærici, ad motum segmenti Cylindrici, ut duplum trianguli EFH & simul triplum areæ circularis $AEFC$, ad quadruplum rectanguli $AGFC$.

Accedente puncto E ad punctum B, evanescet recta IA & triangulum EFH. Erit igitur motus Sphæræ totius, ad motum Cylindri circumscripti, ut triplum quadrantalis arcus AB ad octuplum radii CB, sive ut triplum quadrantis ABC ad quadruplum quadrati circumscripti $ADBC$.

PROBLEMA IX.

Invenire vim attractivam corporis Sphærici homogenei, cujus particularum vires sunt reciproce in triplicata ratione distantiarum.

Sit $AEBK$ corpus Sphæricum attrahens cujus centrum S, diameter AB; P corpusculum attractum, in hac diametro locatum ad distantiam a centro PS. Ponatur EK diameter laminæ circularis tenuissimæ, quæ sit ad AB normalis eamque secet in D; & ex innumeris hujusmodi laminis intelligatur Sphæra tota componi: & similiter composita erit vis attractiva Sphæræ totius ex viribus attractivis laminarum omnium



Jungatur PE: & erit vis laminæ uniuscujusque EDK ut crassities hujus ducta in $\frac{1}{PD} - \frac{PD}{PEq}$, per Corol. 2. Prop. XC. Lib. I. Princip. *Newtoni*. Itaque si PA vocetur a; PS, b; AD, x; laminæ crassities

crassities erit ut $\dot{x}$, atque vis ejus ut $\frac{\dot{x}}{a+x} - \frac{a\dot{x}}{aa+2bx} - \frac{\dot{x}x}{aa+2bx}$. PARS TERTIA.

Hujus Fluxionis partes singulæ referendæ sunt ad FORMAM I: atque ita pars ejus prima $\frac{\dot{x}}{a+x}$ Fluentem habebit $1 \left| \frac{a+x}{x} \right.$, pars secunda $- \frac{a\dot{x}}{aa+2bx}$ Fluentem $- \frac{a}{2b} \left| \frac{aa+2bx}{aa} \right.$, pars tertia $- \frac{\dot{x}x}{aa+2bx}$ Fluentem $- \frac{x}{2b} + \frac{aa}{4bb} \left| \frac{aa+2bx}{aa} \right.$. In unam summam colligantur istæ Fluentes: & vis attractiva segmenti Sphærici $AEDKA$ erit ut $1 \left| \frac{a+x}{a} \right. + \frac{aa-2ab}{4bb} \left| \frac{aa+2bx}{aa} \right. - \frac{x}{2b}$, sive ut hujus duplum $\frac{1}{b}$ in $2b \left| \frac{a+x}{a} \right. + \frac{aa-2ab}{2b} \left| \frac{aa+2bx}{aa} \right. - x$, hoc est si ponatur SI tertia proportionalis ad SP, SA, ut $\frac{1}{PS}$ in $2PS \left| \frac{PD}{PA} \right. - \frac{1}{2} PI \left| \frac{PEq}{PAq} \right. - AD$, sive ut $\frac{1}{PS}$ in $2PS \left| \frac{PD}{PA} \right. - PI \left| \frac{PE}{PA} \right. - AD$.

Augeatur segmentum $AEDKA$, donec tandem in integram Sphæram evadat; & punctis E, D, K coincidentibus cum puncto B, erit vis Sphæræ totius ut $\frac{1}{PS}$ in $2PS \left| \frac{PB}{PA} \right. - PI \left| \frac{PB}{PA} \right. - AB$, hoc est, ut $\frac{1}{PS}$ in $\overline{PS+SI} \left| \frac{PB}{PA} \right. - AB$.

Unde talis emergit Problematis constructio, sive extra sive intra Sphæram locetur corpusculum. Per punctum P & Sphæræ centrum S agatur PS recta, quæ superficiei Sphæricæ occurrat in A & B;



ita ut PA sit corpusculi a Sphæra distantia minima, PB maxima. Capiatur SQ ad SB ut SA ad SP, sitque PQ aggregatum linearum SP, SQ: & erit vis attractiva Sphæræ reciproce ut PS distantia

 corpusculi

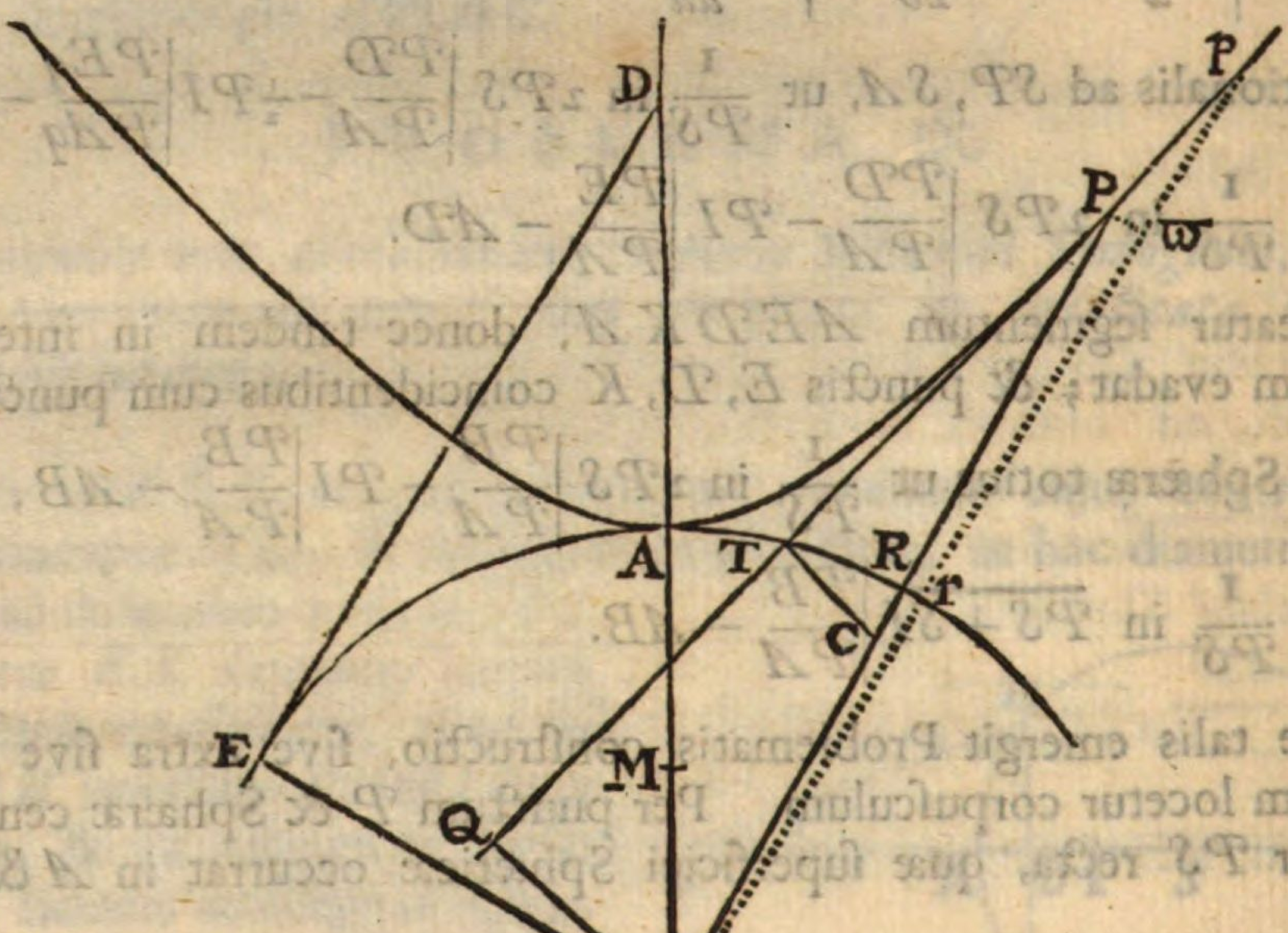
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 corpusculi a centro, & directe ut defectus diametri *AB* a mensura rationis inter *PB* & *PA* ad Modulum *PQ*.

PROBLEMA X.

Invenire Trajectoriam quam describet corpus a viribus centrifugis in ratione distantiarum triplicata decrescentibus agitatum, deque loco dato cum data velocitate secundum datam rectam egressum.

SIT *S* centrum virium, exeatque corpus de loco *P* secundum rectam *PQ* vel *QP* ea cum velocitate quam acquirere posset ab iisdem viribus libere ascendendo de loco *C* & ascensu suo describendo altitudinem *CP*: oportet invenire describendam vel descriptam Trajectoriam *AP*.



Sit *p* punctum hujus Trajectoriæ puncto *P* vicinissimum, connectatur *Sp*, & in eam demittatur perpendicularis $P\varpi$, centroque *S* & dato quovis intervallo *SR* describatur circulus *ARr*, qui rectis *SP*, *Sp* in *R* & *r* occurrat. Jam si *SC* vocetur c; *SR*, r; *SP*, x; $p\varpi$, $\dot{x}$; *Rr*, $\dot{z}$; corporis in puncto *P* velocitas, v: tempus quo describitur arcus *Pp* erit ut area *SPp*, sive ut $SP \times P\varpi$, hoc

hoc est ut $\frac{xx\dot{z}}{r}$; atque inde arcus ille Pp erit ut $\frac{vxx\dot{z}}{r}$, & quadratum ejus $P\varpi^2 + p\varpi^2$ sive $\dot{x}\dot{x} + \frac{xx\dot{z}\dot{z}}{rr}$ erit ut $\frac{vvx^4\dot{z}\dot{z}}{rr}$, adeoq; $rr\dot{x}\dot{x} + xx\dot{z}\dot{z}$ ut $vvx^4\dot{z}\dot{z}$. PARS TERTIA.

Est autem vv ut $\frac{xx-cc}{ccxx}$ per Prop. XXXIX & XL Lib. I Princip. *Newtoni*. Itaque $rr\dot{x}\dot{x} + xx\dot{z}\dot{z}$ erit ut $\frac{xx-cc}{ccxx}x^4\dot{z}\dot{z}$, hoc est (ob datum cc) ut $x^4\dot{z}\dot{z} - ccxx\dot{z}\dot{z}$. Assumatur quantitas invariabilis a, quæ magnitudinem habeat idoneam: & erit $aarr\dot{x}\dot{x} + aaxx\dot{z}\dot{z} = x^4\dot{z}\dot{z} - ccxx\dot{z}\dot{z}$, indeque $\dot{z} = \frac{ar\dot{x}}{x\sqrt{xx-aa-cc}}$; vel si ponatur $aa+cc=bb$, erit $\dot{z} = \frac{ar\dot{x}}{x\sqrt{xx-bb}}$.

Hæc Fluxio referri potest ad FORMAM V, & prodibit $z = \frac{ar}{b}(\mathrm{R}:\mathrm{T}:\mathrm{S}::b:\sqrt{xx-bb}:x)$. Ponamus $r=b$, & exsurget Æquatio paullo simplicior $z = a(\mathrm{R}:\mathrm{T}:\mathrm{S}::b:\sqrt{xx-bb}:x)$.

Ad Æquationis hujus constructionem, inveniendæ sunt quantitates a & b: illæ vero demissis ad tangentem PQ perpendicularibus SQ, CT, determinantur in hunc modum ex dato situ punctorum S, C, P, T, Q. Propter similia triangula SRr, $SP\varpi$; erit $\dot{z}$ ad $P\varpi$ ut r ad $\dot{x}$: rursus propter similia triangula $P\varpi p$, SQP; erit $P\varpi$ ad $\dot{x}$ ut SQ ad PQ: ex æquo igitur $\dot{z}$ ad $\dot{x}$ ut $r\times SQ$ ad $x\times PQ$. Inde $\dot{z} = \frac{SQ\times r\dot{x}}{x\times PQ}$, erat autem $\dot{z} = \frac{ar\dot{x}}{x\sqrt{xx-bb}}$, itaque $\frac{SQ}{PQ} = \frac{a}{\sqrt{xx-bb}} = \sqrt{\frac{bb-cc}{xx-bb}}$, atque adeo $\frac{bb-SCq}{SPq-bb} = \frac{SQq}{PQq}$. Hinc vero $\overline{PQq+SQq}\times bb = SPq\times SQq + PQq\times SCq$, & $bb = SQq + \frac{PQq\times SCq}{SPq} = SQq + QTq$: unde $SR = r = b = \sqrt{SQq+QTq}$, & $a = \sqrt{bb-cc} = \sqrt{SRq-SCq}$. Itaque cum z

sit

PROBLEMATA fit arcus aliquis AR qui interjicitur immobili suo termino A mobilique R, habebimus tandem $AR = \sqrt{SRq - SCq}$ (R : T : S :: SR : $\sqrt{SPq - SRq} : SP$).

Hinc ex dato situ punctorum S, P, C, T, Q per conditiones Problematis, primo dabitur punctum A; eoque semel invento, deinde dabuntur alia quotlibetcunque Trajectoriæ puncta P. Hinc etiam intelligitur, immobile illud punctum A ipsam esse Apsidem imam Trajectoriæ. Nam si punctum R versus punctum A revertatur & in illud incidat, necesse est cum arcu AR simul evanescere angulum (R : T : S :: SR : $\sqrt{SPq - SRq} : SP$) ideoque æquales fieri SR & SP, atque ita punctum P coincidere cum punctis R & A. Transibit igitur Tajectoria per A punctum: atqui non ulterius descendet, ne quantitas $\sqrt{SPq - SRq}$ fiat impossibilis.

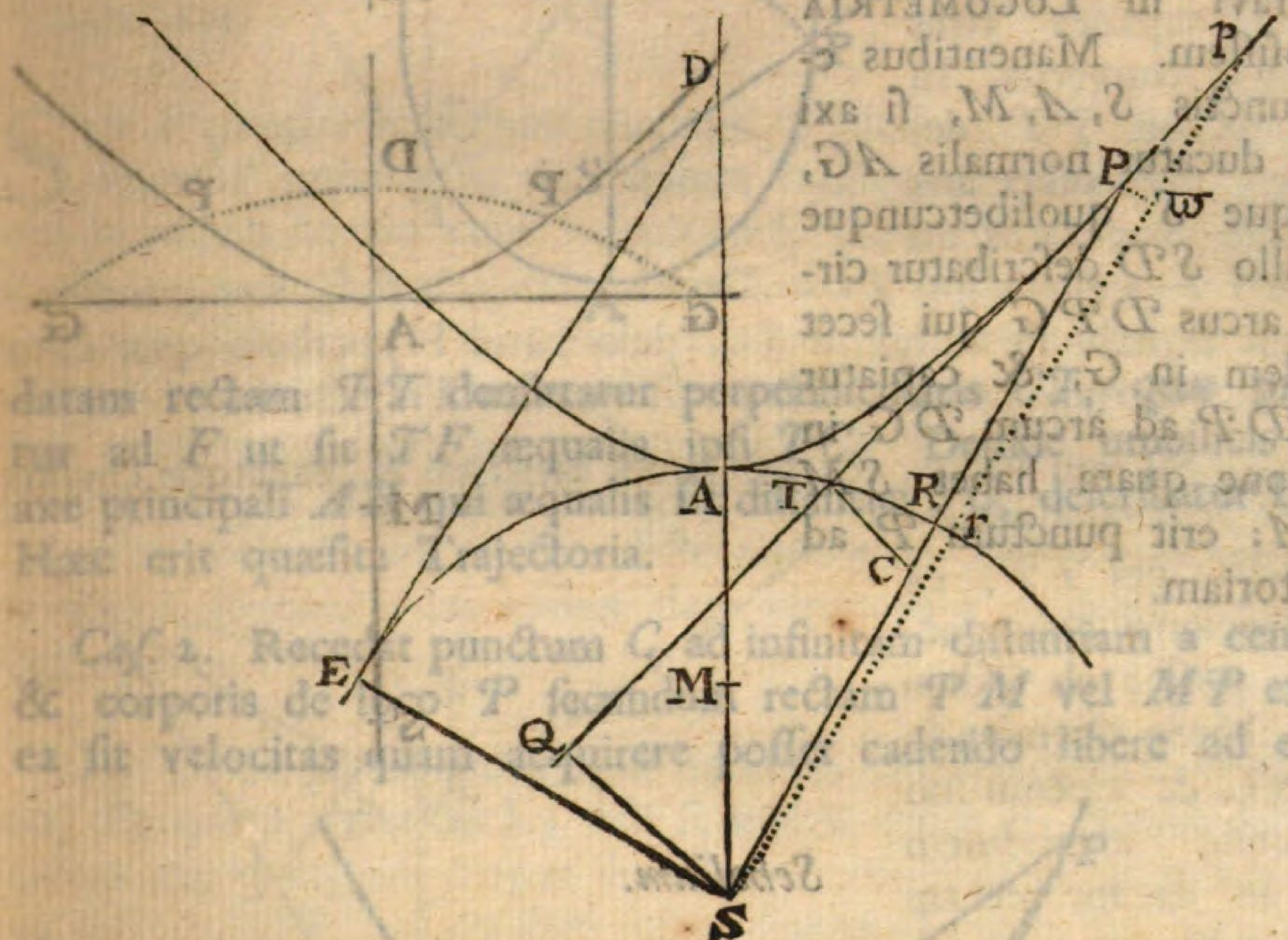
Si desideretur tempus quo describi possit arcus AP: invenienda erit area PAS ad hunc modum. Erat supra Rr sive $\dot{z} = \frac{ar\dot{x}}{x\sqrt{xx-bb}}$, itaque $P\varpi = \frac{a\dot{x}}{\sqrt{xx-bb}}$, & $\frac{1}{2}P\varpi \times SP = \frac{ax\dot{x}}{2\sqrt{xx-bb}}$. Hæc autem quantitas ad Formam quintam relata, vel etiam ipso intuitu, Fluentem exhibet $\frac{1}{2}a\sqrt{xx-bb}$ æqualem Areæ PAS ac proinde proportionalem tempori quo describi possit arcus AP.

Denique si conferenda sit velocitas corporis in loco P cum velocitate qua in Circulo ad eandem distantiam Sp cum iisdem viribus in centripetas conversis revolvi posset: quoniam velocitas in Trajectoria ea est quæ acquiratur ascendendo a puncto C ad locum P, & velocitas in Circulo ea est quæ acquiratur descendendo ab infinita distantia ad eundem locum P; erit (per Prop. XXXIX & XL Lib. I Princip. *Newtoni*) velocitas prior ad velocitatem posteriorem, ut $\sqrt{\frac{xx-cc}{ccxx}}$ ad $\sqrt{\frac{1}{xx}}$, hoc est, ut $\sqrt{xx-cc}$ ad c.

Hisce positis Problematis constructio sic procedet. In datam rectam PTQ demittantur perpendicula CT, SQ; centroque S & intervallo $\sqrt{SQq + TQq}$ describatur circulus $RTAE$ rectæ SP occurrens in R; deinde sit arcus RA, ad ejusdem circuli arcum

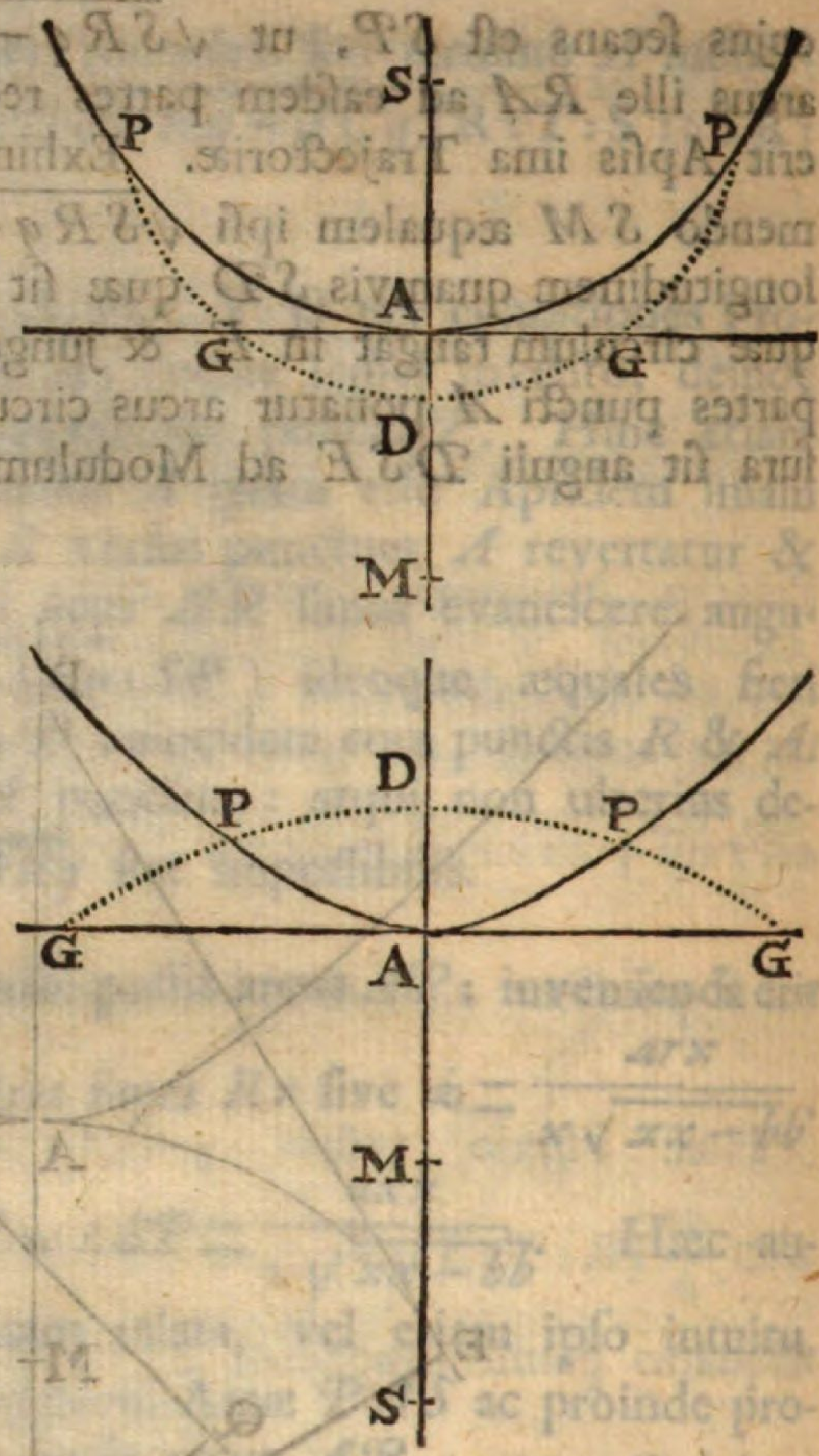
cujus

cujus ſecans eſt SP, ut $\sqrt{SRq - SCq}$ ad SR; ponatur autem arcus ille RA ad eaſdem partes rectæ SP cum puncto Q: & A erit Apſis ima Trajectoriæ. Exhinc vero Trajectoria dabitur, ſumendo SM æqualem ipſi $\sqrt{SRq - SCq}$, in recta SA capiendo longitudinem quamvis SD quæ ſit major quam SA, ducendo DE quæ circulum tangat in E, & jungendo SE. Nam ſi ad utraſque partes puncti A ponatur arcus circularis AR, cujus longitudo menſura ſit anguli DSE ad Modulum SM, & in ſemidiametris SR PARS TERTIA.



capiantur diſtantiæ SP æquales ipſi SD: erunt puncta P ad Trajectoriam deſcribendam. Tempus autem quo radius SP, a centro ad corpus motum ductus, percurret aream quamvis SAP, erit ut recta DE: nam area percurſa æquatur ipſi DE in Modulum dimidiatum $\frac{1}{2}SM$ ductæ. Velocitas vero corporis in loco quovis P, erit ad velocitatem qua in Circulo, ad eandem diſtantiam SP, cum iiſdem viribus in centripetas converſis revolvi poſſet, ut $\sqrt{SPq - SCq}$ ad SC.

PROBLEMATA

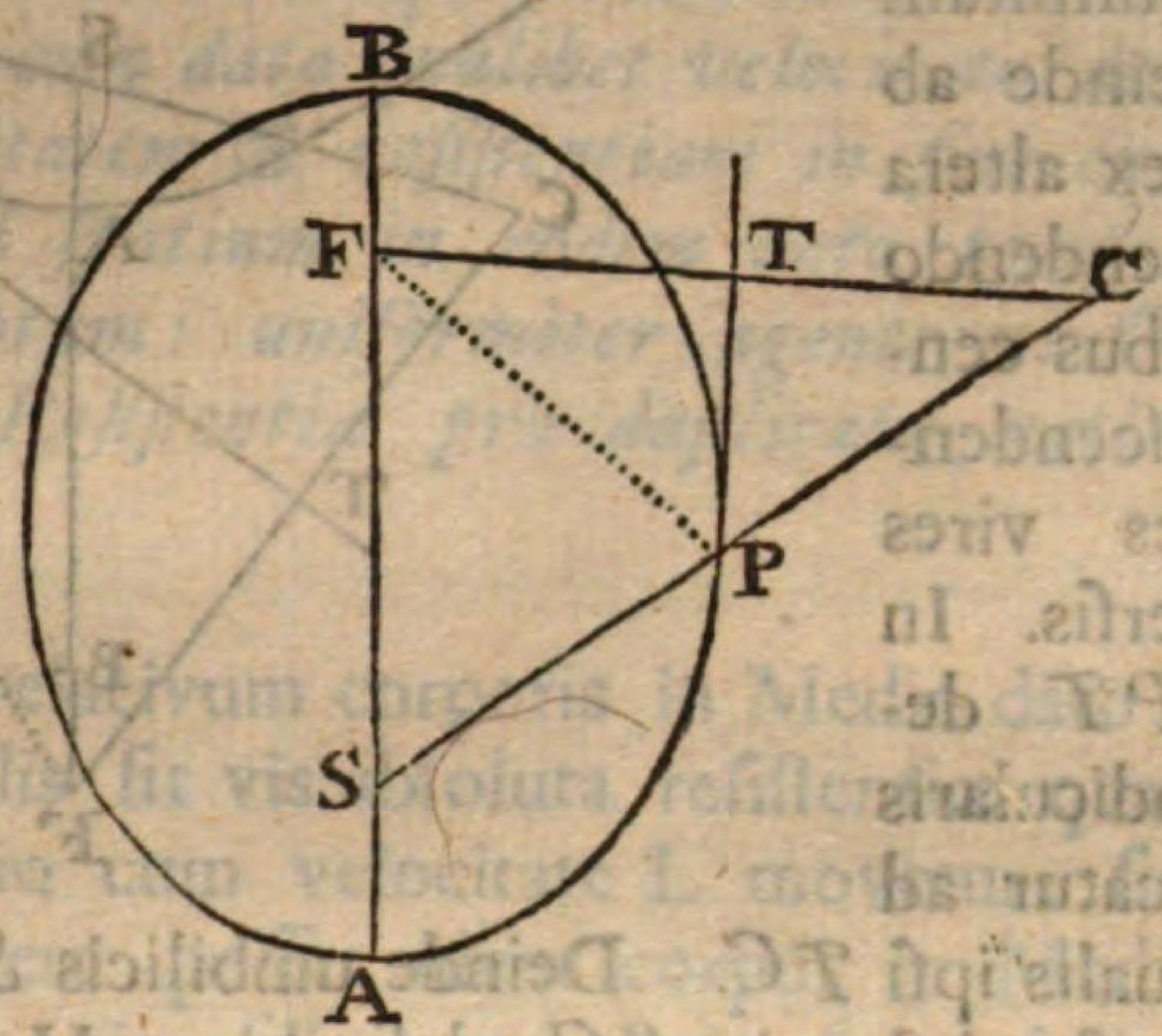


Per hujusmodi constructionem commodissime demonstratur analogia Mensuratum. Quod ni mihi propositum esset, aliter hanc Trajectoriam & ultimam Spiralium quas enumeravi in LOGOMETRIA descripsissem. Manentibus enim punctis S, A, M, si axi SAD ducatur normalis AG, centroque S quolibetcunque intervallo SD describatur circularis arcus DPG qui secet normalem in G, & capiatur arcus DP ad arcum DG in ea ratione quam habet SM ad SA: erit punctum P ad Trajectoriam.

Scholium.

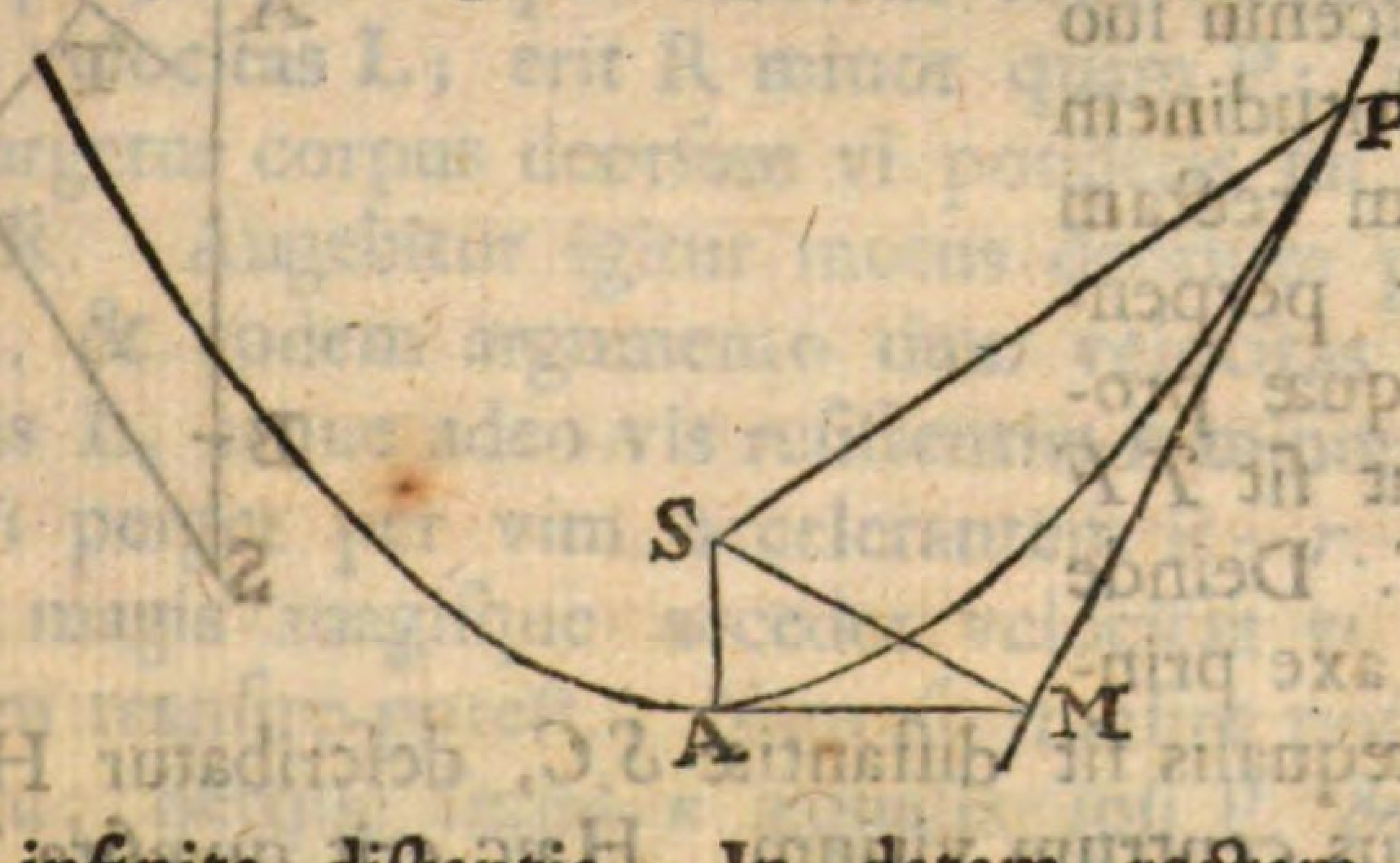
Ad eundem modum & per idem Formæ quintæ Theorema facillime mihi se obtulit inventio Primæ, Tertiæ, & Quintæ Spiralium, quas modo memoravi: erant enim Secunda & Quarta simplicioris adhuc indaginis. Quinetiam si Vis centralis sit reciproce ut quadratum distantiæ: per eandem Methodum prodibunt constructiones perelegantes, quales olim annotatas in adversariis meis reperio. Has itaque cum nondum observatas videam ab iis qui nuper idem Problema pertractarunt non gravabor hic apponere; omissa tamen earum demonstratione, quæ sane perquam facilis est, sed non est hujus loci.

Cas. 1. Sit *S* virium centripetarum centrum, exeatque corpus de loco *P* secundum rectam *PT* vel *TP*, ea cum velocitate quam acquirere posset ab iisdem viribus libere cadendo versus centrum *S* de loco *C*, & casu suo describendo altitudinem *CP*. In



datam rectam *PT* demittatur perpendicularis *CT*, quæ producatur ad *F* ut sit *TF* æqualis ipsi *TC*. Deinde umbilicis *S*, *F*, axe principali *AB* qui æqualis sit distantiæ *SC*, describatur Ellipsis. Hæc erit quæsita Trajectoria.

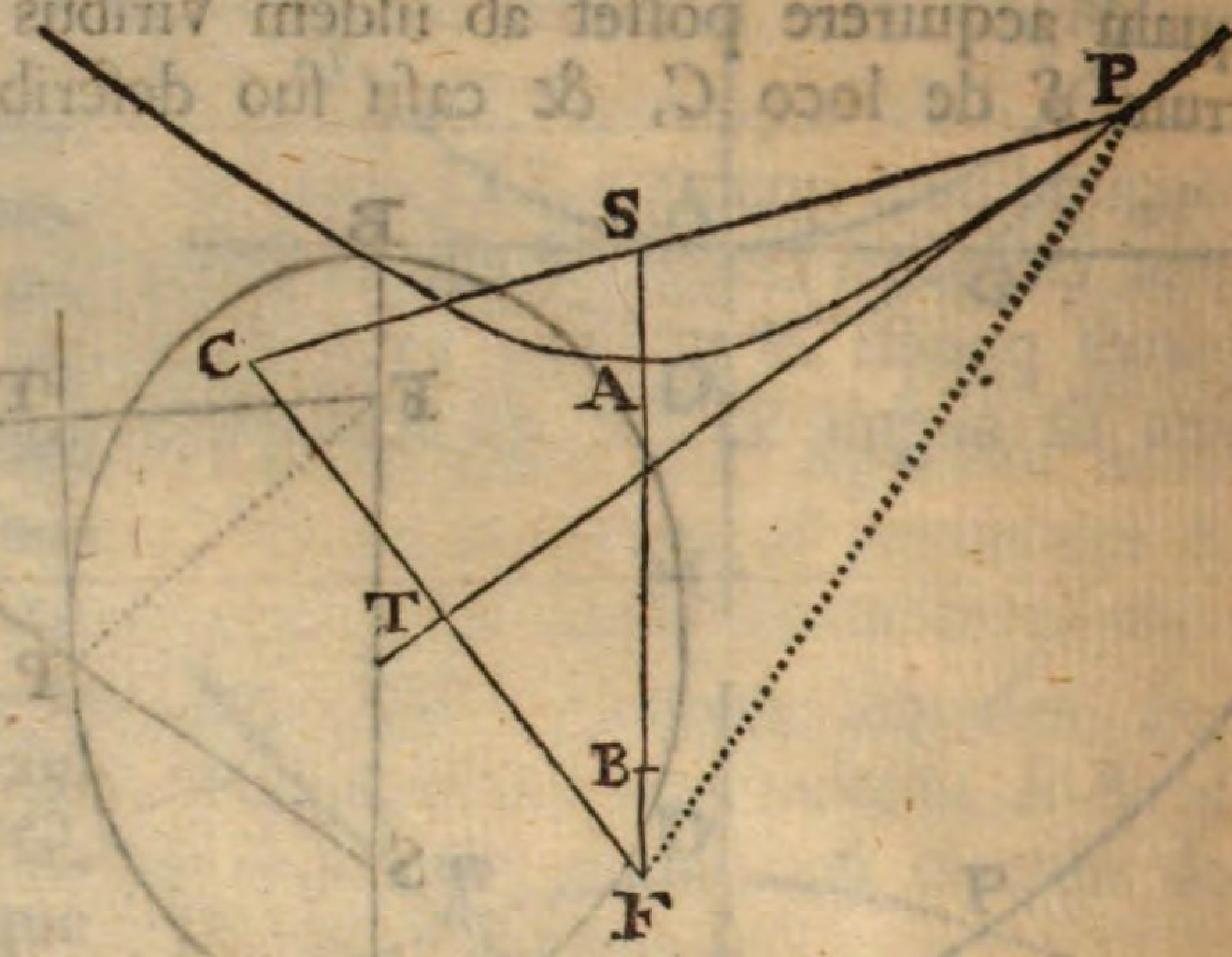
Cas. 2. Recedat punctum *C* ad infinitam distantiam a centro *S*; & corporis de loco *P* secundum rectam *PM* vel *MP* exeuntis ea sit velocitas quam acquirere posset cadendo libere ad eundem



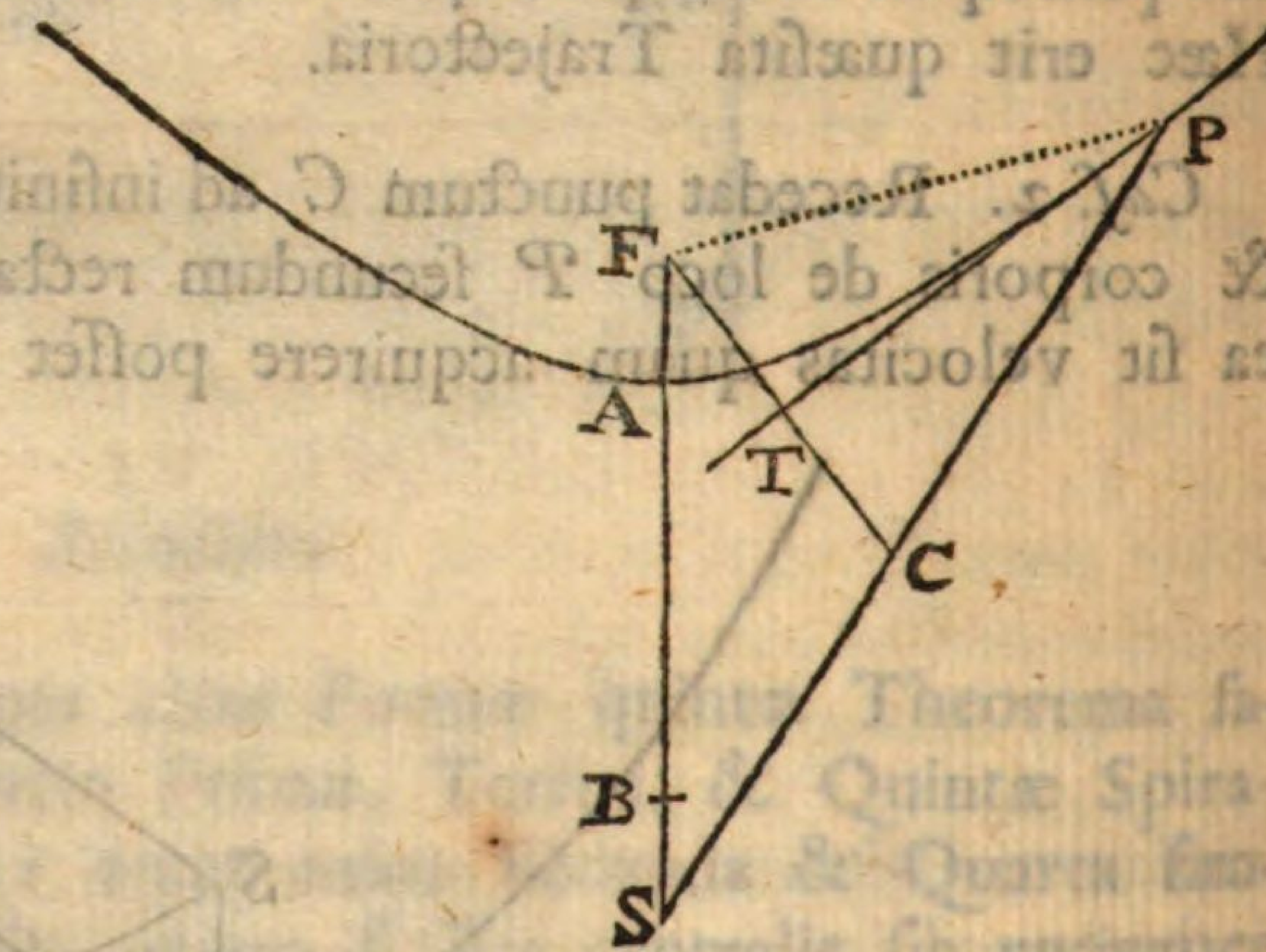
locum *P* ab infinita distantia. In datam rectam *PM* demittatur perpendicularis *SM*, & fiant anguli *MSA*, *SMA* æquales angulis *PSM*, *SPM*. Deinde umbilico *S*, vertice principali *A* describatur Parabola. Hæc erit quæsita Trajectoria.

Cas. 3. Ut velocitas sit adhuc major, abeat jam punctum *C* ad distantiam plusquam infinitam a centro *S*, vel (quod perinde est) accedat

accedat a parte contraria eidem centro, ad finitam distantiam; & corporis de loco *P* secundum rectam *PT* vel *TP* exeuntis, ea sit velocitas, quam acquirere posset ascendendo libere de loco *C* ad infinitam distantiam, & deinde ab infinita distantia ex altera centri parte descendendo ad locum *P*, viribus centripetis inter ascendendum in æquales vires centrifugas conversis. In datam rectam *PT* demittatur perpendicularis *CT*, quæ producatur ad *F* ut sit *TF* æqualis ipsi *TC*. Deinde umbilicis *SF*, axe principali *AB* qui æqualis sit distantiæ *SC*, describatur Hyperbola quæ cava sit versus centrum virium. Hæc erit quæsita Trajectoria.



Cas. 4 Sit jam virium centrifugarum centrum *S*, exeatque corpus de loco *P* secundum rectam *PT* vel *TP*, ea cum velocitate quam acquirere posset ab iisdem viribus, libere ascendendo de loco *C* & ascensu suo describendo altitudinem *CP*. In datam rectam *PT* demittatur perpendicularis *CT*, quæ producatur ad *F* ut sit *TF* æqualis ipsi *TC*. Deinde umbilicis *S*, *F*, axe principali *AB* qui æqualis sit distantiæ *SC*, describatur Hyperbola quæ convexa sit versus centrum virium. Hæc erit quæsita Trajectoria.



Notatu dignum est, si V sit quantitas vis absolutæ centralis; Velocitatem in hisce omnibus fore ubique ut $\sqrt{\frac{V \times PF}{AB \times PS}}$. Hæc autem quantitas in Circulo fit $\sqrt{\frac{V}{AB}}$, in Parabola fit $\sqrt{\frac{V}{PS}}$.

PRO-

PROBLEMA XI.

Data vi absoluta resistentiæ, quam patitur corpus datum in dato Medio cum data aliqua velocitate motum; invenire corporis ejusdem, cum data qualibet velocitate deorsum sursumve projecti, velocitatem & resistentiam in fine cujusvis dati temporis, & simul spatium in eodem dato tempore descensu vel ascensu descriptum: uniformiter agente vi Gravitatis, mutata vero vi Resistentiæ pro duplicata ratione mutatæ velocitatis.

SIT P pondus relativum corporis in Medio dato: & vi ponderis hujus P æqualis sit vis absoluta resistentiæ quam patitur corpus ubi in eodem Medio cum velocitate L movetur: sit autem velocitas L ea quam acquirere posset idem corpus in Medio non resistente cadendo libere per vim solam ponderis sui relativi P & casu suo describendo altitudinem A. Præterea sit V velocitas quacum corpus projicitur; huic autem respondeat resistentia R, quam scilicet patitur corpus sub initio descensus vel ascensus: sitque propositum invenire velocitatem v & resistentiam r in fine temporis dati t, simulque spatium s in eodem dato tempore t descensu vel ascensu descriptum.

Cas. 1. Si projiciatur corpus deorsum cum velocitate V quæ minor sit quam velocitas L; erit R minor quam P: itaque sub initio motus magis urgetur corpus deorsum vi ponderis sui P quam sursum vi resistentiæ R. Augebitur igitur motus deorsum per vim accelerantem $P - R$, & eodem argumento dum velocitas v minor fuerit quam velocitas L, atque adeo vis resistentiæ r minor quam vis ponderis P, augeri perget per vim accelerantem $P - r$. Continuo igitur augmento magis magisque accedet velocitas v ad Limitem L quem nunquam transire potest. Nam si transire possit, fiet æqualis in ipso transitu, fietque simul r æqualis ipsi P, & vis accelerans $P - r$ in nihilum redacta erit, atque adeo nulla erit ulterior acceleratio, proinde nec transitus ullus.

Jam cum velocitas omnis, omnisque mutatio velocitatis sit ut vis per quam uniformiter generatur & tempus in quo generatur conjunctim; atque vis uniformiter generans velocitatem L in tempore

PROBLEMATA H sit P, vis autem uniformiter generans velocitatis incrementum $\dot{v}$ in tempore $\dot{t}$ sit $P-r$: erit L ad $\dot{v}$ ut PH ad $\overline{P-r}\times\dot{t}$. Rursus cum omne spatium uniformiter descriptum, sit ut tempus & velocitas conjunctim quibus describitur vel describi possit; atque velocitas qua uniformiter describi possit spatium 2A in tempore H sit L, velocitas autem qua uniformiter describitur spatium $\dot{s}$ in tempore $\dot{t}$ sit v: erit 2A ad $\dot{s}$ ut HL ad $\dot{t}v$. Ex hisce proportionibus (cum præterea sit ex Hypothesi P ad r ut LL ad vv) facile colligitur $\dot{t} = \frac{HL\dot{v}}{LL-vv}$, & $\dot{s} = \frac{A\dot{r}}{P-r}$. Itaque FORMA II dabit $t = \frac{1}{2}H \left| \frac{L+v}{L-v} \right. - \frac{1}{2}H \left| \frac{L+V}{L-V} \right.$, & FORMA I dabit $s = A \left| \frac{P-R}{P-r} \right.$.

Unde si ita capiatur numerus n, ut sit $t + \frac{1}{2}H \left| \frac{L+V}{L-V} \right. = \frac{1}{2}H \left| \frac{n}{1} \right.$: erit $L+v$ ad $L-v$ ut n ad 1, atque adeo v ad L ut $n-1$ ad $n+1$. Datur igitur velocitas v in fine dati temporis t; indeque datur resistentia r in fine temporis ejusdem, sumendo r ad P ut vv ad LL; denique hinc datur spatium in eodem tempore descensu descriptum, sumendo $s = A \left| \frac{P-R}{P-r} \right.$.

Cas. 2. Si projiciatur corpus deorsum cum velocitate V quæ æqualis sit velocitati L; erit R æqualis ipsi P: itaque sub initio motus æqualiter urgetur corpus deorsum vi ponderis sui P atque sursum vi resistentiæ R. Igitur motus nec augebitur nec diminuetur, & eodem argumento perget continuo nec augeri nec diminui. Hinc in fine cujusvis dati temporis t manebit velocitas V vel L & resistentia R vel P; spatium vero s quod in eodem tempore t descensu describitur erit ad spatium 2A ut t ad H.

Cas. 3. Si projiciatur corpus deorsum cum velocitate V quæ major sit quam velocitas L; erit R major quam P: itaque sub initio motus minus urgetur corpus deorsum vi ponderis sui P quam sursum vi resistentiæ R. Diminuetur igitur motus deorsum per vim retardantem $R-P$; & eodem argumento dum velocitas v major fuerit quam velocitas L, atque ita vis resistentiæ r major quam vis ponderis P diminui perget per vim retardantem $r-P$. Continuo igitur decremento magis magisque accedet velocitas v ad Limitem L quem (uti supra) nunquam transire potest.

Jam

Jam cum vis uniformiter generans velocitatis decrementum $-\dot{v}$ in tempore $\dot{t}$ sit $r-\mathrm{P}$: erit (uti supra) L ad $-\dot{v}$ ut PH ad $\overline{r-\mathrm{P}}\times\dot{t}$. Rursus cum velocitas qua uniformiter describitur spatium $\dot{s}$ in tempore $\dot{t}$ sit v: erit (uti supra) 2A ad $\dot{s}$ ut HL ad $\dot{t}v$. Et ex hisce proportionibus (cum sit præterea P ad r ut LL ad vv) facile colligitur $\dot{t} = \frac{\mathrm{HL}\dot{v}}{\mathrm{LL}-vv}$, & $\dot{s} = \frac{\mathrm{A}\dot{r}}{r-\mathrm{P}}$. Itaque FORMA II dabit $t = \frac{1}{2}\mathrm{H}\left|\frac{v+\mathrm{L}}{v-\mathrm{L}}\right. - \frac{1}{2}\mathrm{H}\left|\frac{\mathrm{V}+\mathrm{L}}{\mathrm{V}-\mathrm{L}}\right.$, & FORMA I dabit $s = \mathrm{A}\left|\frac{\mathrm{R}-\mathrm{P}}{r-\mathrm{P}}\right.$. PARS TERTIA.

Unde si ita capiatur numerus n, ut sit $t+\frac{1}{2}\mathrm{H}\left|\frac{\mathrm{V}+\mathrm{L}}{\mathrm{V}-\mathrm{L}}\right. = \frac{1}{2}\mathrm{H}\left|\frac{n}{1}\right.$: erit $v+\mathrm{L}$ ad $v-\mathrm{L}$ ut n ad 1, atque adeo v ad L ut $n+1$ ad $n-1$. Datur igitur velocitas v in fine dati temporis t; indeque datur resistentia r in fine temporis ejusdem, sumendo r ad P ut vv ad LL; denique hinc datur spatium in eodem tempore descensu descriptum, sumendo $s = \mathrm{A}\left|\frac{\mathrm{R}-\mathrm{P}}{r-\mathrm{P}}\right.$.

Cas. 4. Si jam sursum projiciatur corpus cum velocitate quacunque V, ita ut R ad P quamlibet rationem habeat: perspicuum est sub initio motus urgeri corpus deorsum summa virium P & R, & similiter elapso tempore quovis t urgeri deorsum summa virium P & r. Diminuetur igitur motus sursum per vim retardantem P+R, continuoque perget diminui per vim retardantem $\mathrm{P}+r$, donec ad quietem pervenerit.

Jam cum vis uniformiter generans velocitatis decrementum $-\dot{v}$ in tempore $\dot{t}$ sit $\mathrm{P}+r$: erit (ut supra) L ad $-\dot{v}$ ut PH ad $\overline{\mathrm{P}+r}, \dot{t}$. Rursus cum velocitas qua uniformiter describitur spatium $\dot{s}$ in tempore $\dot{t}$ sit v: erit (ut supra) 2A ad $\dot{s}$ ut HL ad $\dot{t}v$. Et ex hisce proportionibus (cum sit præterea P ad r ut LL ad vv) facile colligitur $\dot{t} = -\frac{\mathrm{HL}\dot{v}}{\mathrm{LL}+vv}$, & $\dot{s} = -\frac{\mathrm{A}\dot{r}}{\mathrm{P}+r}$. Itaque FORMA II dabit $t =$ H (Rad : Tang : : L : V) − H (Rad : Tang : : L : v), & FORMA I dabit $s = \mathrm{A}\left|\frac{\mathrm{P}+\mathrm{R}}{\mathrm{P}+r}\right.$.

Unde si ita capiatur numerus n, ut sit H (Rad : Tang : : L : V) − t = H (Rad : Tang : : 1 : n) : erit v ad L ut n ad 1. Datur igitur

velocitas

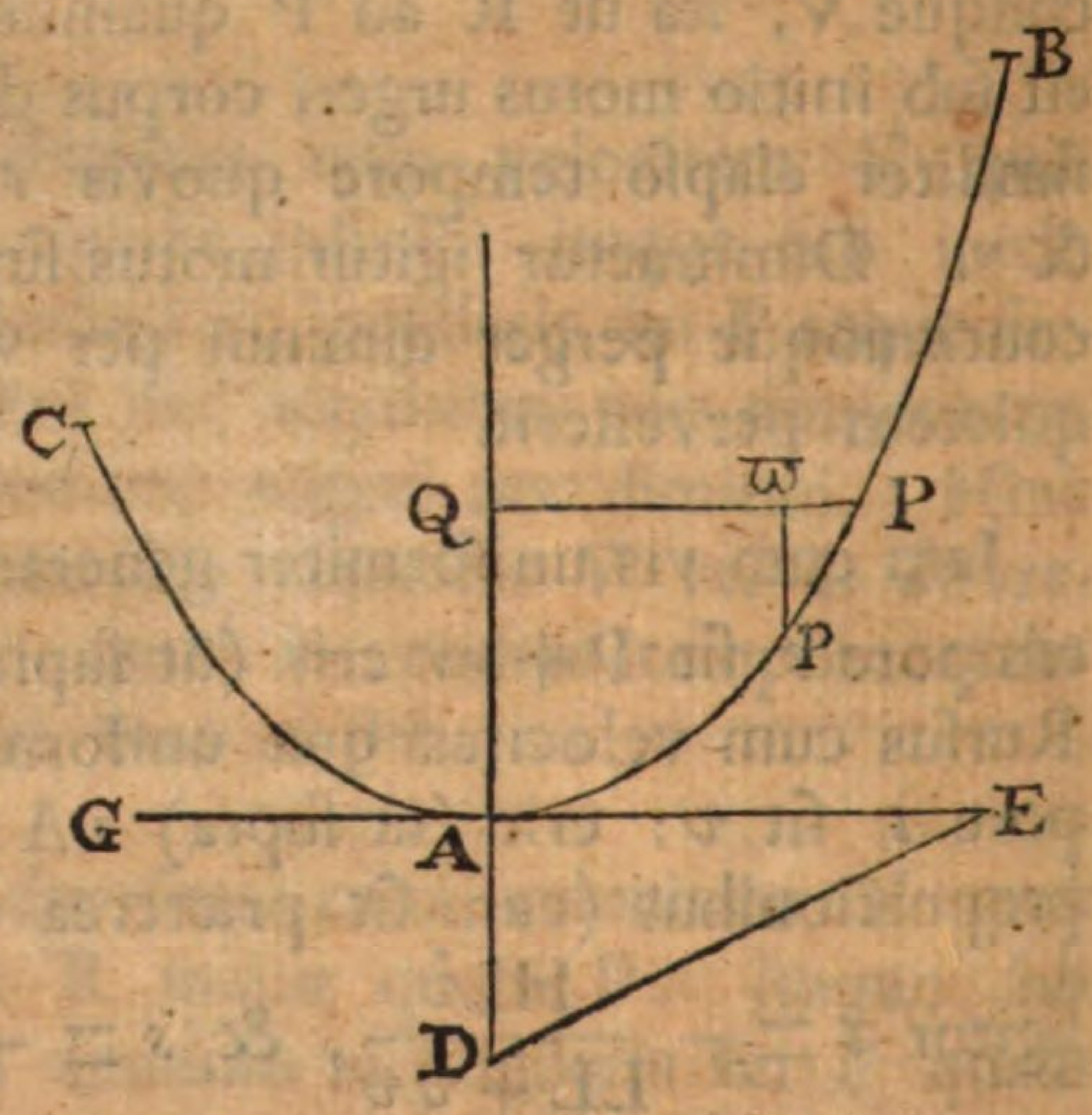
PROBLEMATA velocitas v in fine dati temporis t; indeque datur resistentia r in fine temporis ejusdem, sumendo r ad P ut vv ad LL; denique hinc datur spatium in eodem tempore ascensu descriptum, sumendo $s = A \left| \frac{P+R}{P+r} \right.$.

Notandum tamen, si H (Rad : Tang : : L : V) dicatur T, & $A \left| \frac{P+R}{P} \right.$ dicatur S, pervenire corpus ad quietem & ad altitudinem summam S elapso tempore T. Unde si t major sit quam T, adhibenda erit solutio casus primi.

PROBLEMA XII.

Invenire quotlibetcunque puncta Catenariæ.

ESTO *BAC* Catena tenuissima, seu potius Linea Mathematica per totam longitudinem vi quavis minima flexibilis, quæ neque extendi possit neque contrahi. Hæc a terminis *B* & *C* dependens vi proprii ponderis, per omnes sui particulas æquales æqualiter diffusi, componatur in Figuram curvam *BPpAC*: oportet hujusce Figuræ quotlibetcunque puncta *P* invenire.



Si per terminos *B* & *C* transire intelligatur planum quod sit ad Horizontem erectum: perspicuum est in hoc plano sita esse puncta omnia Curvæ propositæ, scilicet ut descendant quousque fieri potest. Per infimum punctum *A* ducatur ad Horizontem perpendicularis *AQ*, huic normalis sit *PϖQ* a quovis alio puncto *P*, & per punctum *p* puncto *P* vicinissimum agatur *pϖ* parallela rectæ *AQ*, vocentur autem *AQ*, x; *PQ*, y; arcus *AP*, z: & lineolæ *pϖ*, *Pϖ*, *Pp* erunt ad invicem ut $\dot{x}$, $\dot{y}$, $\dot{z}$. Jam cum arcus *AP* in æquilibrio sustineatur per vim ponderis sui cujus directio parallela est lineolæ *ϖp*, per vim

vim contigui arcus AC trahentis ſecundum directionem tangentis ad A lineolæ $P\varpi$ parallelam, perque vim contigui arcus PB trahentis in directione lineolæ pP: patet ex Mechanicis, eſſe has vires ad invicem ut ϖp, $P\varpi$, pP, ſive ut $\dot{x}$, $\dot{y}$, $\dot{z}$. Itaque ſi pondus arcus AP exponatur per longitudinem ejus z, & vis data trahentis arcus AC exponatur per datam longitudinem a: erit $\dot{x}$ ad $\dot{y}$ ut z ad a, indeque $\dot{x}$ ad $\sqrt{\dot{x}\dot{x}+\dot{y}\dot{y}}$ ſive $\dot{z}$ ut z ad $\sqrt{aa+zz}$.

Eſt igitur $\dot{x}=\frac{z\dot{z}}{\sqrt{aa+zz}}$, unde $a+x=\sqrt{aa+zz}$, atque adeo $z=\sqrt{\overline{a+x}|^2-aa}=\sqrt{2ax+xx}$. Itaque ſi in recta QA deorſum producta capiatur AD æqualis ipſi a, & in tangente AE capiatur longitudo AE æqualis arcui AP, & jungatur DE: erit hæc æqualis ipſi DQ. Unde ſi detur AE longitudo alicujus arcus AP & ſimul AQ altitudo ejuſdem arcus: dabitur AD æqualis ipſi a, jungendo QE & eandem biſecando perpendiculariter; tranſibit enim perpendicularis per punctum D. Data vero ſemel AD, deinde dabitur AE longitudo cujuſvis arcus AP cujus datur altitudo AQ, centro D intervallo DQ deſcribendo circulum qui ſecet AE in E. Atque hæ quidem ſunt relationes mutuæ Parametri AD, arcus AP & altitudinis ejus AQ. Videamus nunc de amplitudine PQ.

Ex ſupra poſitis efficitur $\dot{y}=\frac{a\dot{x}}{z}=\frac{a\dot{x}}{x^{\frac{1}{2}}\sqrt{2a+x}}$; adeoque ſecundum FORMAM VI erit $y=2a\left|\frac{\sqrt{2a+x}+\sqrt{x}}{\sqrt{2a}}\right.=a\left|\frac{2a+2x+2\sqrt{2ax+xx}}{2a}\right.$ $=a\left|\frac{a+x+z}{a}\right.$. Eſt ergo PQ menſura rationis inter $DE+EA$ & AD vel rationis huic æqualis quæ eſt inter $AP+AQ$ & $AP-AQ$ ad Modulum AD. Dato itaque vel invento, ut ſupra, Modulo AD, ſi aſſumantur in axe AQ quotlibetcunque puncta Q, invenientur totidem Curvæ correſpondentia puncta P. $Q.E.I.$

THEOREMATA

TUM

LOGOMETRICA

TUM

TRIGONOMETRICA

Datarum Fluxionum Fluentes exhibentia,

per

METHODUM MENSURARUM

Ulterius extensam.

Operis Materiam a ROGERO COTESIO accepit,

Opus ipsum composuit

ROBERTUS SMITH

Astronomiæ & Philosophiæ Experimentalis

apud *Cantabrigienses* Professor *Plumianus*,

& Collegii S. *Trinitatis* Socius.

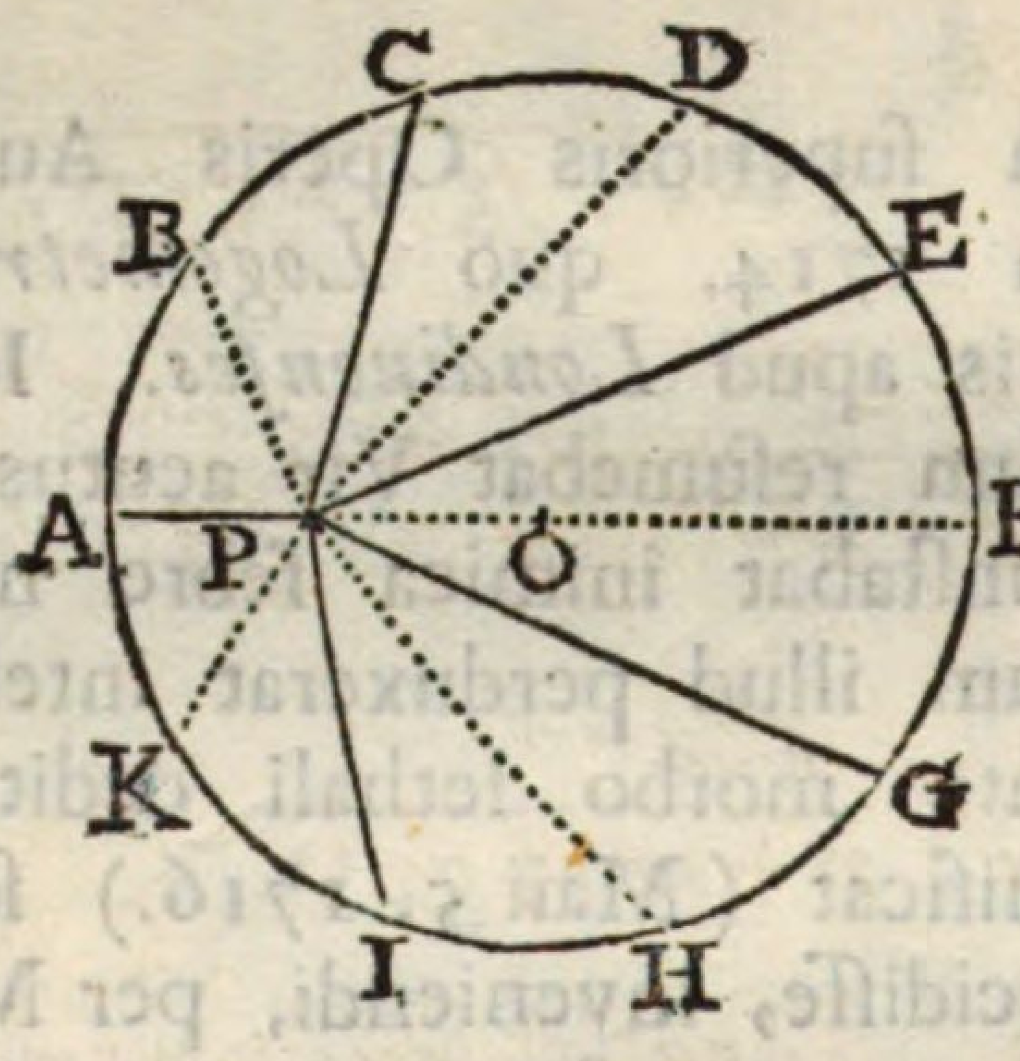
PRÆFATIO.

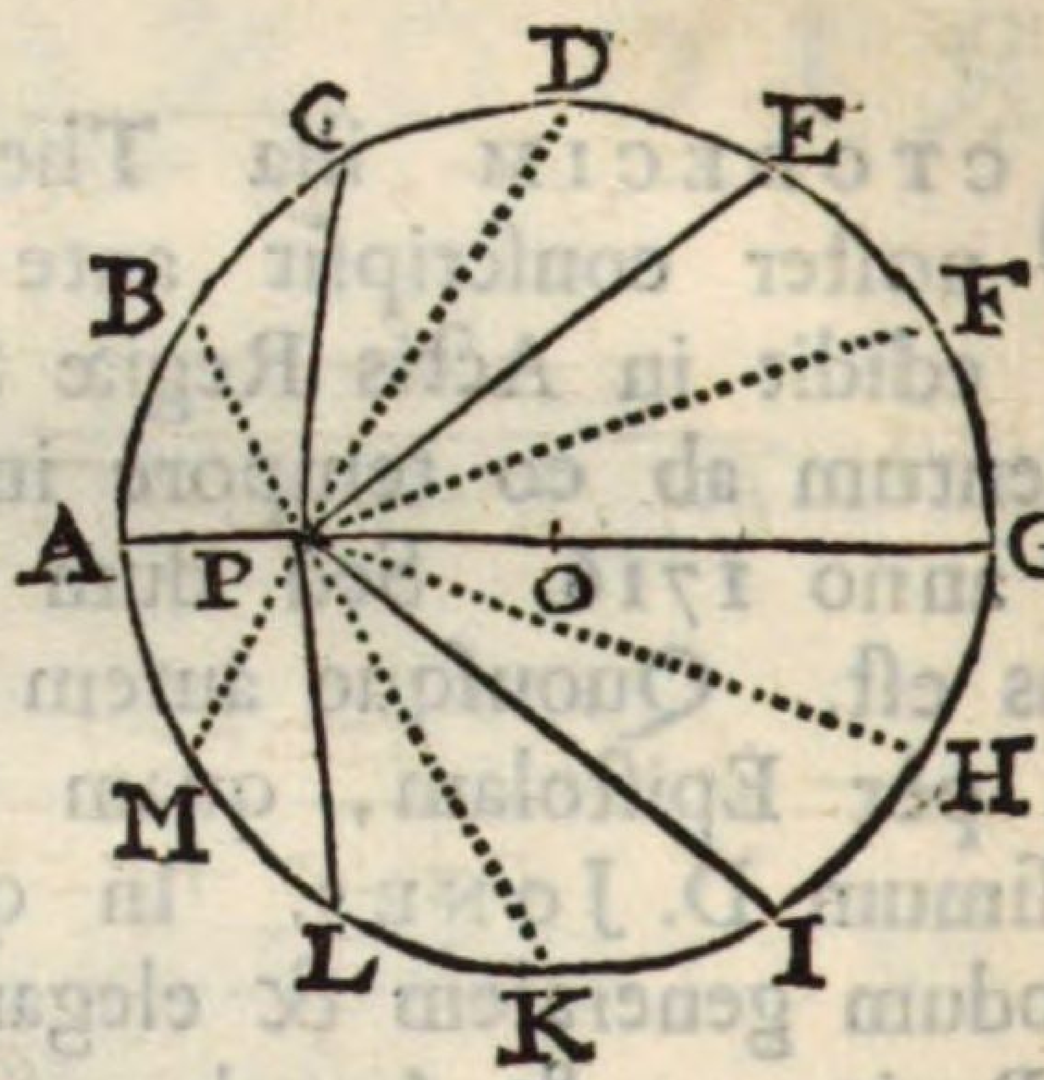
OCTODECIM illa Theoremata ſuperioris Operis Auctor noſter conſcripſit ante annum 1714, quo *Logometriam* edidit in Actis Regiæ Societatis apud *Londinenſes*. Idem argumentum ab eo tempore intermiſſum reſumebat Vir acutus ineunte anno 1716. Sed dum operi inſtabat inimica febre nobis ereptus eſt. Quouſque autem inceptum illud perduxerat intellexi ſtatim per Epiſtolam, quam ingruente morbo lethali dedit ad doctiſſimum D. JONES. In qua ſignificat (Maii 5. 1716.) ſe in Methodum generalem & elegantem incidiſſe, inveniendi, per Menſuras Rationum & Angulorum, Fluentem cujuſcunque quantitatis quæ cum hac Forma comparari poteſt $\frac{d\dot{z}z^{\theta\eta+\frac{\delta}{\lambda}\eta-1}}{e+fz^{\eta}}$: ubi d, e, f denotant quantitates quaſlibet invariabiles, z variabilem, η indicem quemvis, θ numerum quemvis integrum affirmativum aut negativum, $\frac{\delta}{\lambda}$ quemlibet fractum. Præclari hujus inventi indicio excitatus, ne cariſſimi amici famæ optimiſque Scientiis deeſſem, contuli me continuo ad ejus adverſaria. Quæ quanquam primo intuitu, Sibyllæ foliis obſcuriora videbantur, quod nullo ordine nec verbo erant explicata: multiplices tamen conjectandi occaſiones præbendo, ſpem fortiter conceptam non fefellerunt. Quippe cum ea ſæpius evolverem aliamque ex alia rem lucrarer, revocavi tandem ab interitu Theorema pulcherrimum; fundamentum ſcilicet methodi modo memoratæ: quod ut candide agam ſic deſcribo.

Si quærantur Factores Binomii $a^{\lambda} \pm x^{\lambda}$, Indice λ exiſtente quolibet integro: dividatur circuli circumferentia $ABCD$ cujus cen-

 trum

trum O, in totidem partes æquales AB, BC, CD, DE, EF, &c. quot sint unitates in 2λ; & ab omnibus divisionibus ad punctum quodvis P in OA radio si opus producto situm, ducantur rectæ AP, BP, CP, DP, EP, FP, &c; deinde positis $OA=a$, $OP=x$, contentum sub omnibus AP, CP, EP, &c sumptis a divisionibus alternis per integrum circuitum, adæquabit $a^\lambda - x^\lambda$ vel $x^\lambda - a^\lambda$, prout P fuerit intra vel extra circulum: & contentum sub reliquis BP, DP, FP, &c in locis reliquis alternis adæquabit $a^\lambda + x^\lambda$.





Exempli gratia si λ sit 5, dividatur circumferentia in 10 partes æquales, eritque $AP \times CP \times EP \times GP \times IP = OA^5 - OP^5$ existente P intra circulum: & $BP \times DP \times FP \times HP \times KP = OA^5 + OP^5$. Similiter si λ sit 6, divisa circumferentia in 12 partes æquales: erit $AP \times CP \times EP \times GP \times IP \times LP = OA^6 - OP^6$, existente P intra circulum; & $BP \times DP \times FP \times HP \times KP \times MP = OA^6 + OP^6$.

Narrat insuper in iisdem literis se in Methodo Fluxionum Inversa ulteriores progressus fecisse, & ostendere posse quomodo per mensuras rationum & angulorum exhibendi sint Fluentes fluxionum hujus Formæ $\frac{dzz^{\theta n+\frac{\delta}{\lambda}n-1}}{e+fz^n+gz^{2n}}$ vel etiam hujus $\frac{dzz^{\theta n+\frac{\delta}{\lambda}n-1}}{e+fz^n+gz^{2n}+hz^{3n}}$ absque ulla exceptione aut limitatione: si θ ut supra denotaverit numerum quemvis integrum affirmativum aut negativum, & λ denominator

minator fractionis $\frac{\delta}{\lambda}$ denotaverit numerum quemlibet hujus seriei 2. 4. 8. 16. 32. &c, scripto quolibet numero integro pro ejusdem numeratore δ. Et deinde admonet sibi videri maxime verisimile, Fluentem cujuscunque fluxionis rationalis pendere a mensuris rationum & angulorum, exceptis iis quæ terminis finitis etiam absque mensuris designari possunt. Denique sub finem ejusdem epistolæ sequentes Regulas addit, quas huc spectantes memoriæ causa annotaverat.

Positis
$$p = \tfrac{1}{2}f + \sqrt{\tfrac{1}{4}ff - eg}. \qquad s = \sqrt{-fg + 2\sqrt{eg^3}}.$$
$$r = \sqrt{eg}.$$
$$q = \tfrac{1}{2}f - \sqrt{\tfrac{1}{4}ff - eg}. \qquad t = \sqrt{-fg - 2\sqrt{eg^3}}.$$

I. Erit $$\frac{d\dot{z}z^{\theta n-1}}{e+fz^{2n}+gz^{4n}} = \frac{g}{q-p} \times \frac{d\dot{z}z^{\theta n-1}}{p+gz^{2n}} + \frac{g}{p-q} \times \frac{d\dot{z}z^{\theta n-1}}{q+gz^{2n}}$$

II. vel $$= \frac{g}{2r} \times \frac{d\dot{z}z^{\theta n-1}}{r+sz^{n}+gz^{2n}} + \frac{gg}{2rs} \times \frac{d\dot{z}z^{\theta n+n-1}}{r+sz^{n}+gz^{2n}}$$
$$+ \frac{g}{2r} \times \frac{d\dot{z}z^{\theta n-1}}{r-sz^{n}+gz^{2n}} - \frac{gg}{2rs} \times \frac{d\dot{z}z^{\theta n+n-1}}{r-sz^{n}+gz^{2n}}$$

III. vel $$= \frac{g}{2r} \times \frac{d\dot{z}z^{\theta n-1}}{r+tz^{n}-gz^{2n}} - \frac{gg}{2rt} \times \frac{d\dot{z}z^{\theta n+n-1}}{r+tz^{n}-gz^{2n}}$$
$$+ \frac{g}{2r} \times \frac{d\dot{z}z^{\theta n-1}}{r-tz^{n}-gz^{2n}} + \frac{gg}{2rt} \times \frac{d\dot{z}z^{\theta n+n-1}}{r-tz^{n}-gz^{2n}}$$

Si e, g sunt signis dissimilibus: solvitur per I.
Si e, g sunt signis similibus & f dissimili: solvitur per II.
Si e, f, g sunt signis similibus & $ff \sqsubset 4eg$: solvitur per I.
Si e, f, g sunt signis similibus & $ff \sqsupset 4eg$: solvitur per II.
Si e, g sunt signis similibus, f dissimili & $ff \sqsubset 4eg$: solvitur per I. II. III.

Hæc exposui ut appareat quod fuerit Auctoris fundamentum quo superstruxi hæc Theoremata. Index Formarum huic præ-

præfationi annexus Operis Synopſin exhibet: & videor mihi ad umbilicum perduxiſſe quicquid tunc temporis Auctori in animo erat. Formæ illæ diſponuntur in certas Series: & ne deeſſet ordinis ornamentum, locis ſuis inſeruntur octodecim Formæ ab Opere ſuperiore huc tranſlatæ & novo hoc cultu indutæ. Singulæ formamarum ſeries præter ultimam ad infinitum produci poſſunt: & ſeries prima reliquis longius producitur ut methodi lex & progreſſio clarius appareat. His inſuper adjicere potuiſſem Fluentes formarum & ſerierum ſequentium: niſi calculi faſtidium me cepiſſet.

$$\frac{d\dot{z}z^{\theta\eta-1}}{e+fz^{\eta}+gz^{2\eta}+hz^{3\eta}+kz^{4\eta}};\quad \frac{d\dot{z}z^{\theta\eta-1}}{e+fz^{\eta}+gz^{2\eta}+hz^{3\eta}+kz^{4\eta}+lz^{5\eta}}$$

$$\frac{d\dot{z}z^{\theta\eta-1}\times\overline{k+lz^{\eta}}|^{\frac{\delta}{\lambda}}}{e+fz^{\eta}+gz^{2\eta}+hz^{3\eta}};\quad \frac{d\dot{z}z^{\theta\eta-1}}{e+fz^{\eta}+gz^{2\eta}+hz^{3\eta}}\times\left.\frac{k+lz^{\eta}}{m+nz^{\eta}}\right|^{\frac{\delta}{\lambda}}.$$

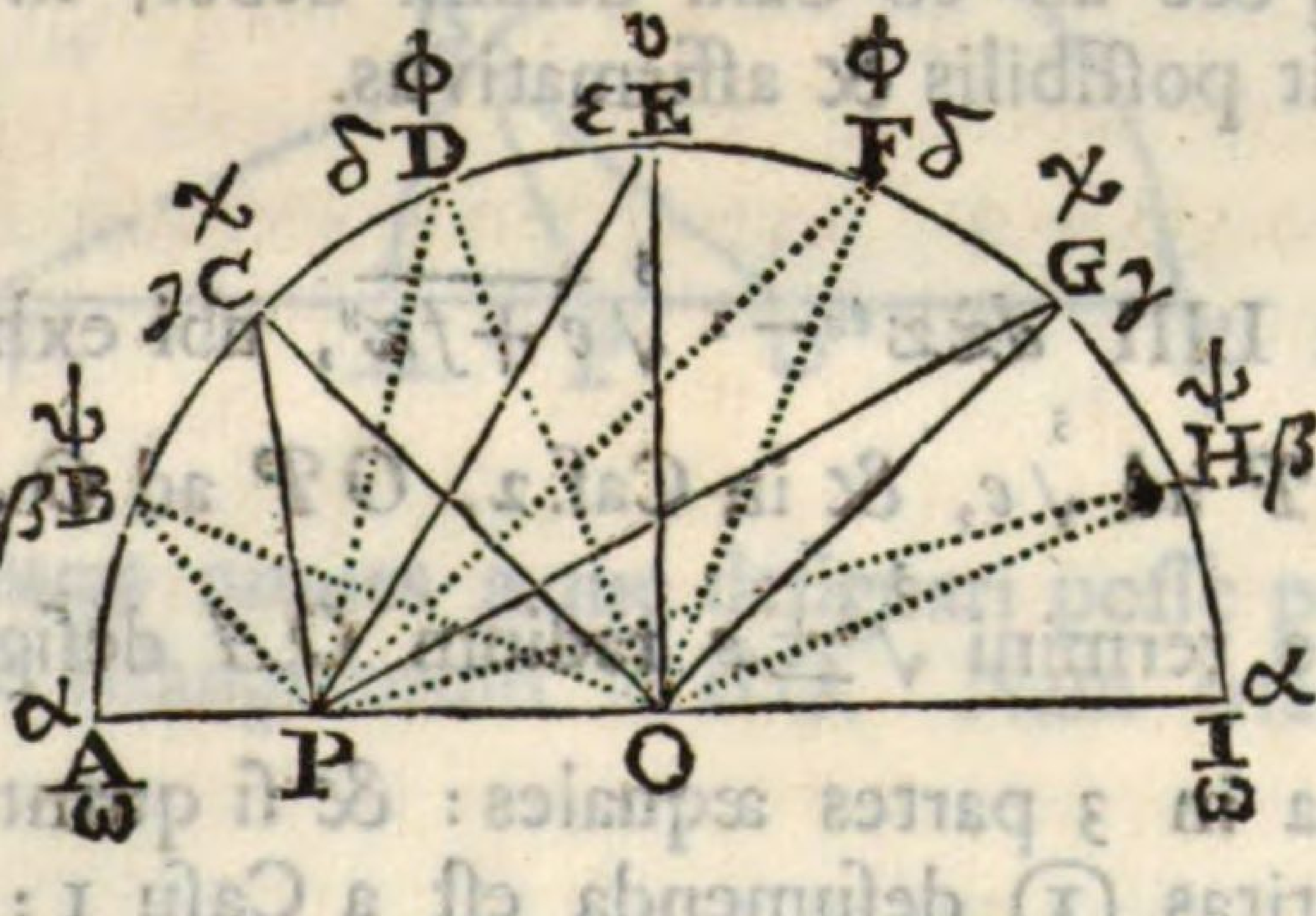
In his quantitates θ, δ, λ denotant numeros quoſlibet integros affirmativos vel negativos.

Erunt autem fortaſſe qui me vel hactenus progreſſum otio abuſum eſſe dicant, quique tanti apparatus utilitatem requirant. At vero cum viderint, ſummorum Geometrarum inventa Fluentibus ſimplicioribus potiſſimum deberi: ideone cenſebunt magis compoſitas utilitate carere? Caveant ſane, ſi qui ſunt qui hoc cenſent, ne in rebus inveſtigandis parum verſati eſſe videantur. Quanto rectius Newtonus in eximio Tractatu ſuo de Quadratura Curvarum, *Ex his principiis via ſternitur ad majora.* Summis itaque Geometris qualiacunque optimæ artis incrementa non ingrata fore confido. Norunt illi multa ſupereſſe quæ Geometriam meliorem etiamnum effugiunt. Vix tamen effugerent, ſi vel illam Newtoni vel hanc Cotesii methodum aliaſve conſimiles, ulteriore cultura univerſales reddere poſſemus. Supereſt ut pauca adjiciantur de explicatione generali Notarum & Figurarum.

In omnibus Fluxionum Formis unica eſt variabilis & indeterminata quantitas z, ejuſque Fluxio $\dot{z}$: reliquæ notæ deſignant quantitates conſtantes & prorſus immutabiles. Quantitates η, $\theta\eta-1$, $\theta\eta+\frac{1}{2}\eta-1$, &c. ſunt indices poteſtatum ipſius z. Valores quantitatis θ ſcribuntur in ſiniſtro margine paginarum, & e regione apponitur Theorema vel Fluens cujuſque Fluxionis, quam quiſque valor conſtituit.

In omnibus Theorematis valores quantitatum ①, ②, $\boxed{1}$, $\boxed{2}$, &c. exponuntur in calce cujusque paginæ per Mensuras rationum & angulorum. Hæ mensuræ ubique determinantur mirabili simplicitate per circuli Semicircumferentiam in partes æquales divisam, ad modum sequentis exempli in Forma XX $\frac{dzz^{6n+\frac{7}{8}n-1}}{e+fz^n}$. Pag. 149.

In data recta AO, si opus producta, sumatur OP ad OA ut $\sqrt[8]{z^n}$ ad $\sqrt[8]{\frac{e}{f}}$, si quantitas $\frac{e}{f}$ sit affirmativa; aliter ut $\sqrt[8]{z^n}$ ad $\sqrt[8]{\frac{-e}{f}}$. In utroque casu dividatur semicircumferentia AEI, radio AO descripta, in 8 partes æquales AB, BC, CD, DE, EF, FG, GH, HI; totidem nempe quot unitates habeat radicalis nota termini $\sqrt[8]{\frac{e}{f}}$ Radium OA exponentis; & ad centrum O ducantur rectæ BO, CO, DO, EO, FO, GO, HO, IO, itemque ad punctum P



totidem aliæ BP, CP, DP, EP, FP, GP, HP, IP: deinde in calce paginæ citatæ exhibetur in Casu 1, $① = {}^{+}\frac{1}{8}\sqrt[8]{\frac{f}{e}}$ in $\beta(BP:BO) + \delta(DP:DO) - \delta(FP:FO) - \beta(HP:HO) + \psi(PBO) + \varphi(PDO) + \varphi(PFO) + \psi(PHO)$; & in Casu 2, $① = {}^{-}\frac{1}{8}\sqrt[8]{\frac{-f}{e}}$ in $-(AP:AO) - \gamma(CP:CO) + \gamma(GP:GO) + (IP:IO) - \chi(PCO) - \upsilon(PEO) - \chi(PGO)$. Notarum sensus est, quantitatem ${}^{+}\frac{1}{8}\sqrt[8]{\frac{f}{e}}$ ducendam esse in mensuras rationum BP ad BO, DP ad DO, FP ad FO, HP ad HO, ad modulos respectivos β, δ, $-\delta$, $-\beta$; itemque in mensuras angulorum PBO,

 PDO,

PDO, PFO, PHO, ad modulos reſpectivos $\psi, \varphi, \varphi, \psi$. Moduli autem β, γ, δ, &c ſunt numeri, quos reſpective deſignabunt arcuum chordæ a punctis B, C, D, &c ad radium primum AO parallelas ductæ, poſita unitate pro iſto radio: & moduli ψ, χ, φ, &c ſunt etiam numeri, quos reſpective deſignabunt aliæ chordæ ab iiſdem punctis B, C, D, &c ad eundem radium AO perpendiculariter ductæ & in altera ſemicircumferentia terminatæ, poſita itidem unitate pro radio. Dantur itaque hi moduli commodiſſime per Canonem Sinuum. Quod ſi menſuræ nullus præfigatur modulus, ibi ſemper ſubintelligitur unitas. Menſuras autem communem modulum habentes componere licebit.

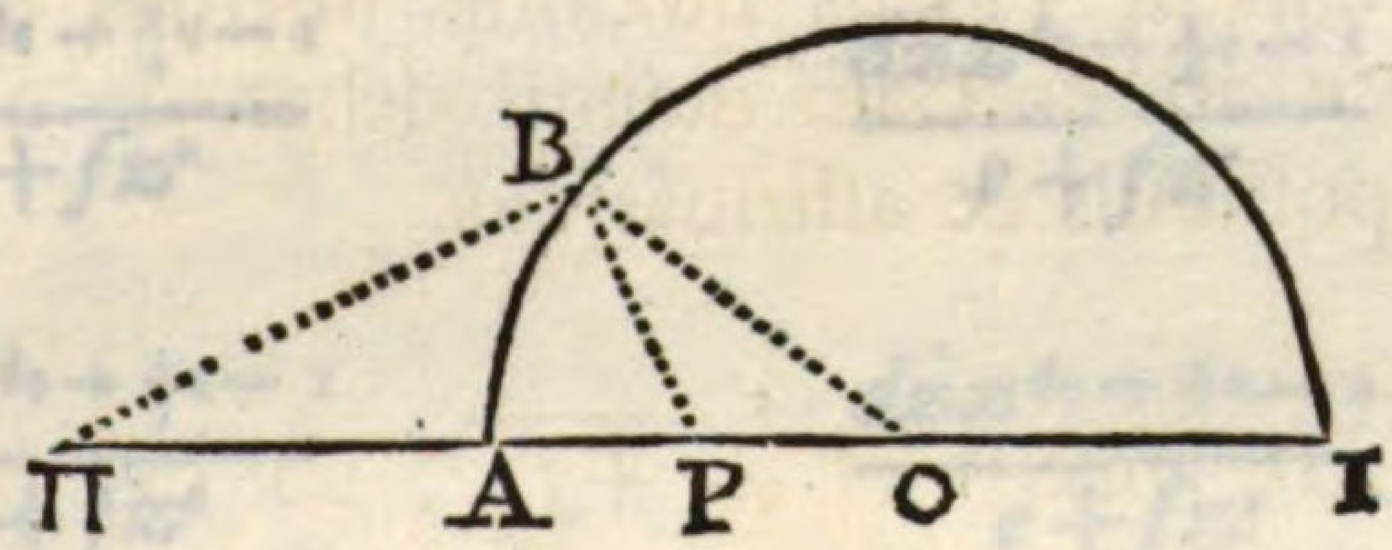
Omnino ſimilis eſt conſtructio Figurarum & determinatio Menſurarum per omnia Theoremata: dividendo ſcilicet circuli ſemicircumferentiam in totidem partes æquales quot ſint unitates in indice termini dati qui Radium AO deſignat; & quoties Theoremata conſulenda ſunt, ut inveniatur Fluens fluxionis alicujus datæ, quantitas $\textcircled{1}$ vel $\textcircled{2}$ vel $\boxed{1}$ &c ab eo Caſu deſumi debet, in quo valor iſtius radii OA prodibit poſſibilis & affirmativus.

Sic in Forma LIII $d\dot{z}z^{\theta n-1}\sqrt[3]{e+fz^n}$, ubi exhibetur in Caſ. 1. OP ad OA ut P ad $\sqrt[3]{e}$, & in Caſ. 2. OP ad OA ut P ad $\sqrt[3]{-e}$, exiſtente 3 indice termini $\sqrt[3]{\pm e}$ radium OA deſignantis; dividitur ſemicircumferentia in 3 partes æquales: & ſi quantitas $\sqrt[3]{e}$ prodibit affirmativa, quantitas $\textcircled{1}$ deſumenda eſt a Caſu 1; aliter a Caſu 2. Sic etiam in Forma I $\frac{d\dot{z}z^{\theta n-1}}{e+fz^n}$ ubi exhibetur in Caſ. 1. OP ad OA ut z^n ad $\frac{e}{f}$, & in Caſ. 2. OP ad OA ut z^n ad $\frac{-e}{f}$, exiſtente 1 indice termini $\pm\frac{e}{f}$ vel $\sqrt[1]{\pm\frac{e}{f}}$ radium OA deſignantis; dividitur ſemicircumferentia ut ita dicam in 1 partem, id eſt integra manet: & prout $\frac{e}{f}$ prodierit affirmativa vel negativa, quantitas $\textcircled{1}$ deſumenda eſt vel a Caſu 1, vel a Caſu 2.

Notatu

Notatu dignum eſt, cujuſcunque quantitatis hujus Formæ $\frac{d\dot{z}z^{\theta\eta+\frac{\delta}{\lambda}\eta-1}}{e+fz^{\eta}}$ & aliarum inde pendentium Fluentem exprimi poſſe ope Semicircumferentiæ in totidem partes æquales diviſæ quot ſint unitates in denominatore Fractionis $\frac{\delta}{\lambda}$, ſive termini ejus ſint Primi inter ſe ſive utcunque compoſiti. Adeo ut eadem Fluens infinitis modis exponi poſſit per menſuras diverſas, quas tamen omnes eadem ſemicircumferentia mirabili elegantia deſignabit. Sed ad uſum Theorematum, de quo jam agitur, hæc exponere non attinet.

Nec indignum eſt animadverſione, ſi in OA radio producto, ipſis $O\mathcal{P}$, OA ſumatur tertia proportionalis $O\Pi$ & ab omnibus ſemicircumferentiæ diviſionibus ducantur rectæ $B\Pi$, prout ſupra ductæ



erant $B\mathcal{P}$: menſuras omnes æque deſignari poſſe per partes triangulorum $B\Pi O$ ac triangulorum $\mathcal{P}BO$. Nam triangula illa ſunt ſimilia.

[illegible] chordæ ab ulteri[illegible]

[illegible] ejusdem unitate pro-

[illegible]

[illegible] exemplis in indice ter-

[illegible] & quoties Theoremata con-

[illegible] aliquot datæ, quantitas

[illegible] debet, in quo valor istius

[illegible]

Sic in Forma [illegible] ut exhibetur in Cas. 1.

[illegible] OP ad OA ut P ad $\sqrt{-e}$,

[illegible] & si quantitas $\sqrt{e}$ prodiit

affirmativa, quantitas [illegible] desumenda est a Casu 1; aliter a Casu 2.

Sic etiam in Forma 2 [illegible] exhibetur in Cas. 1. OP ad OA

ut [illegible] ad [illegible], & in Cas. 2. OP ad OA ut [illegible] ad [illegible], existente 1 in-

dice termini [illegible] vel [illegible] radium OA designantis; dividitur semi-

circumferentia ut ita dicam in 1 partem, id est integra manet: &

prout [illegible] prodierit affirmativa vel negativa, quantitas (1) desumenda

est vel a Casu 1, vel a Casu 2.

Notatu

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29. 30.	$\frac{d\dot{z}z^{\theta n-1}}{e+fz^n+gz^{2n}}$	$\frac{d\dot{z}z^{\theta n+\frac{1}{2}n-1}}{e+fz^n+gz^{2n}}$	159
31. 32.	$\frac{d\dot{z}z^{\theta n+\frac{1}{3}n-1}}{e+fz^n+gz^{2n}}$	$\frac{d\dot{z}z^{\theta n+\frac{2}{3}n-1}}{e+fz^n+gz^{2n}}$	162
33. 34.	$\frac{d\dot{z}z^{\theta n+\frac{1}{4}n-1}}{e+fz^n+gz^{2n}}$	$\frac{d\dot{z}z^{\theta n+\frac{3}{4}n-1}}{e+fz^n+gz^{2n}}$	164

Series Quarta.

Formæ			Pag.
35. 36.	$\frac{d\dot{z}z^{\theta n-1}}{\overline{e+fz^n+gz^{2n}}\times\overline{k+lz^n}}$	$\frac{d\dot{z}z^{\theta n+\frac{1}{2}n-1}}{\overline{e+fz^n+gz^{2n}}\times\overline{k+lz^n}}$	169
37. 38.	$\frac{d\dot{z}z^{\theta n+\frac{1}{3}n-1}}{\overline{e+fz^n+gz^{2n}}\times\overline{k+lz^n}}$	$\frac{d\dot{z}z^{\theta n+\frac{2}{3}n-1}}{\overline{e+fz^n+gz^{2n}}\times\overline{k+lz^n}}$	172
39. 40.	$\frac{d\dot{z}z^{\theta n+\frac{1}{4}n-1}}{\overline{e+fz^n+gz^{2n}}\times\overline{k+lz^n}}$	$\frac{d\dot{z}z^{\theta n+\frac{3}{4}n-1}}{\overline{e+fz^n+gz^{2n}}\times\overline{k+lz^n}}$	174

Series Quinta.

41. 42.	$\frac{d\dot{z}z^{\theta n-1}}{e+fz^n+gz^{2n}+hz^{3n}}$	$\frac{d\dot{z}z^{\theta n+\frac{1}{2}n-1}}{e+fz^n+gz^{2n}+hz^{3n}}$	178
43. 44.	$\frac{d\dot{z}z^{\theta n+\frac{1}{3}n-1}}{e+fz^n+gz^{2n}+hz^{3n}}$	$\frac{d\dot{z}z^{\theta n+\frac{2}{3}n-1}}{e+fz^n+gz^{2n}+hz^{3n}}$	182
45. 46.	$\frac{d\dot{z}z^{\theta n+\frac{1}{4}n-1}}{e+fz^n+gz^{2n}+hz^{3n}}$	$\frac{d\dot{z}z^{\theta n+\frac{3}{4}n-1}}{e+fz^n+gz^{2n}+hz^{3n}}$	184

Series Sexta.

47. 48.	$\frac{d\dot{z}z^{\theta n-1}}{\sqrt{e+fz^n}}$	$\frac{d\dot{z}z^{\theta n+\frac{1}{2}n-1}}{\sqrt{e+fz^n}}$	188
49. 50.	$d\dot{z}z^{\theta n-1}\sqrt{e+fz^n}$	$d\dot{z}z^{\theta n+\frac{1}{2}n-1}\sqrt{e+fz^n}$	190

Formæ			Pag.
51. 52.	$\frac{d\dot{z}z^{\theta n-1}}{\sqrt[3]{e+fz^n}}$	$\frac{d\dot{z}z^{\theta n+\frac{1}{3}n-1}}{\sqrt[3]{e+fz^n}}$	192
53. 54.	$d\dot{z}z^{\theta n-1}\sqrt[3]{e+fz^n}$	$d\dot{z}z^{\theta n+\frac{2}{3}n-1}\sqrt[3]{e+fz^n}$	194
55. 56.	$\frac{d\dot{z}z^{\theta n-1}}{\sqrt[4]{e+fz^n}}$	$\frac{d\dot{z}z^{\theta n+\frac{1}{4}n-1}}{\sqrt[4]{e+fz^n}}$	196
57. 58.	$d\dot{z}z^{\theta n-1}\sqrt[4]{e+fz^n}$	$d\dot{z}z^{\theta n+\frac{3}{4}n-1}\sqrt[4]{e+fz^n}$	198

Series Septima.

59. 60.	$\frac{d\dot{z}z^{\theta n-1}}{g+hz^n\sqrt{e+fz^n}}$	$\frac{d\dot{z}z^{\theta n+\frac{1}{2}n-1}}{g+hz^n\sqrt{e+fz^n}}$	200
61. 62.	$\frac{d\dot{z}z^{\theta n-1}\sqrt{e+fz^n}}{g+hz^n}$	$\frac{d\dot{z}z^{\theta n+\frac{1}{2}n-1}\sqrt{e+fz^n}}{g+hz^n}$	202
63. 64.	$\frac{d\dot{z}z^{\theta n-1}}{g+hz^n\sqrt[3]{e+fz^n}}$	$\frac{d\dot{z}z^{\theta n+\frac{1}{3}n-1}}{g+hz^n\sqrt[3]{e+fz^n}}$	204
65. 66.	$\frac{d\dot{z}z^{\theta n-1}\sqrt[3]{e+fz^n}}{g+hz^n}$	$\frac{d\dot{z}z^{\theta n+\frac{2}{3}n-1}\sqrt[3]{e+fz^n}}{g+hz^n}$	206
67. 68.	$\frac{d\dot{z}z^{\theta n-1}}{g+hz^n\sqrt[4]{e+fz^n}}$	$\frac{d\dot{z}z^{\theta n+\frac{1}{4}n-1}}{g+hz^n\sqrt[4]{e+fz^n}}$	208
69. 70.	$\frac{d\dot{z}z^{\theta n-1}\sqrt[4]{e+fz^n}}{g+hz^n}$	$\frac{d\dot{z}z^{\theta n+\frac{3}{4}n-1}\sqrt[4]{e+fz^n}}{g+hz^n}$	210

Series Octava.

Formæ			Pag.
71. 72.	$\dfrac{d\dot{z}z^{\theta n-1}}{e+fz^n+gz^{2n}\sqrt{k+lz^n}}$	$\dfrac{d\dot{z}z^{\theta n+\frac{1}{2}n-1}}{e+fz^n+gz^{2n}\sqrt{k+lz^n}}$	212
73. 74.	$\dfrac{d\dot{z}z^{\theta n-1}\sqrt{k+lz^n}}{e+fz^n+gz^{2n}}$	$\dfrac{d\dot{z}z^{\theta n+\frac{1}{2}n-1}\sqrt{k+lz^n}}{e+fz^n+gz^{2n}}$	216
75. 76.	$\dfrac{d\dot{z}z^{\theta n-1}}{e+fz^n+gz^{2n}\sqrt[3]{k+lz^n}}$	$\dfrac{d\dot{z}z^{\theta n+\frac{1}{3}n-1}}{e+fz^n+gz^{2n}\sqrt[3]{k+lz^n}}$	220
77. 78.	$\dfrac{d\dot{z}z^{\theta n-1}\sqrt[3]{k+lz^n}}{e+fz^n+gz^{2n}}$	$\dfrac{d\dot{z}z^{\theta n+\frac{2}{3}n-1}\sqrt[3]{k+lz^n}}{e+fz^n+gz^{2n}}$	222
79. 80.	$\dfrac{d\dot{z}z^{\theta n-1}}{e+fz^n+gz^{2n}\sqrt[4]{k+lz^n}}$	$\dfrac{d\dot{z}z^{\theta n+\frac{1}{4}n-1}}{e+fz^n+gz^{2n}\sqrt[4]{k+lz^n}}$	224
81. 82.	$\dfrac{d\dot{z}z^{\theta n-1}\sqrt[4]{k+lz^n}}{e+fz^n+gz^{2n}}$	$\dfrac{d\dot{z}z^{\theta n+\frac{3}{4}n-1}\sqrt[4]{k+lz^n}}{e+fz^n+gz^{2n}}$	230

Series Nona et Decima.

83. 84.	$d\dot{z}z^{\theta n-1}\sqrt[2]{\dfrac{e+fz^n}{g+hz^n}}$	$d\dot{z}z^{\theta n-1}\sqrt[3]{\dfrac{e+fz^n}{g+hz^n}}$	236
85. 86.	$d\dot{z}z^{\theta n-1}\sqrt[4]{\dfrac{e+fz^n}{g+hz^n}}$	$\dfrac{d\dot{z}z^{\theta n-1}}{k+lz^n}\sqrt[2]{\dfrac{e+fz^n}{g+hz^n}}$	238
87. 88.	$\dfrac{d\dot{z}z^{\theta n-1}}{k+lz^n}\sqrt[3]{\dfrac{e+fz^n}{g+hz^n}}$	$\dfrac{d\dot{z}z^{\theta n-1}}{k+lz^n}\sqrt[4]{\dfrac{e+fz^n}{g+hz^n}}$	240

SERIES ULTIMA.

Formæ			Pag.
89. 90.	$\frac{d\dot{z}z^{\theta n-1}}{\sqrt{e+fz^{n}+gz^{2n}}}$	$d\dot{z}z^{\theta n-1}\sqrt{e+fz^{n}+gz^{2n}}$	242
91. 92.	$\frac{d\dot{z}z^{\theta n-1}}{\overline{k+lz^{n}}\sqrt{e+fz^{2n}+gz^{2n}}}$	$\frac{d\dot{z}z^{\theta n-1}\sqrt{e+fz^{n}+gz^{2n}}}{k+lz^{n}}$	244
93. 94.	$\frac{d\dot{z}z^{\theta n-1}}{\overline{k+lz^{n}+mz^{2n}}\sqrt{e+fz^{n}+gz^{2n}}}$	$\frac{d\dot{z}z^{\theta n-1}\sqrt{e+fz^{n}+gz^{2n}}}{k+lz^{n}+mz^{2n}}$	246

CORRI-

CORRIGENDA in THEOREMATIS ſequentibus.

P. 131. l. 2. pro $\frac{2de^4}{ef^5}$ ⓘ lege $\frac{2de^4}{\eta f^5}$ ⓘ

P. 132. l. 9. pro $\frac{3df}{5\eta eez^{\frac{2n}{3}}}$ lege $\frac{3df}{5\eta eez^{\frac{5n}{3}}}$

P. 164. l. 6. pro $\frac{-4d}{3\eta egz^{\frac{3n}{4}}}$ lege $\frac{-4d}{3\eta ez^{\frac{3n}{4}}}$

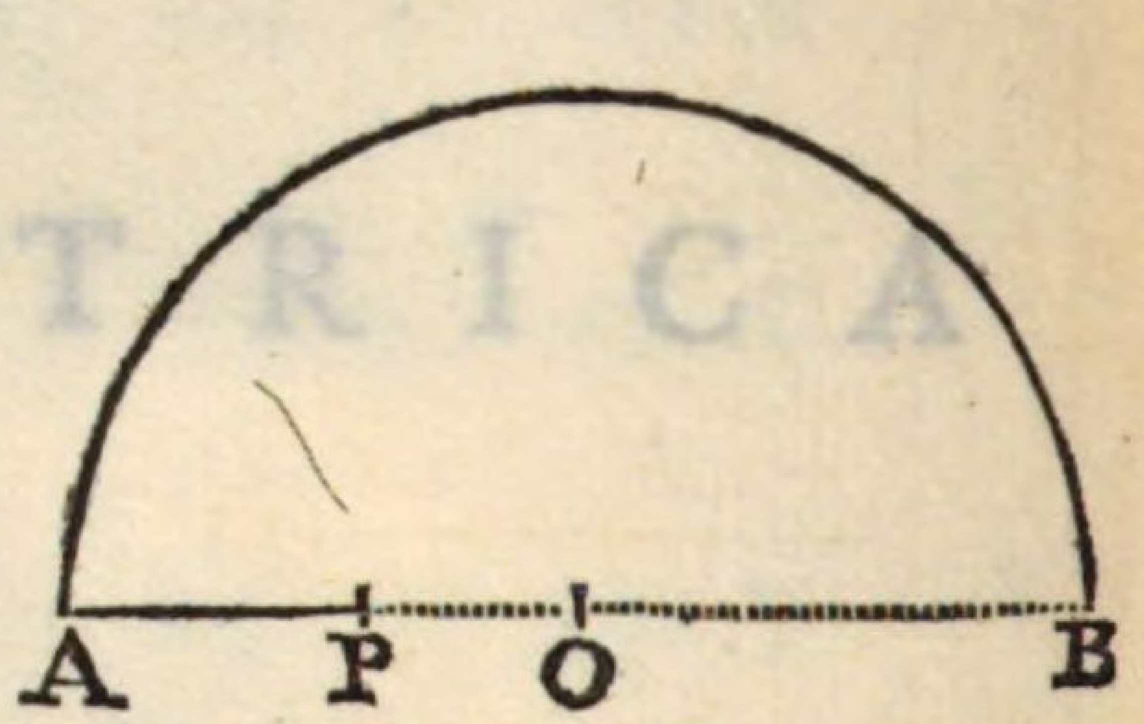
P. 177. conſule Figuram in pag. 175.

P. 231. l. 1. pro $\theta\eta + \frac{1}{4}\eta - 1$ lege $\theta\eta + \frac{3}{4}\eta - 1$.

THEOREMATA
TUM
LOGOMETRICA
TUM
TRIGONOMETRICA.

D d

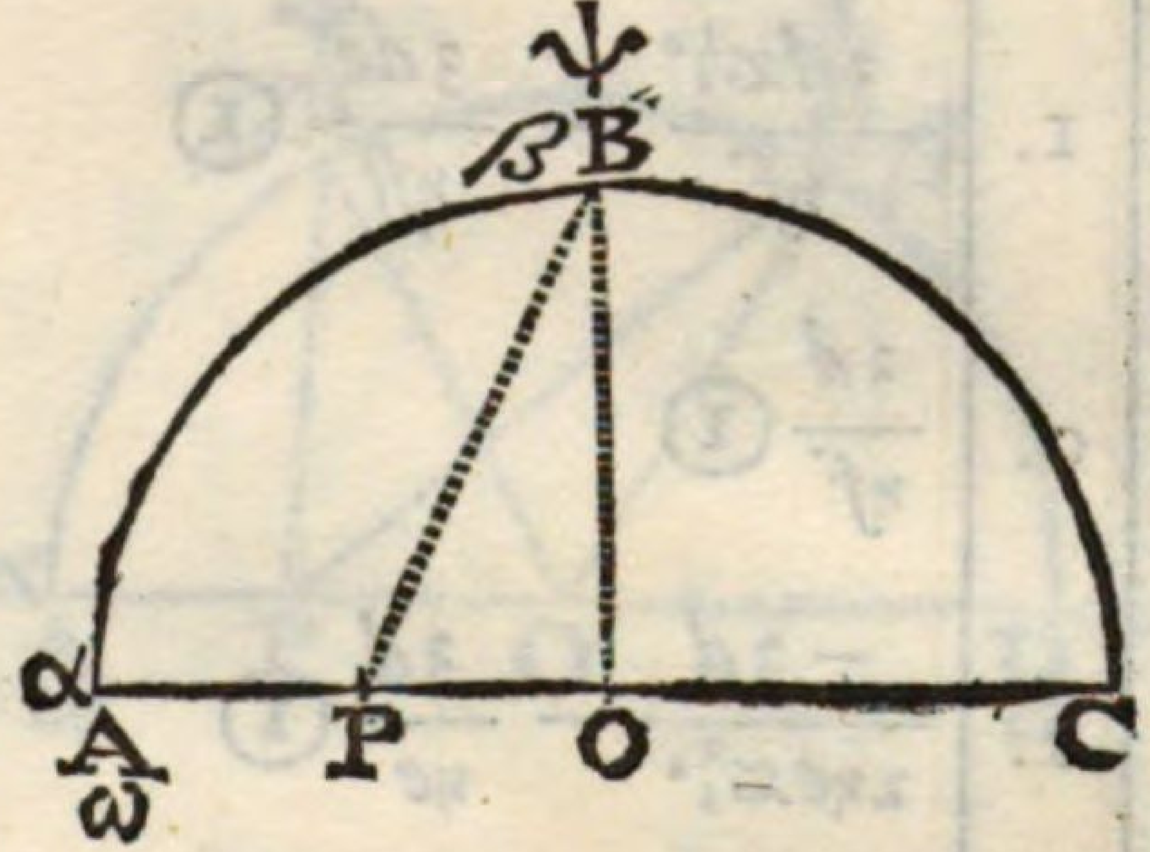
θ	FORMA I. $\dfrac{d\dot{z}z^{\theta n-1}}{e+fz^n}$
&c.	
5.	$\frac{dz^{4n}}{4\eta f}-\frac{dez^{3n}}{3\eta ff}+\frac{deez^{2n}}{2\eta f^3}-\frac{de^3z^n}{\eta f^4}+\frac{de^4}{\eta f^5}\textcircled{I}$
4.	$\frac{dz^{3n}}{3\eta f}-\frac{dez^{2n}}{2\eta ff}+\frac{deez^n}{\eta f^3}-\frac{de^3}{\eta f^4}\textcircled{I}$
3.	$\frac{dz^{2n}}{2\eta f}-\frac{dez^n}{\eta ff}+\frac{dee}{\eta f^3}\textcircled{I}$
2.	$\frac{dz^n}{\eta f}-\frac{de}{\eta ff}\textcircled{I}$
1.	$\frac{d}{\eta f}\textcircled{I}$
0.	$\frac{-d}{\eta e}\boxed{I}$
$\bar{1}$.	$\frac{-d}{\eta ez^n}+\frac{df}{\eta ee}\boxed{I}$
$\bar{2}$.	$\frac{-d}{2\eta ez^{2n}}+\frac{df}{\eta eez^n}-\frac{dff}{\eta e^3}\boxed{I}$
$\bar{3}$.	$\frac{-d}{3\eta ez^{3n}}+\frac{df}{2\eta eez^{2n}}-\frac{dff}{\eta e^3z^n}+\frac{df^3}{\eta e^4}\boxed{I}$
$\bar{4}$.	$\frac{-d}{4\eta ez^{4n}}+\frac{df}{3\eta eez^{3n}}-\frac{dff}{2\eta e^3z^{2n}}+\frac{df^3}{\eta e^4z^n}-\frac{df^4}{\eta e^5}\boxed{I}$
&c.	



Cas. 1. $\begin{cases} OP:OA::z^n:\frac{e}{f}; & \textcircled{I}=1\,(BP:BO) \\ OP:OA::\frac{1}{z^n}:\frac{f}{e}; & \boxed{I}=1\,(BP:BO)\end{cases}$

Cas. 2. $\begin{cases} OP:OA::z^n:\frac{-e}{f}; & \textcircled{I}=1\,(AP:AO) \\ OP:OA::\frac{1}{z^n}:\frac{-f}{e}; & \boxed{I}=1\,(AP:AO)\end{cases}$

θ	FORMA II. $\dfrac{dz z^{\theta n + \frac{1}{2}n - 1}}{e + fz^n}$
&c.	
4.	$\frac{2dz^{\frac{7}{2}n}}{7\eta f} - \frac{2dez^{\frac{5}{2}n}}{5\eta ff} + \frac{2deez^{\frac{3}{2}n}}{3\eta f^3} - \frac{2de^3 z^{\frac{1}{2}n}}{\eta f^4} + \frac{2de^4}{ef^5}\textcircled{I}$
3.	$\frac{2dz^{\frac{5}{2}n}}{5\eta f} - \frac{2dez^{\frac{3}{2}n}}{3\eta ff} + \frac{2deez^{\frac{1}{2}n}}{\eta f^3} - \frac{2de^3}{\eta f^4}\textcircled{I}$
2.	$\frac{2dz^{\frac{3}{2}n}}{3\eta f} - \frac{2dez^{\frac{1}{2}n}}{\eta ff} + \frac{2dee}{\eta f^3}\textcircled{I}$
1.	$\frac{2dz^{\frac{1}{2}n}}{\eta f} - \frac{2de}{\eta ff}\textcircled{I}$
0.	$\frac{2d}{\eta f}\textcircled{I}$
$\bar{1}$.	$\frac{-2d}{\eta ez^{\frac{1}{2}n}} - \frac{2d}{\eta e}\textcircled{I}$
$\bar{2}$.	$\frac{-2d}{3\eta ez^{\frac{3}{2}n}} + \frac{2df}{\eta eez^{\frac{1}{2}n}} + \frac{2df}{\eta ee}\textcircled{I}$
$\bar{3}$.	$\frac{-2d}{5\eta ez^{\frac{5}{2}n}} + \frac{2df}{3\eta eez^{\frac{3}{2}n}} - \frac{2dff}{\eta e^3 z^{\frac{1}{2}n}} - \frac{2dff}{\eta e^3}\textcircled{I}$
$\bar{4}$.	$\frac{-2d}{7\eta ez^{\frac{7}{2}n}} + \frac{2df}{5\eta eez^{\frac{5}{2}n}} - \frac{2dff}{3\eta e^3 z^{\frac{3}{2}n}} + \frac{2df^3}{\eta e^4 z^{\frac{1}{2}n}} + \frac{2df^3}{\eta e^4}\textcircled{I}$
&c.	



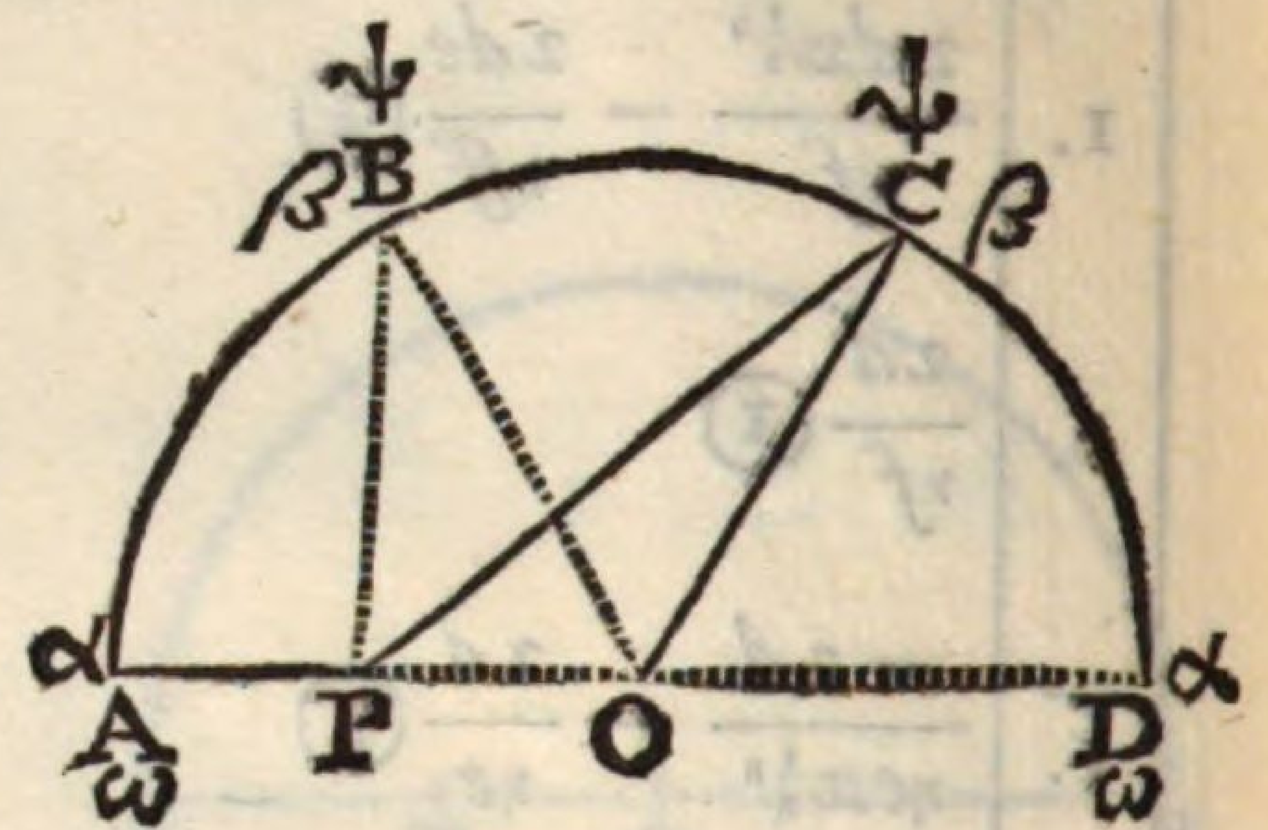
Cas. 1. $OP : OA :: \sqrt[2]{z^n} : \sqrt[2]{\frac{e}{f}}$;

$\textcircled{I} = \frac{1}{2}\sqrt[2]{\frac{f}{e}}$ in $\psi(PBO)$.

Cas. 2. $OP : OA :: \sqrt[2]{z^n} : \sqrt[2]{\frac{-e}{f}}$;

$\textcircled{I} = \frac{-1}{2}\sqrt[2]{\frac{-f}{e}}$ in $-(AP : AO) + (CP : CO)$

θ	FORMA III. $\dfrac{dzz^{\theta n+\frac{1}{3}n-1}}{e+fz^{n}}$
&c.	
4.	$\frac{3dz^{\frac{10}{3}n}}{10\eta f}-\frac{3dez^{\frac{7}{3}n}}{7\eta ff}+\frac{3deez^{\frac{4}{3}n}}{4\eta f^{3}}-\frac{3de^{3}z^{\frac{1}{3}n}}{\eta f^{4}}+\frac{3de^{4}}{\eta f^{5}}\textcircled{I}$
3.	$\frac{3dz^{\frac{7}{3}n}}{7\eta f}-\frac{3dez^{\frac{4}{3}n}}{4\eta ff}+\frac{3deez^{\frac{1}{3}n}}{\eta f^{3}}-\frac{3de^{3}}{\eta f^{4}}\textcircled{I}$
2.	$\frac{3dz^{\frac{4}{3}n}}{4\eta f}-\frac{3dez^{\frac{1}{3}n}}{\eta ff}+\frac{3dee}{\eta f^{3}}\textcircled{I}$
1.	$\frac{3dz^{\frac{1}{3}n}}{\eta f}-\frac{3de}{\eta ff}\textcircled{I}$
0.	$\frac{3d}{\eta f}\textcircled{I}$
$\overline{1}$.	$\frac{-3d}{2\eta ez^{\frac{2}{3}n}}-\frac{3d}{\eta e}\textcircled{I}$
$\overline{2}$.	$\frac{-3d}{5\eta ez^{\frac{5}{3}n}}+\frac{3df}{2\eta eez^{\frac{2}{3}n}}+\frac{3df}{\eta ee}\textcircled{I}$
$\overline{3}$.	$\frac{-3d}{8\eta ez^{\frac{8}{3}n}}+\frac{3df}{5\eta eez^{\frac{2}{3}n}}-\frac{3dff}{2\eta e^{3}z^{\frac{2}{3}n}}-\frac{3dff}{\eta e^{3}}\textcircled{I}$
$\overline{4}$.	$\frac{-3d}{11\eta ez^{\frac{11}{3}n}}+\frac{3df}{8\eta eez^{\frac{8}{3}n}}-\frac{3dff}{5\eta e^{3}z^{\frac{5}{3}n}}+\frac{3df^{3}}{2\eta e^{4}z^{\frac{2}{3}n}}+\frac{3df^{3}}{\eta e^{4}}\textcircled{I}$
&c.	



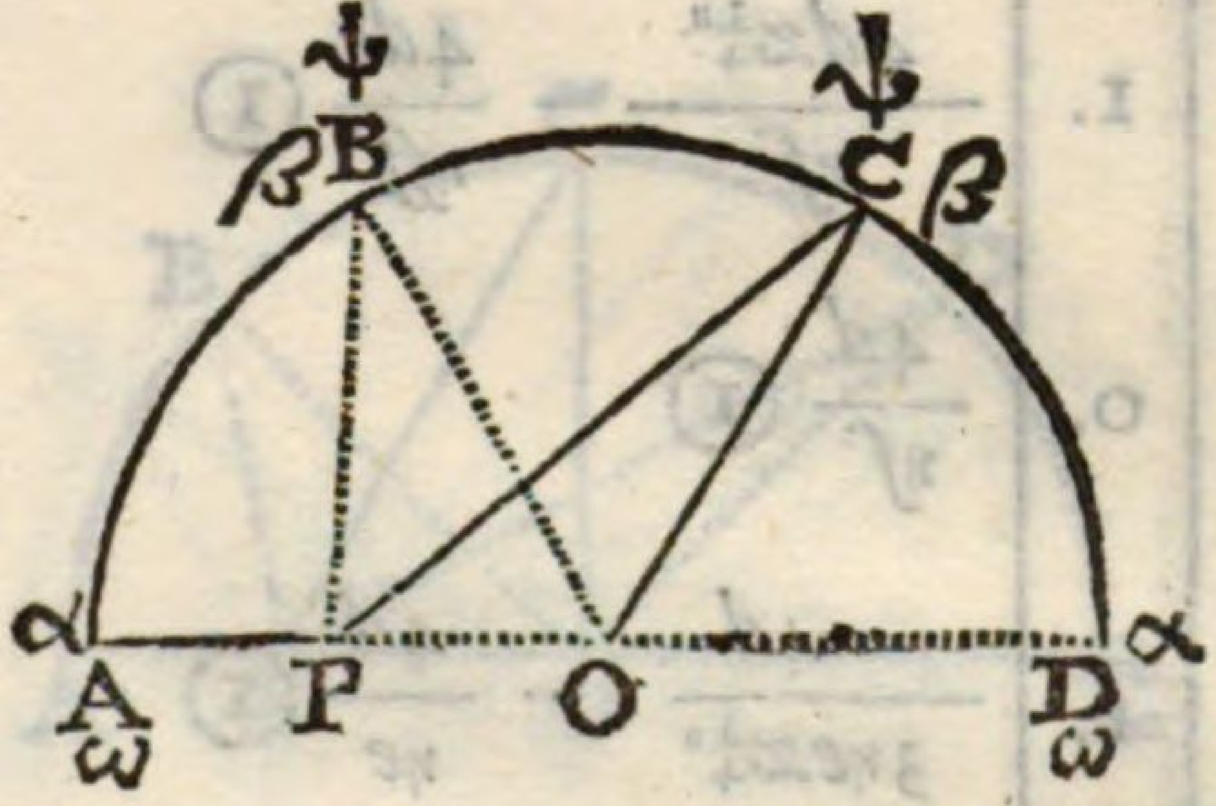
Cas. 1. $OP:OA::\sqrt[3]{z^{n}}:\sqrt[3]{\frac{e}{f}}$;

$$\textcircled{I}=\tfrac{1}{3}\sqrt[3]{\frac{ff}{ee}}\ \text{in}\ -\beta\,(BP:BO)+(DP:DO)+\psi(PBO)$$

Cas. 2. $OP:OA::\sqrt[3]{z^{n}}:\sqrt[3]{\frac{-e}{f}}$;

$$\textcircled{I}=\tfrac{-1}{3}\sqrt[3]{\frac{ff}{ee}}\ \text{in}\ -(AP:AO)+\beta(CP:CO)+\psi(PCO)$$

θ	FORMA IV. $\dfrac{d\dot{z}z^{\theta n+\frac{2}{3}n-1}}{e+fz^{n}}$
&c.	
4.	$\dfrac{3dz^{\frac{11}{3}n}}{11\eta f}-\dfrac{3dez^{\frac{8}{3}n}}{8\eta ff}+\dfrac{3deez^{\frac{5}{3}n}}{5\eta f^{3}}-\dfrac{3de^{3}z^{\frac{2}{3}n}}{2\eta f^{4}}+\dfrac{3de^{4}}{\eta f^{5}}\textcircled{I}$
3.	$\dfrac{3dz^{\frac{8}{3}n}}{8\eta f}-\dfrac{3dez^{\frac{5}{3}n}}{5\eta ff}+\dfrac{3deez^{\frac{2}{3}n}}{2\eta f^{3}}-\dfrac{3de^{3}}{\eta f^{4}}\textcircled{I}$
2.	$\dfrac{3dz^{\frac{5}{3}n}}{5\eta f}-\dfrac{3dez^{\frac{2}{3}n}}{2\eta ff}+\dfrac{3dee}{\eta f^{3}}\textcircled{I}$
1.	$\dfrac{3dz^{\frac{2}{3}n}}{2\eta f}-\dfrac{3de}{\eta ff}\textcircled{I}$
0.	$\dfrac{3d}{\eta f}\textcircled{I}$
$\overline{1}$.	$\dfrac{-3d}{\eta ez^{\frac{1}{3}n}}-\dfrac{3d}{\eta e}\textcircled{I}$
$\overline{2}$.	$\dfrac{-3d}{4\eta ez^{\frac{4}{3}n}}+\dfrac{3df}{\eta eez^{\frac{1}{3}n}}+\dfrac{3df}{\eta ee}\textcircled{I}$
$\overline{3}$.	$\dfrac{-3d}{7\eta ez^{\frac{7}{3}n}}+\dfrac{3df}{4\eta eez^{\frac{4}{3}n}}-\dfrac{3dff}{\eta e^{3}z^{\frac{1}{3}n}}-\dfrac{3dff}{\eta e^{3}}\textcircled{I}$
$\overline{4}$.	$\dfrac{-3d}{10\eta ez^{\frac{10}{3}n}}+\dfrac{3df}{7\eta eez^{\frac{7}{3}n}}-\dfrac{3dff}{4\eta e^{3}z^{\frac{4}{3}n}}+\dfrac{3df^{3}}{\eta e^{4}z^{\frac{1}{3}n}}+\dfrac{3df^{3}}{\eta e^{4}}\textcircled{I}$
&c.	



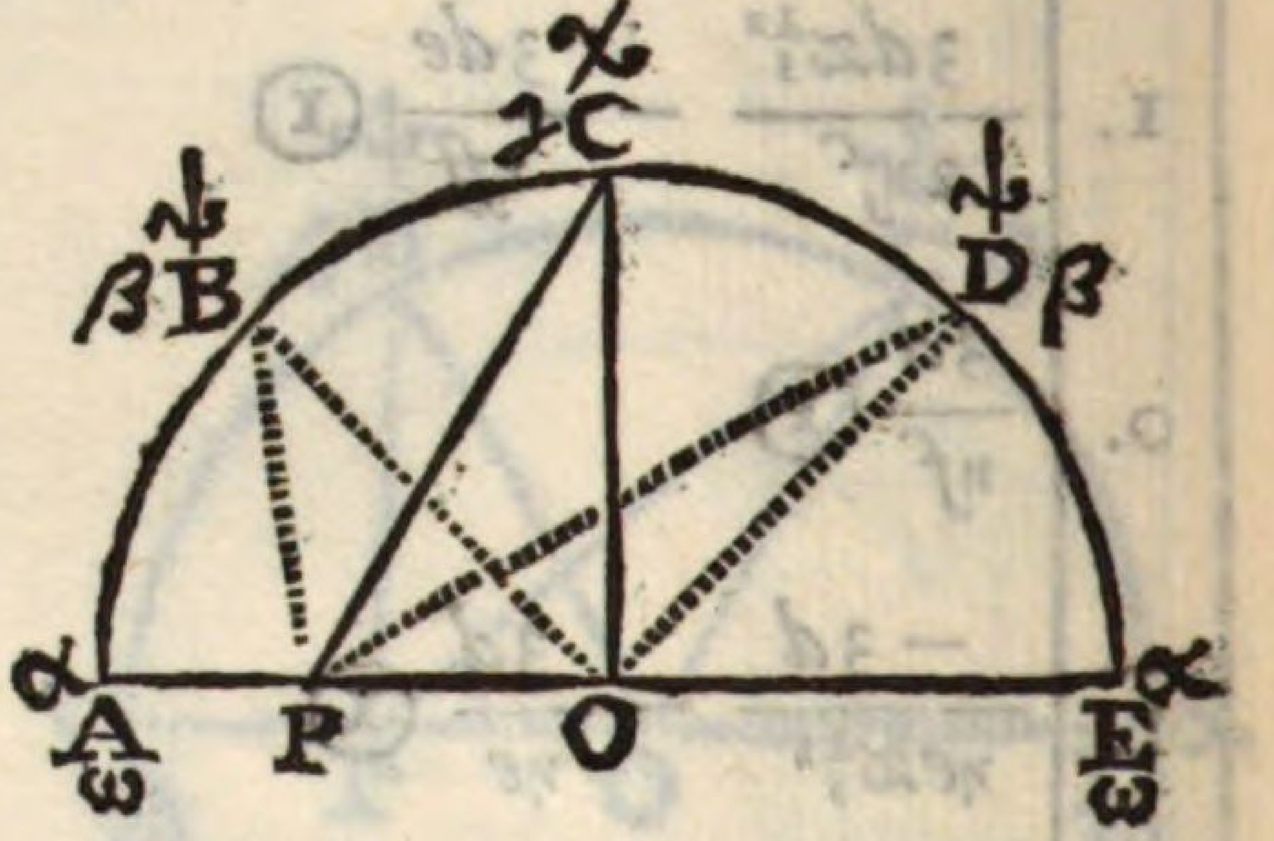
Caſ. 1. $OP:OA::\sqrt[3]{z^{n}}:\sqrt[3]{\dfrac{e}{f}}$;

$\textcircled{I}=\frac{1}{3}\sqrt[3]{\dfrac{f}{e}}$ in $+\beta(BP:BO)-(DP:DO)+\psi(PBO)$

Caſ. 2. $OP:OA::\sqrt[3]{z^{n}}:\sqrt[3]{\dfrac{-e}{f}}$;

$\textcircled{I}=-\frac{1}{3}\sqrt[3]{\dfrac{-f}{e}}$ in $-(AP:AO)+\beta(CP:CO)-\psi(PCO)$

θ	FORMA V. $\frac{d\dot{z}z^{\theta n+\frac{1}{4}n-1}}{e+fz^n}$
&c.	
4.	$\frac{4dz^{\frac{13}{4}n}}{13\eta f}-\frac{4dez^{\frac{9}{4}n}}{9\eta ff}+\frac{4deez^{\frac{5}{4}n}}{5\eta f^3}-\frac{4de^3z^{\frac{1}{4}n}}{\eta f^4}+\frac{4de^4}{\eta f^5}\textcircled{I}$
3.	$\frac{4dz^{\frac{9}{4}n}}{9\eta f}-\frac{4dez^{\frac{5}{4}n}}{5\eta ff}+\frac{4deez^{\frac{1}{4}n}}{\eta f^3}-\frac{4de^3}{\eta f^4}\textcircled{I}$
2.	$\frac{4dz^{\frac{5}{4}n}}{5\eta f}-\frac{4dez^{\frac{1}{4}n}}{\eta ff}+\frac{4dee}{\eta f^3}\textcircled{I}$
1.	$\frac{4dz^{\frac{1}{4}n}}{\eta f}-\frac{4de}{\eta ff}\textcircled{I}$
0.	$\frac{4d}{\eta f}\textcircled{I}$
$\bar{1}$.	$\frac{-4d}{3\eta ez^{\frac{3}{4}n}}-\frac{4d}{\eta e}\textcircled{I}$
$\bar{2}$.	$\frac{-4d}{7\eta ez^{\frac{7}{4}n}}+\frac{4df}{3\eta eez^{\frac{3}{4}n}}+\frac{4df}{\eta ee}\textcircled{I}$
$\bar{3}$.	$\frac{-4d}{11\eta ez^{\frac{11}{4}n}}+\frac{4df}{7\eta eez^{\frac{7}{4}n}}-\frac{4dff}{3\eta e^3z^{\frac{3}{4}n}}-\frac{4dff}{\eta e^3}\textcircled{I}$
$\bar{4}$.	$\frac{-4d}{15\eta ez^{\frac{15}{4}n}}+\frac{4df}{11\eta eez^{\frac{11}{4}n}}-\frac{4dff}{7\eta e^3z^{\frac{7}{4}n}}+\frac{4df^3}{3\eta e^4z^{\frac{3}{4}n}}+\frac{4df^3}{\eta e^4}\textcircled{I}$
&c.	



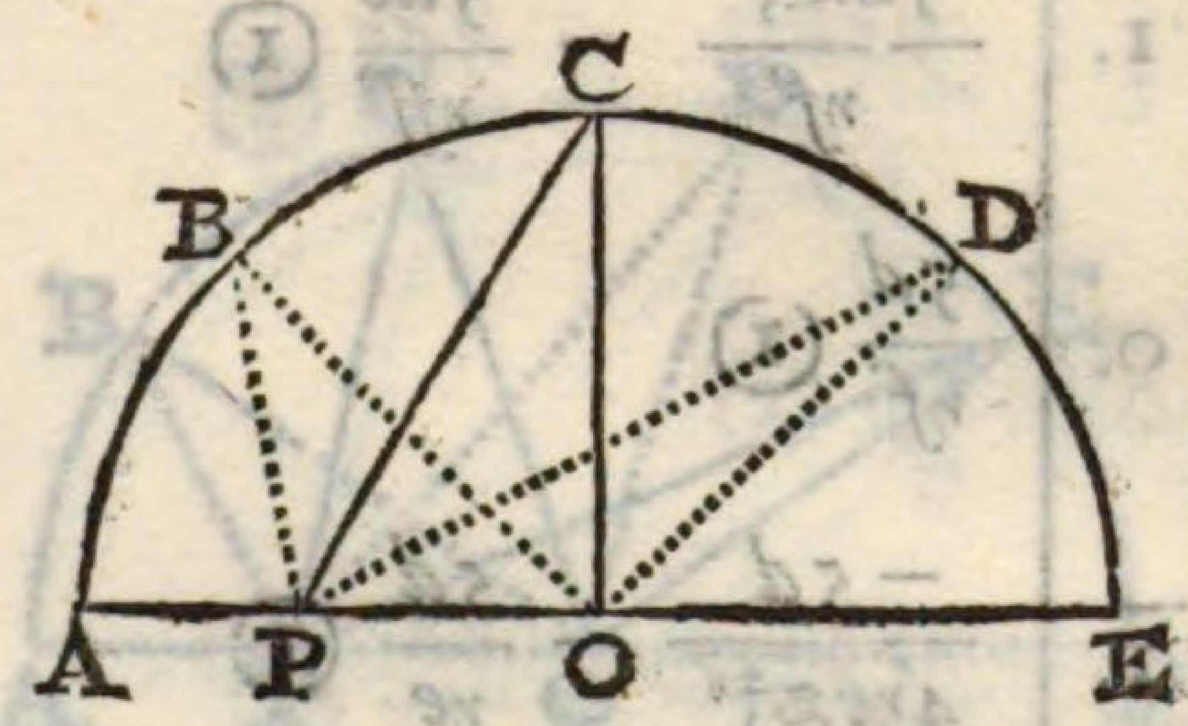
Cas. 1. $OP:OA::\sqrt[4]{z^n}:\sqrt[4]{\frac{e}{f}}$;

$\textcircled{I}=\frac{1}{4}\sqrt[4]{\frac{f^3}{e^3}}\text{ in }-\beta(BP:BO)+\beta(DP:DO)+\downarrow(PBO)+\downarrow(PDO)$

Cas. 2. $OP:OA::\sqrt[4]{z^n}:\sqrt[4]{\frac{-e}{f}}$;

$\textcircled{I}=-\frac{1}{4}\sqrt[4]{\frac{-f^3}{e^3}}\text{ in }-(AP:AO)+(EP:EO)+\chi(PCO)$

θ	FORMA VI. $\frac{dz\,z^{\theta n+\frac{3}{4}n-1}}{e+fz^n}$
&c.	
4.	$\frac{4dz^{\frac{15}{4}n}}{15\eta f}-\frac{4dez^{\frac{11}{4}n}}{11\eta ff}+\frac{4deez^{\frac{7}{4}n}}{7\eta f^3}-\frac{4de^3z^{\frac{3}{4}n}}{3\eta f^4}+\frac{4de^4}{\eta f^5}$ Ⓘ
3.	$\frac{4dz^{\frac{11}{4}n}}{11\eta f}-\frac{4dez^{\frac{7}{4}n}}{7\eta ff}+\frac{4deez^{\frac{3}{4}n}}{3\eta f^3}-\frac{4de^3}{\eta f^4}$ Ⓘ
2.	$\frac{4dz^{\frac{7}{4}n}}{7\eta f}-\frac{4dez^{\frac{3}{4}n}}{3\eta ff}+\frac{4dee}{\eta f^3}$ Ⓘ
1.	$\frac{4dz^{\frac{3}{4}n}}{3\eta f}-\frac{4de}{\eta ff}$ Ⓘ
0.	$\frac{4d}{\eta f}$ Ⓘ
$\overline{1}$.	$\frac{-4d}{\eta ez^{\frac{1}{4}n}}-\frac{4d}{\eta e}$ Ⓘ
$\overline{2}$.	$\frac{-4d}{5\eta ez^{\frac{5}{4}n}}+\frac{4df}{\eta eez^{\frac{1}{4}n}}+\frac{4df}{\eta ee}$ Ⓘ
$\overline{3}$.	$\frac{-4d}{9\eta ez^{\frac{9}{4}n}}+\frac{4df}{5\eta eez^{\frac{5}{4}n}}-\frac{4dff}{\eta e^3z^{\frac{1}{4}n}}-\frac{4dff}{\eta e^3}$ Ⓘ
$\overline{4}$.	$\frac{-4d}{13\eta ez^{\frac{13}{4}n}}+\frac{4df}{9\eta eez^{\frac{9}{4}n}}-\frac{4dff}{5\eta e^3z^{\frac{5}{4}n}}+\frac{4df^3}{\eta e^4z^{\frac{1}{4}n}}+\frac{4df^3}{\eta e^4}$ Ⓘ
&c.	



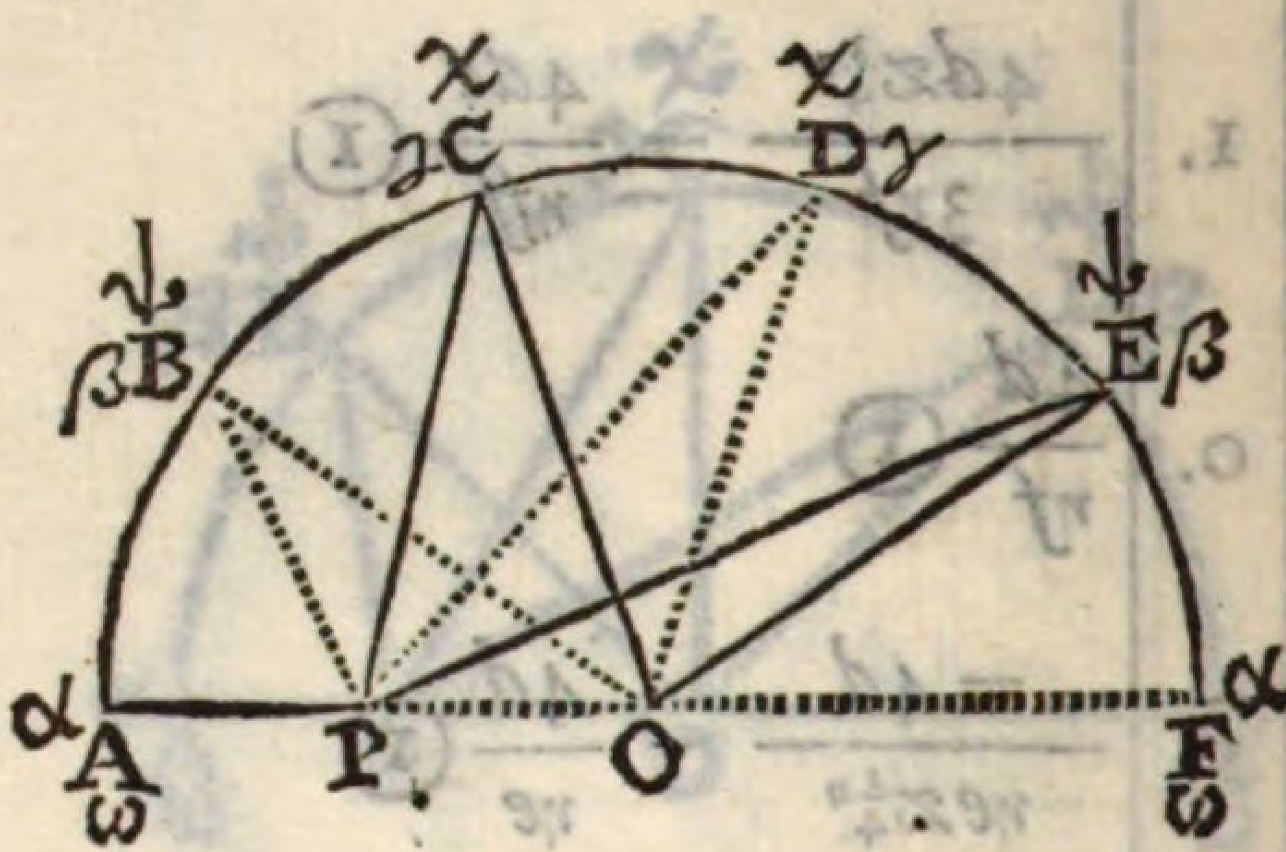
Cas. 1. $OP:OA::\sqrt[4]{z^n}:\sqrt[4]{\frac{e}{f}}$;

Ⓘ $=\frac{1}{4}\sqrt[4]{\frac{f}{e}}$ in $+\beta(BP:BO)-\beta(DP:DO)+\psi(PBO)+\psi(PDO)$

Cas. 2. $OP:OA::\sqrt[4]{z^n}:\sqrt[4]{\frac{-e}{f}}$;

Ⓘ $=\frac{-1}{4}\sqrt[4]{\frac{-f}{e}}$ in $-(AP:AO)+(EP:EO)-\chi(PCO)$

θ	FORMA VII. $\frac{d\dot{z}z^{\theta\eta+\frac{1}{5}\eta-1}}{e+fz^{\eta}}$
&c.	
4.	$\frac{5dz^{\frac{16}{5}\eta}}{16\eta f}-\frac{5dez^{\frac{11}{5}\eta}}{11\eta ff}+\frac{5deez^{\frac{6}{5}\eta}}{6\eta f^{3}}-\frac{5de^{3}z^{\frac{1}{5}\eta}}{\eta f^{4}}+\frac{5de^{4}}{\eta f^{5}}\textcircled{I}$
3.	$\frac{5dz^{\frac{11}{5}\eta}}{11\eta f}-\frac{5dez^{\frac{6}{5}\eta}}{6\eta ff}+\frac{5deez^{\frac{1}{5}\eta}}{\eta f^{3}}-\frac{5ae^{3}}{\eta f^{4}}\textcircled{I}$
2.	$\frac{5dz^{\frac{6}{5}\eta}}{6\eta f}-\frac{5dez^{\frac{1}{5}\eta}}{\eta ff}+\frac{5dee}{\eta f^{3}}\textcircled{I}$
1.	$\frac{5dz^{\frac{1}{5}\eta}}{\eta f}-\frac{5de}{\eta ff}\textcircled{I}$
0.	$\frac{5d}{\eta f}\textcircled{I}$
$\overline{1}$.	$\frac{-5d}{4\eta ez^{\frac{4}{5}\eta}}-\frac{5d}{\eta e}\textcircled{I}$
$\overline{2}$.	$\frac{-5d}{9\eta ez^{\frac{9}{5}\eta}}+\frac{5df}{4\eta eez^{\frac{4}{5}\eta}}+\frac{5df}{\eta ee}\textcircled{I}$
$\overline{3}$.	$\frac{-5d}{14\eta ez^{\frac{14}{5}\eta}}+\frac{5df}{9\eta eez^{\frac{9}{5}\eta}}-\frac{5dff}{4\eta e^{3}z^{\frac{4}{5}\eta}}-\frac{5dff}{\eta e^{3}}\textcircled{I}$
$\overline{4}$.	$\frac{-5d}{19\eta ez^{\frac{19}{5}\eta}}+\frac{5df}{14\eta eez^{\frac{14}{5}\eta}}-\frac{5dff}{9\eta e^{3}z^{\frac{9}{5}\eta}}+\frac{5df^{3}}{4\eta e^{4}z^{\frac{4}{5}\eta}}+\frac{5df^{3}}{\eta e^{4}}\textcircled{I}$
&c.	



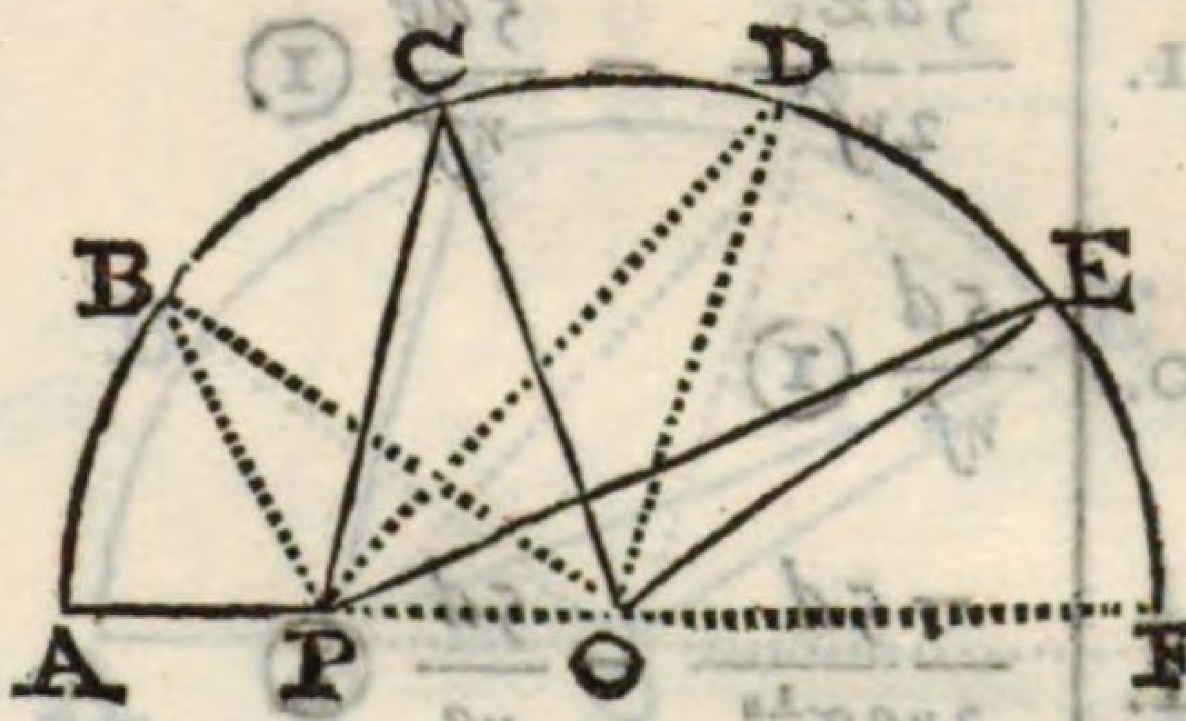
Cas. 1. $OP:OA::\sqrt[5]{z^{\eta}}:\sqrt[5]{\frac{e}{f}}$;

$\textcircled{I}=\frac{1}{5}\sqrt[5]{\frac{f^{4}}{e^{4}}}$ in $-\beta(BP:BO)+\gamma(DP:DO)+(FP:FO)$ $+\psi(PBO)+\chi(PDO)$

Cas. 2. $OP:OA::\sqrt[5]{z^{\eta}}:\sqrt[5]{\frac{-e}{f}}$;

$\textcircled{I}=-\frac{1}{5}\sqrt[5]{\frac{f^{4}}{e^{4}}}$ in $-(AP:AO)-\gamma(CP:CO)+\beta(EP:EO)$ $+\chi(PCO)+\psi(PEO)$

θ	FORMA VIII. $\frac{d\dot{z}z^{\theta n+\frac{4}{5}n-1}}{e+fz^{n}}$
&c.	
4.	$\frac{5dz^{\frac{19}{5}n}}{19nf}-\frac{5dez^{\frac{14}{5}n}}{14nff}+\frac{5deez^{\frac{9}{5}n}}{9nf^3}-\frac{5de^3z^{\frac{4}{5}n}}{4nf^4}+\frac{5de^4}{nf^5}\textcircled{I}$
3.	$\frac{5dz^{\frac{14}{5}n}}{14nf}-\frac{5dez^{\frac{9}{5}n}}{9nff}+\frac{5deez^{\frac{4}{5}n}}{4nf^3}-\frac{5de^3}{nf^4}\textcircled{I}$
2.	$\frac{5dz^{\frac{9}{5}n}}{9nf}-\frac{5dez^{\frac{4}{5}n}}{4nff}+\frac{5dee}{nf^3}\textcircled{I}$
1.	$\frac{5dz^{\frac{4}{5}n}}{4nf}-\frac{5de}{nff}\textcircled{I}$
0.	$\frac{5d}{nf}\textcircled{I}$
$\overline{1}$.	$\frac{-5d}{nez^{\frac{1}{5}n}}-\frac{5d}{ne}\textcircled{I}$
$\overline{2}$.	$\frac{-5d}{6nez^{\frac{6}{5}n}}+\frac{5df}{neez^{\frac{1}{5}n}}+\frac{5df}{nee}\textcircled{I}$
$\overline{3}$.	$\frac{-5d}{11nez^{\frac{11}{5}n}}+\frac{5df}{6neez^{\frac{6}{5}n}}-\frac{5dff}{ne^3z^{\frac{1}{5}n}}-\frac{5dff}{ne^3}\textcircled{I}$
$\overline{4}$.	$\frac{-5d}{16nez^{\frac{16}{5}n}}+\frac{5df}{11neez^{\frac{11}{5}n}}-\frac{5dff}{6ne^3z^{\frac{6}{5}n}}+\frac{5df^3}{ne^4z^{\frac{1}{5}n}}+\frac{5df^3}{ne^4}\textcircled{I}$
&c.	



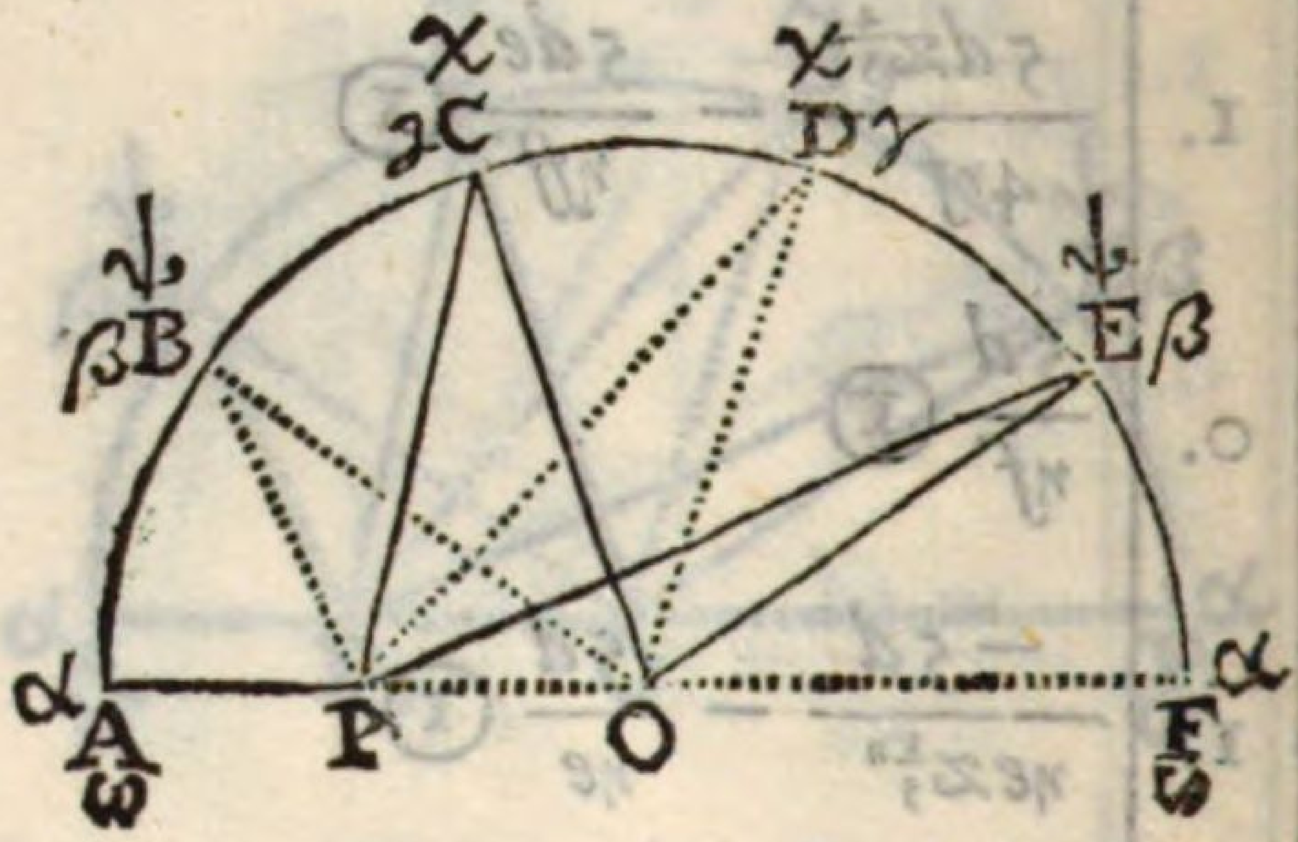
Cas. 1. $OP:OA::\sqrt[5]{z^n}:\sqrt[5]{\frac{e}{f}}$;

$\textcircled{I}=\frac{1}{5}\sqrt[5]{\frac{f}{e}}$ in $+\beta(BP:BO)-\gamma(DP:DO)-(FP:FO)$ $+\psi(PBO)+\chi(PDO)$

Cas. 2. $OP:OA::\sqrt[5]{z^n}:\sqrt[5]{\frac{-e}{f}}$;

$\textcircled{I}=\frac{-1}{5}\sqrt[5]{\frac{-f}{e}}$ in $-(AP:AO)-\gamma(CP:CO)+\beta(EP:EO)$ $-\chi(PCO)-\psi(PEO)$

θ	FORMA IX. $\dfrac{dz z^{\theta n+\frac{2}{5}n-1}}{e+fz^{n}}$
&c.	
4.	$\dfrac{5dz^{1\frac{7}{5}n}}{17\eta f}-\dfrac{5dez^{1\frac{2}{5}n}}{12\eta ff}+\dfrac{5deez^{\frac{7}{5}n}}{7\eta f^{3}}-\dfrac{5de^{3}z^{\frac{2}{5}n}}{2\eta f^{4}}+\dfrac{5de^{4}}{\eta f^{5}}$ Ⓘ
3.	$\dfrac{5dz^{1\frac{2}{5}n}}{12\eta f}-\dfrac{5dez^{\frac{7}{5}n}}{7\eta ff}+\dfrac{5deez^{\frac{2}{5}n}}{2\eta f^{3}}-\dfrac{5de^{3}}{\eta f^{4}}$ Ⓘ
2.	$\dfrac{5dz^{\frac{7}{5}n}}{7\eta f}-\dfrac{5dez^{\frac{2}{5}n}}{2\eta ff}+\dfrac{5dee}{\eta f^{3}}$ Ⓘ
1.	$\dfrac{5dz^{\frac{2}{5}n}}{2\eta f}-\dfrac{5de}{\eta ff}$ Ⓘ
0.	$\dfrac{5d}{\eta f}$ Ⓘ
$\overline{1}$.	$\dfrac{-5d}{3\eta ez^{\frac{3}{5}n}}-\dfrac{5d}{\eta e}$ Ⓘ
$\overline{2}$.	$\dfrac{-5d}{8\eta ez^{\frac{8}{5}n}}+\dfrac{5df}{3\eta eez^{\frac{3}{5}n}}+\dfrac{5df}{\eta ee}$ Ⓘ
$\overline{3}$.	$\dfrac{-5d}{13\eta ez^{1\frac{3}{5}n}}+\dfrac{5df}{8\eta eez^{\frac{8}{5}n}}-\dfrac{5dff}{3\eta e^{3}z^{\frac{3}{5}n}}-\dfrac{5dff}{\eta e^{3}}$ Ⓘ
$\overline{4}$.	$\dfrac{-5d}{18\eta ez^{1\frac{8}{5}n}}+\dfrac{5df}{13\eta eez^{1\frac{3}{5}n}}-\dfrac{5dff}{8\eta e^{3}z^{\frac{8}{5}n}}+\dfrac{5df^{3}}{3\eta e^{4}z^{\frac{3}{5}n}}+\dfrac{5df^{3}}{\eta e^{4}}$ Ⓘ
&c.	



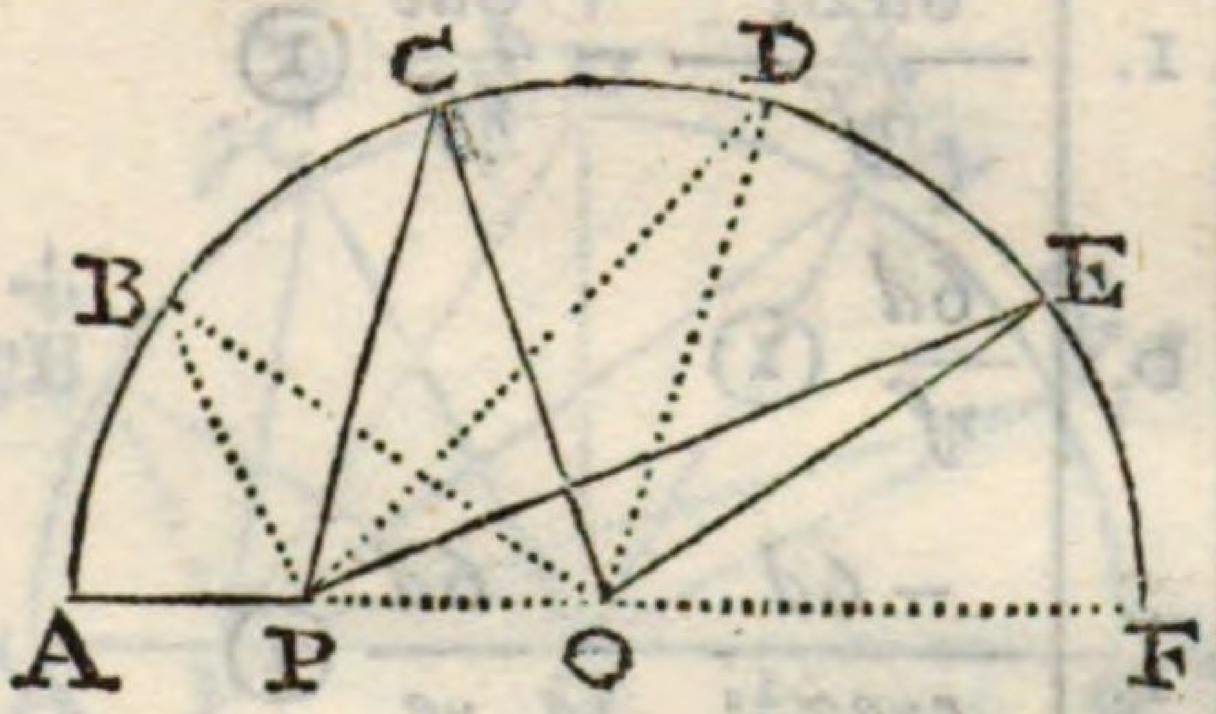
Caſ. 1. $OP : OA :: \sqrt[5]{z^{n}} : \sqrt[5]{\dfrac{e}{f}}$;

Ⓘ $= \frac{1}{5}\sqrt[5]{\dfrac{f^{3}}{e^{3}}}$ in $-\gamma(BP:BO)+\beta(DP:DO)-(FP:FO)$
$+\chi(PBO)-\psi(PDO)$

Caſ. 2. $OP : OA :: \sqrt[5]{z^{n}} : \sqrt[5]{\dfrac{-e}{f}}$;

Ⓘ $= \frac{-1}{5}\sqrt[5]{\dfrac{-f^{3}}{e^{3}}}$ in $-(AP:AO)+\beta(CP:CO)-\gamma(EP:EO)$
$+\psi(PCO)+\chi(PEO)$

θ	FORMA X. $\frac{dz z^{\theta n + \frac{3}{5}n - 1}}{e + f z^n}$
&c.	
4.	$\frac{5dz^{3\frac{3}{5}n}}{18nf} - \frac{5dez^{2\frac{3}{5}n}}{13nff} + \frac{5deez^{\frac{8}{5}n}}{8nf^3} - \frac{5de^3 z^{\frac{3}{5}n}}{3nf^4} + \frac{5de^4}{nf^5}$ Ⓘ
3.	$\frac{5dz^{2\frac{3}{5}n}}{13nf} - \frac{5dez^{\frac{8}{5}n}}{8nff} + \frac{5deez^{\frac{3}{5}n}}{3nf^3} - \frac{5de^3}{nf^4}$ Ⓘ
2.	$\frac{5dz^{\frac{8}{5}n}}{8nf} - \frac{5dez^{\frac{3}{5}n}}{3nff} + \frac{5dee}{nf^3}$ Ⓘ
1.	$\frac{5dz^{\frac{3}{5}n}}{3nf} - \frac{5de}{nff}$ Ⓘ
0.	$\frac{5d}{nf}$ Ⓘ
$\overline{1}$.	$\frac{-5d}{2nez^{\frac{2}{5}n}} - \frac{5d}{ne}$ Ⓘ
$\overline{2}$.	$\frac{-5d}{7nez^{\frac{7}{5}n}} + \frac{5df}{2neez^{\frac{2}{5}n}} + \frac{5df}{nee}$ Ⓘ
$\overline{3}$.	$\frac{-5d}{12nez^{2\frac{2}{5}n}} + \frac{5df}{7neez^{\frac{7}{5}n}} - \frac{5dff}{2ne^3 z^{\frac{2}{5}n}} - \frac{5dff}{ne^3}$ Ⓘ
$\overline{4}$.	$\frac{-5d}{17nez^{3\frac{2}{5}n}} + \frac{5df}{12neez^{2\frac{2}{5}n}} - \frac{5dff}{7ne^3 z^{\frac{7}{5}n}} + \frac{5df^3}{2ne^4 z^{\frac{2}{5}n}} + \frac{5df^3}{ne^4}$ Ⓘ
&c.	



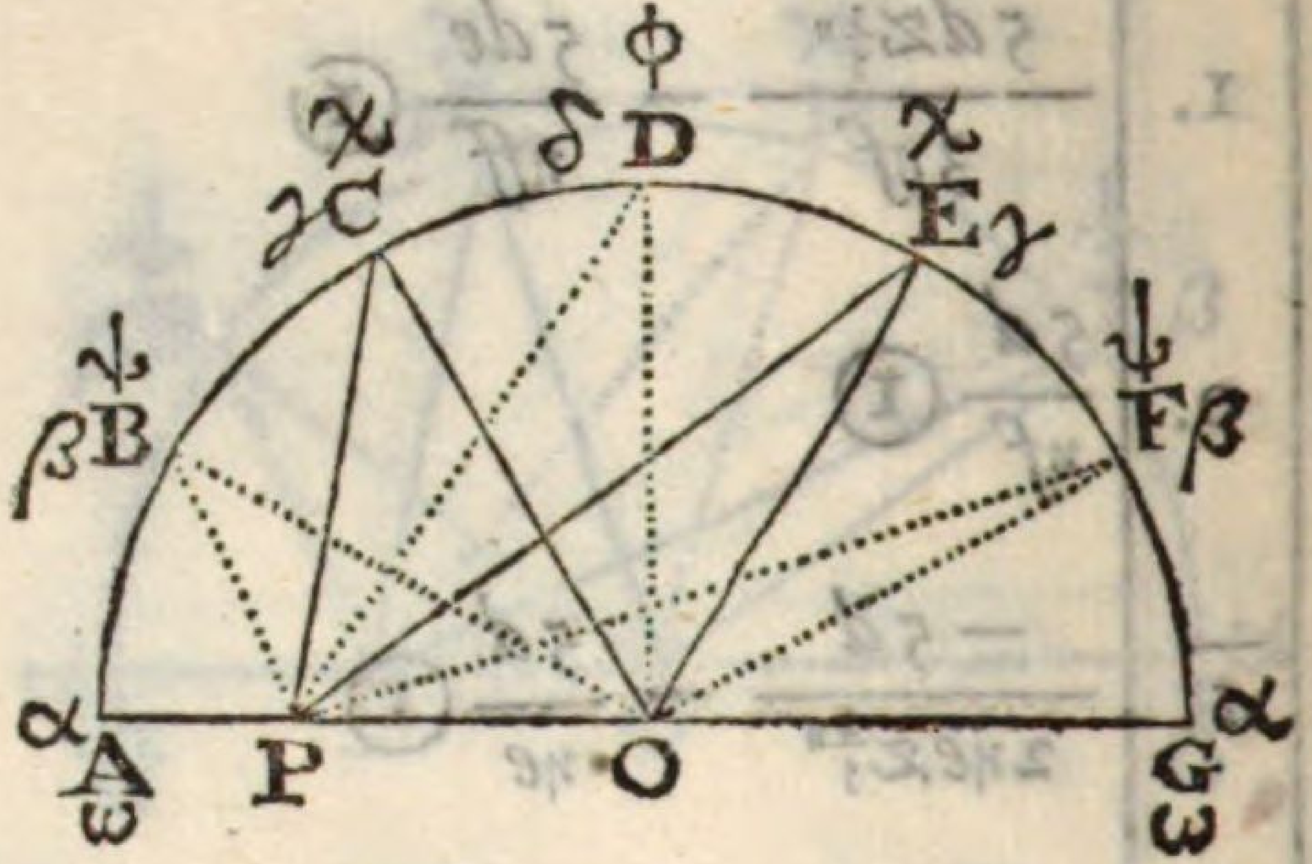
Caf. 1. $OP : OA :: \sqrt[5]{z^n} : \sqrt[5]{\frac{e}{f}}$;

Ⓘ $= \frac{1}{5}\sqrt[5]{\frac{ff}{ee}}$ in $+\gamma(BP:BO) - \beta(DP:DO) + (FP:FO)$ $+\chi(PBO) - \psi(PDO)$

Caf. 2. $OP : OA :: \sqrt[5]{z^n} : \sqrt[5]{\frac{-e}{f}}$;

Ⓘ $= \frac{-1}{5}\sqrt[5]{\frac{ff}{ee}}$ in $-(AP:AO) + \beta(CP:CO) - \gamma(EP:EO)$ $-\psi(PCO) + \chi(PEO)$

θ	FORMA XI. $\frac{d\dot{z}z^{\theta n+\frac{1}{6}n-1}}{e+fz^n}$
&c.	
4.	$\frac{6dz^{\frac{19}{6}n}}{19\eta f}-\frac{6dez^{\frac{13}{6}n}}{13\eta ff}+\frac{6deez^{\frac{7}{6}n}}{7\eta f^3}-\frac{6de^3z^{\frac{1}{6}n}}{\eta f^4}+\frac{6de^4}{\eta f^5}\textcircled{I}$
3.	$\frac{6dz^{\frac{13}{6}n}}{13\eta f}-\frac{6dez^{\frac{7}{6}n}}{7\eta ff}+\frac{6deez^{\frac{1}{6}n}}{\eta f^3}-\frac{6de^3}{\eta f^4}\textcircled{I}$
2.	$\frac{6dz^{\frac{7}{6}n}}{7\eta f}-\frac{6dez^{\frac{1}{6}n}}{\eta ff}+\frac{6dee}{\eta f^3}\textcircled{I}$
1.	$\frac{6dz^{\frac{1}{6}n}}{\eta f}-\frac{6de}{\eta ff}\textcircled{I}$
0.	$\frac{6d}{\eta f}\textcircled{I}$
$\overline{1}$.	$\frac{-6d}{5\eta ez^{\frac{5}{6}n}}-\frac{6d}{\eta e}\textcircled{I}$
$\overline{2}$.	$\frac{-6d}{11\eta ez^{\frac{11}{6}n}}+\frac{6df}{5\eta eez^{\frac{5}{6}n}}+\frac{6df}{\eta ee}\textcircled{I}$
$\overline{3}$.	$\frac{-6d}{17\eta ez^{\frac{17}{6}n}}+\frac{6df}{11\eta eez^{\frac{11}{6}n}}-\frac{6dff}{5\eta e^3z^{\frac{5}{6}n}}-\frac{6dff}{\eta e^3}\textcircled{I}$
$\overline{4}$.	$\frac{-6d}{23\eta ez^{\frac{23}{6}n}}+\frac{6df}{17\eta eez^{\frac{17}{6}n}}-\frac{6dff}{11\eta e^3z^{\frac{11}{6}n}}+\frac{6df^3}{5\eta e^4z^{\frac{5}{6}n}}+\frac{6df^3}{\eta e^4}\textcircled{I}$
&c.	



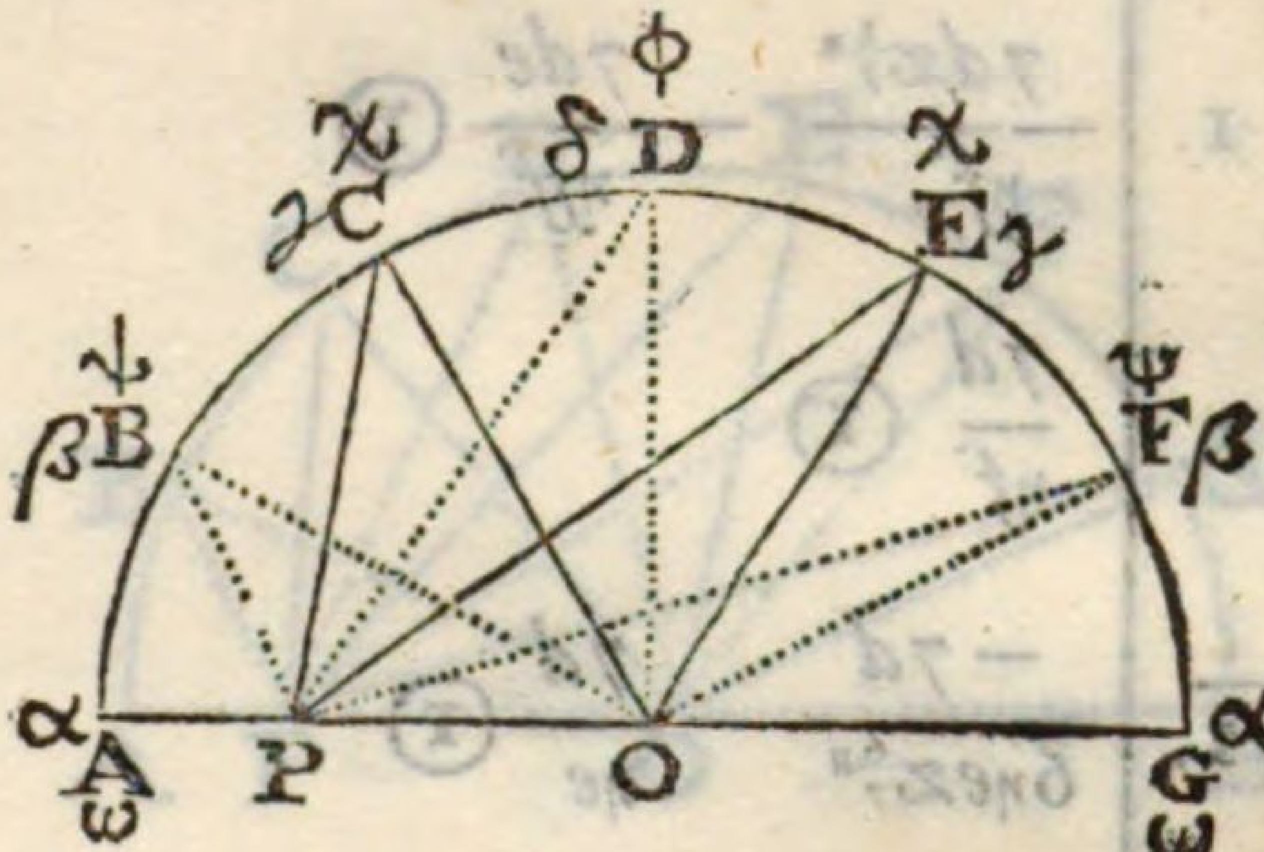
Caf. 1. $OP:OA::\sqrt[6]{z^n}:\sqrt[6]{\frac{e}{f}}$;

$\textcircled{I}=\frac{1}{6}\sqrt[6]{\frac{f^5}{e^5}}$ in $-\beta(BP:BO)+\beta(FP:FO)+\psi(PBO)+\phi(PDO)+\psi(PFO)$

Caf. 2. $OP:OA::\sqrt[6]{z^n}:\sqrt[6]{\frac{-e}{f}}$;

$\textcircled{I}=\frac{1}{6}\sqrt[6]{\frac{-f^5}{e^5}}$ in $-(AP:AO)-\gamma(CP:CO)+\gamma(EP:EO)+(GP:GO)+\chi(PCO)+\chi(PEO)$

θ	FORMA XII. $\frac{d\dot{z}z^{\theta\eta+\frac{1}{6}\eta-1}}{e+fz^{\eta}}$
&c.	
4.	$\frac{6dz^{\frac{23}{6}\eta}}{23\eta f}-\frac{6dez^{\frac{17}{6}\eta}}{17\eta ff}+\frac{6deez^{\frac{11}{6}\eta}}{11\eta f^{3}}-\frac{6de^{3}z^{\frac{5}{6}\eta}}{5\eta f^{4}}+\frac{6de^{4}}{\eta f^{5}}\textcircled{\small I}$
3.	$\frac{6dz^{\frac{17}{6}\eta}}{17\eta f}-\frac{6dez^{\frac{11}{6}\eta}}{11\eta ff}+\frac{6deez^{\frac{5}{6}\eta}}{5\eta f^{3}}-\frac{6de^{3}}{\eta f^{4}}\textcircled{\small I}$
2.	$\frac{6dz^{\frac{11}{6}\eta}}{11\eta f}-\frac{6dez^{\frac{5}{6}\eta}}{5\eta ff}+\frac{6dee}{\eta f^{3}}\textcircled{\small I}$
1.	$\frac{6dz^{\frac{5}{6}\eta}}{5\eta f}-\frac{6de}{\eta ff}\textcircled{\small I}$
0.	$\frac{6d}{\eta f}\textcircled{\small I}$
$\overline{1}$.	$\frac{-6d}{\eta ez^{\frac{1}{6}\eta}}-\frac{6d}{\eta e}\textcircled{\small I}$
$\overline{2}$.	$\frac{-6d}{7\eta ez^{\frac{7}{6}\eta}}+\frac{6df}{\eta eez^{\frac{1}{6}\eta}}+\frac{6df}{\eta ee}\textcircled{\small I}$
$\overline{3}$.	$\frac{-6d}{13\eta ez^{\frac{13}{6}\eta}}+\frac{6df}{7\eta eez^{\frac{7}{6}\eta}}-\frac{6dff}{\eta e^{3}z^{\frac{1}{6}\eta}}-\frac{6dff}{\eta e^{3}}\textcircled{\small I}$
$\overline{4}$.	$\frac{-6d}{19\eta ez^{\frac{19}{6}\eta}}+\frac{6df}{13\eta eez^{\frac{13}{6}\eta}}-\frac{6dff}{7\eta e^{3}z^{\frac{7}{6}\eta}}+\frac{6df^{3}}{\eta e^{4}z^{\frac{1}{6}\eta}}+\frac{6df^{3}}{\eta e^{4}}\textcircled{\small I}$
&c.	



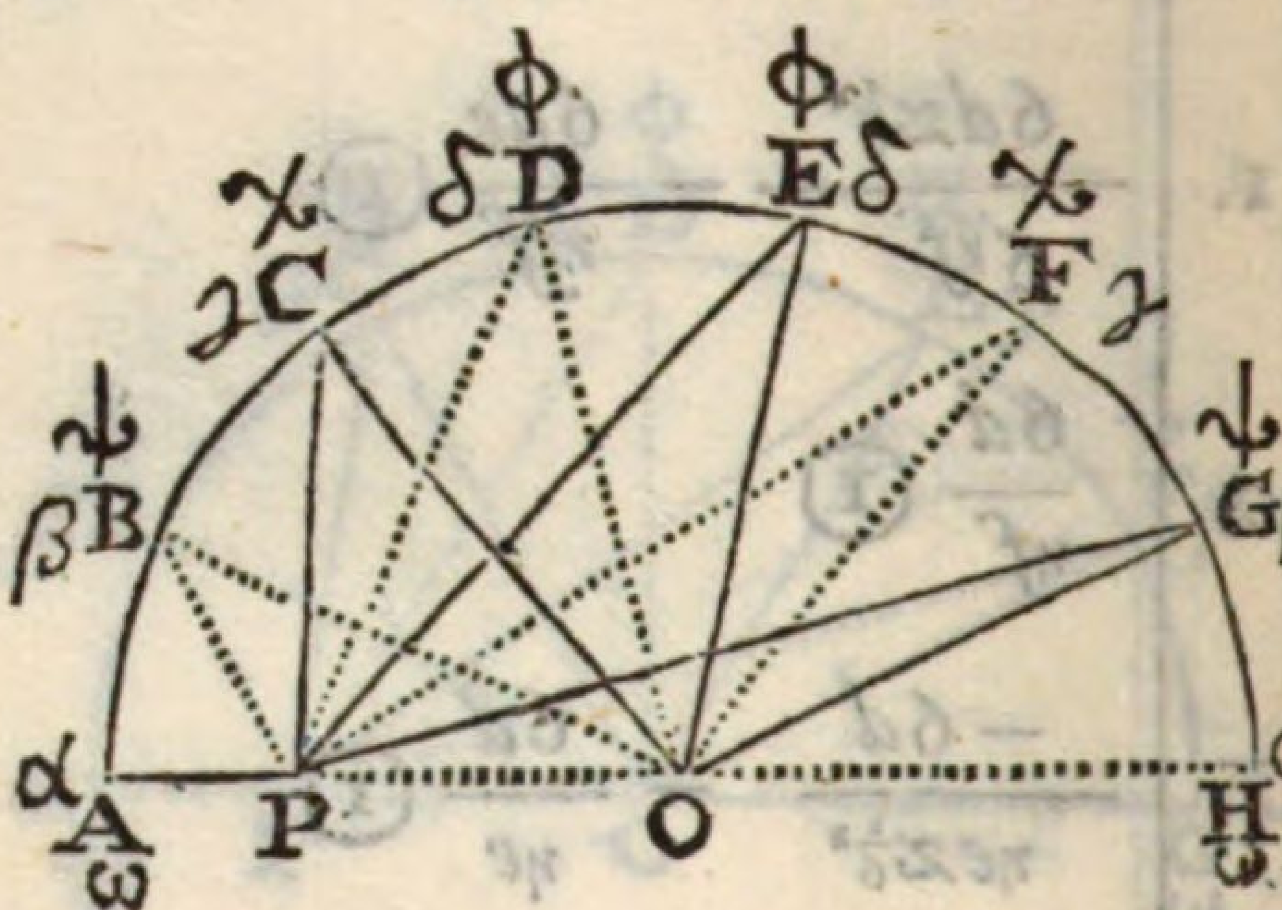
Caf. 1. $OP:OA::\sqrt[6]{z^{\eta}}:\sqrt[6]{\frac{e}{f}}$;

$\textcircled{\small I}=\frac{1}{6}\sqrt[6]{\frac{f}{e}}$ in $+\beta(BP:BO)-\beta(FP:FO)+\psi(PBO)+\phi(PDO)+\psi(PFO)$

Caf. 2. $OP:OA::\sqrt[6]{z^{\eta}}:\sqrt[6]{\frac{-e}{f}}$;

$\textcircled{\small I}=\frac{-1}{6}\sqrt[6]{\frac{-f}{e}}$ in $-(AP:AO)-\gamma(CP:CO)+\gamma(EP:EO)+(GP:GO)-\chi(PCO)-\chi(PEO)$

θ	FORMA XIII. $\dfrac{d\dot{z}z^{\theta\eta+\frac{1}{7}\eta-1}}{e+fz^{\eta}}$
&c.	
4.	$\dfrac{7dz^{\frac{22}{7}\eta}}{22\eta f}-\dfrac{7dez^{\frac{15}{7}\eta}}{15\eta ff}+\dfrac{7deez^{\frac{8}{7}\eta}}{8\eta f^3}-\dfrac{7de^3z^{\frac{1}{7}\eta}}{\eta f^4}+\dfrac{7de^4}{\eta f^5}(\mathrm{I})$
3.	$\dfrac{7dz^{\frac{15}{7}\eta}}{15\eta f}-\dfrac{7dez^{\frac{8}{7}\eta}}{8\eta ff}+\dfrac{7deez^{\frac{1}{7}\eta}}{\eta f^3}-\dfrac{7de^3}{\eta f^4}(\mathrm{I})$
2.	$\dfrac{7dz^{\frac{8}{7}\eta}}{8\eta f}-\dfrac{7dez^{\frac{1}{7}\eta}}{\eta ff}+\dfrac{7dee}{\eta f^3}(\mathrm{I})$
1.	$\dfrac{7dz^{\frac{1}{7}\eta}}{\eta f}-\dfrac{7de}{\eta ff}(\mathrm{I})$
0.	$\dfrac{7d}{\eta f}(\mathrm{I})$
$\overline{1}$.	$\dfrac{-7d}{6\eta ez^{\frac{6}{7}\eta}}-\dfrac{7d}{\eta e}(\mathrm{I})$
$\overline{2}$.	$\dfrac{-7d}{13\eta ez^{\frac{13}{7}\eta}}+\dfrac{7df}{6\eta eez^{\frac{6}{7}\eta}}+\dfrac{7df}{\eta ee}(\mathrm{I})$
$\overline{3}$.	$\dfrac{-7d}{20\eta ez^{\frac{20}{7}\eta}}+\dfrac{7df}{13\eta eez^{\frac{13}{7}\eta}}-\dfrac{7dff}{6\eta e^3z^{\frac{6}{7}\eta}}-\dfrac{7dff}{\eta e^3}(\mathrm{I})$
$\overline{4}$.	$\dfrac{-7d}{27\eta ez^{\frac{27}{7}\eta}}+\dfrac{7df}{20\eta eez^{\frac{20}{7}\eta}}-\dfrac{7dff}{13\eta e^3z^{\frac{13}{7}\eta}}+\dfrac{7df^3}{6\eta e^4z^{\frac{6}{7}\eta}}+\dfrac{7df^3}{\eta e^4}(\mathrm{I})$
&c.	



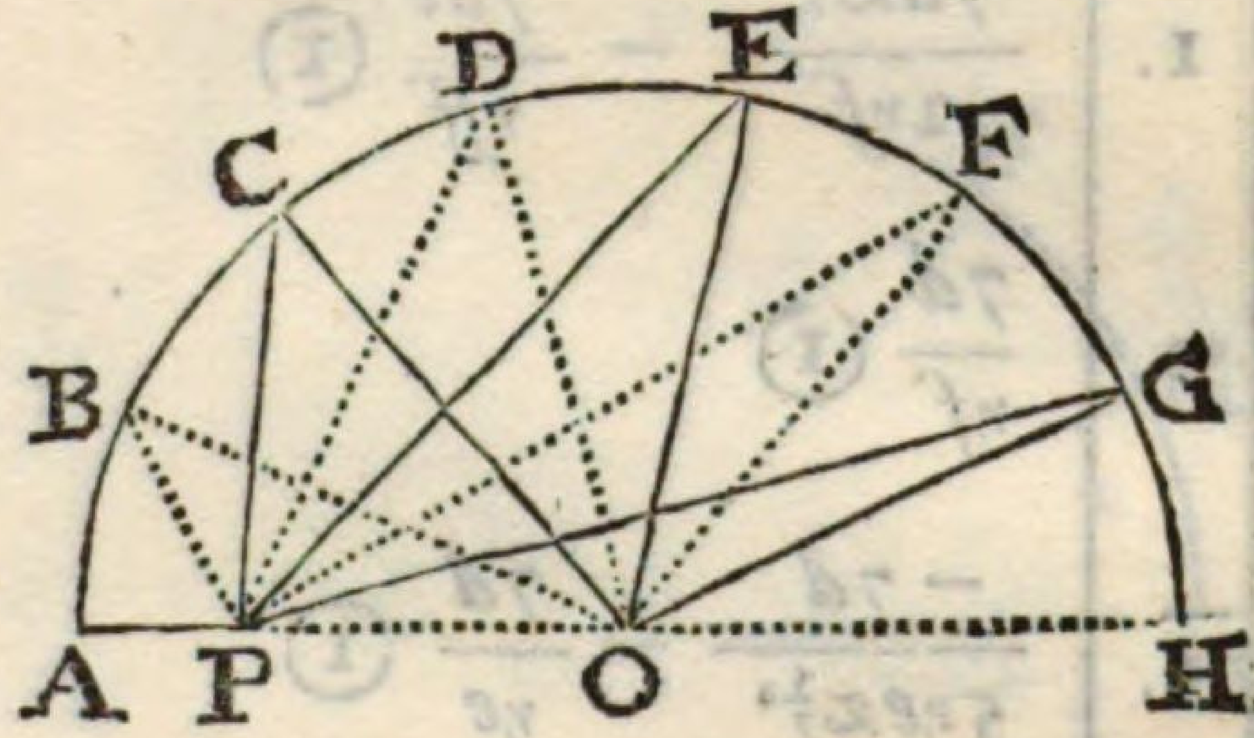
Cas. 1. $OP:OA::\sqrt[7]{z^{\eta}}:\sqrt[7]{\dfrac{e}{f}}$;

$(\mathrm{I}) = \frac{1}{7}\sqrt[7]{\dfrac{f^6}{e^6}}$ in $-\beta(BP:BO)-\delta(DP:DO)+\gamma(FP:FO)$
$+(HP:HO)+\psi(PBO)+\phi(PDO)+\chi(PFO)$

Cas. 2. $OP:OA::\sqrt[7]{z^{\eta}}:\sqrt[7]{\dfrac{-e}{f}}$;

$(\mathrm{I}) = \frac{-1}{7}\sqrt[7]{\dfrac{f^6}{e^6}}$ in $-(AP:AO)-\gamma(CP:CO)+\delta(EP:EO)$
$+\beta(GP:GO)+\chi(PCO)+\phi(PEO)+\psi(PGO)$

θ	FORMA XIV. $\dfrac{dzz^{\theta\eta+\frac{6}{7}\eta-1}}{e+fz^{\eta}}$
&c.	
4.	$\dfrac{7dz^{\frac{27}{7}\eta}}{27\eta f}-\dfrac{7dez^{\frac{20}{7}\eta}}{20\eta ff}+\dfrac{7deez^{\frac{13}{7}\eta}}{13\eta f^{3}}-\dfrac{7de^{3}z^{\frac{6}{7}\eta}}{6\eta f^{4}}+\dfrac{7de^{4}}{\eta f^{5}}\textcircled{\text{I}}$
3.	$\dfrac{7dz^{\frac{20}{7}\eta}}{20\eta f}-\dfrac{7dez^{\frac{13}{7}\eta}}{13\eta ff}+\dfrac{7deez^{\frac{6}{7}\eta}}{6\eta f^{3}}-\dfrac{7de^{3}}{\eta f^{4}}\textcircled{\text{I}}$
2.	$\dfrac{7dz^{\frac{13}{7}\eta}}{13\eta f}-\dfrac{7dez^{\frac{6}{7}\eta}}{6\eta ff}+\dfrac{7dee}{\eta f^{3}}\textcircled{\text{I}}$
1.	$\dfrac{7dz^{\frac{6}{7}\eta}}{6\eta f}-\dfrac{7de}{\eta ff}\textcircled{\text{I}}$
0.	$\dfrac{7d}{\eta f}\textcircled{\text{I}}$
$\overline{1}$.	$\dfrac{-7d}{\eta ez^{\frac{1}{7}\eta}}-\dfrac{7d}{\eta e}\textcircled{\text{I}}$
$\overline{2}$.	$\dfrac{-7d}{8\eta ez^{\frac{8}{7}\eta}}+\dfrac{7df}{\eta eez^{\frac{1}{7}\eta}}+\dfrac{7df}{\eta ee}\textcircled{\text{I}}$
$\overline{3}$.	$\dfrac{-7d}{15\eta ez^{\frac{15}{7}\eta}}+\dfrac{7df}{8\eta eez^{\frac{8}{7}\eta}}-\dfrac{7dff}{\eta e^{3}z^{\frac{1}{7}\eta}}-\dfrac{7dff}{\eta e^{3}}\textcircled{\text{I}}$
$\overline{4}$.	$\dfrac{-7d}{22\eta ez^{\frac{22}{7}\eta}}+\dfrac{7df}{15\eta eez^{\frac{15}{7}\eta}}-\dfrac{7dff}{8\eta e^{3}z^{\frac{8}{7}\eta}}+\dfrac{7df^{3}}{\eta e^{4}z^{\frac{1}{7}\eta}}+\dfrac{7df^{3}}{\eta e^{4}}\textcircled{\text{I}}$
&c.	



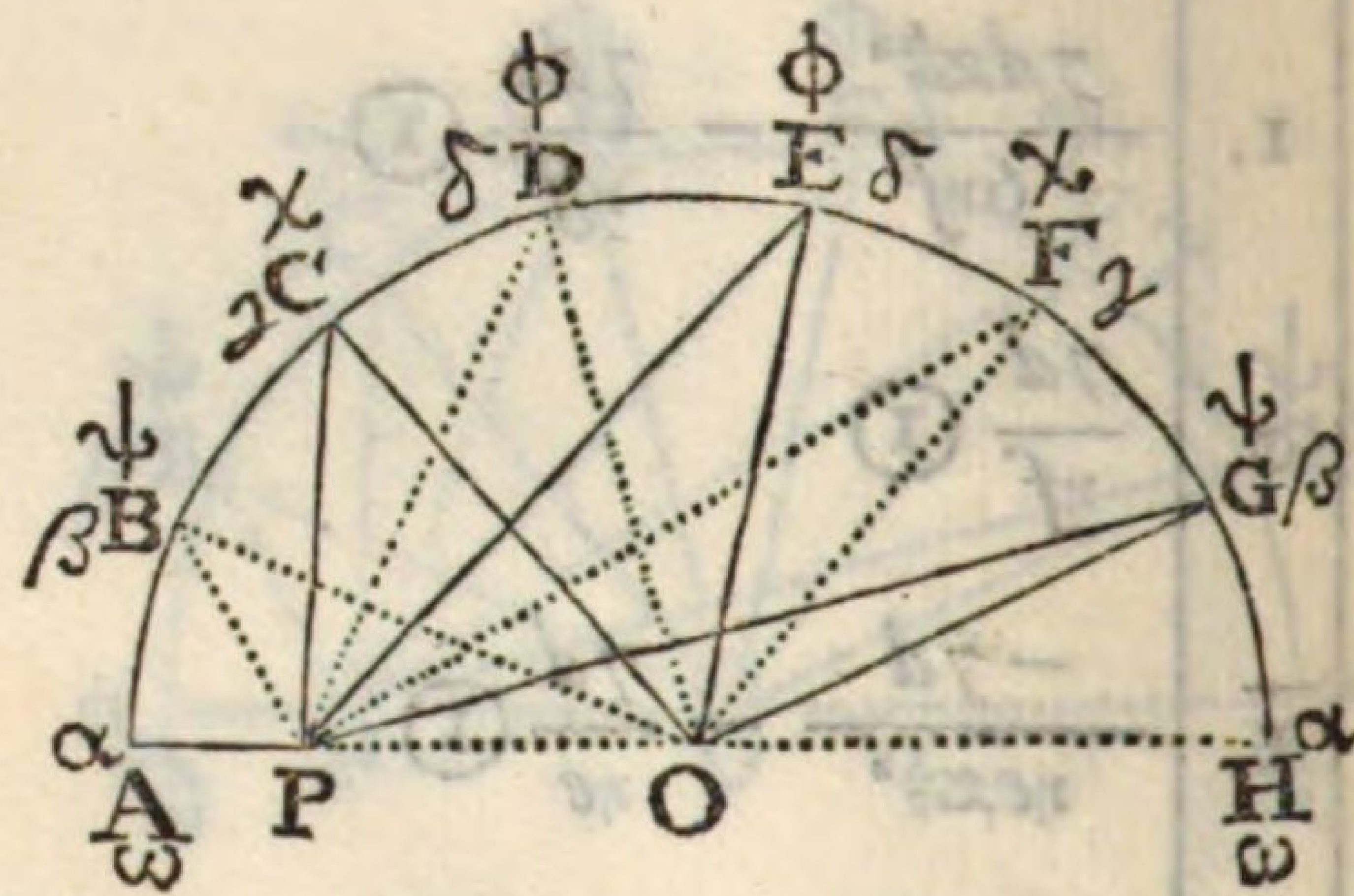
Cas. 1. $OP : OA :: \sqrt[7]{z^{\eta}} : \sqrt[7]{\frac{e}{f}}$;

$\textcircled{\text{I}} = \frac{1}{7}\sqrt[7]{\frac{f}{e}}$ in $+\beta(BP:BO)+\delta(DP:DO)-\gamma(FP:FO)$ $-(HP:HO)+\psi(PBO)+\phi(PDO)+\chi(PFO)$

Cas. 2. $OP : OA :: \sqrt[7]{z^{\eta}} : \sqrt[7]{\frac{-e}{f}}$;

$\textcircled{\text{I}} = \frac{-1}{7}\sqrt[7]{\frac{-f}{e}}$ in $-(AP:AO)-\gamma(CP:CO)+\delta(EP:EO)$ $+\beta(GP:GO)-\chi(PCO)-\phi(PEO)-\psi(PGO)$.

θ	FORMA XV. $\dfrac{dz z^{\theta\eta + \frac{2}{7}\eta - 1}}{e + fz^{\eta}}$
&c.	
4.	$\frac{7dz^{\frac{23}{7}\eta}}{23\eta f} - \frac{7dez^{\frac{16}{7}\eta}}{16\eta ff} + \frac{7deez^{\frac{9}{7}\eta}}{9\eta f^3} - \frac{7de^3 z^{\frac{2}{7}\eta}}{2\eta f^4} + \frac{7de^4}{\eta f^5}\ \textcircled{I}$
3.	$\frac{7dz^{\frac{16}{7}\eta}}{16\eta f} - \frac{7dez^{\frac{9}{7}\eta}}{9\eta ff} + \frac{7deez^{\frac{2}{7}\eta}}{2\eta f^3} - \frac{7de^3}{\eta f^4}\ \textcircled{I}$
2.	$\frac{7dz^{\frac{9}{7}\eta}}{9\eta f} - \frac{7dez^{\frac{2}{7}\eta}}{2\eta ff} + \frac{7dee}{\eta f^3}\ \textcircled{I}$
1.	$\frac{7dz^{\frac{2}{7}\eta}}{2\eta f} - \frac{7de}{\eta ff}\ \textcircled{I}$
0.	$\frac{7d}{\eta f}\ \textcircled{I}$
$\overline{1}$.	$\frac{-7d}{5\eta ez^{\frac{5}{7}\eta}} - \frac{7d}{\eta e}\ \textcircled{I}$
$\overline{2}$.	$\frac{-7d}{12\eta ez^{\frac{12}{7}\eta}} + \frac{7df}{5\eta eez^{\frac{5}{7}\eta}} + \frac{7df}{\eta ee}\ \textcircled{I}$
$\overline{3}$.	$\frac{-7d}{19\eta ez^{\frac{19}{7}\eta}} + \frac{7df}{12\eta eez^{\frac{12}{7}\eta}} - \frac{7dff}{5\eta e^3 z^{\frac{5}{7}\eta}} - \frac{7dff}{\eta e^3}\ \textcircled{I}$
$\overline{4}$.	$\frac{-7d}{26\eta ez^{\frac{26}{7}\eta}} + \frac{7df}{19\eta eez^{\frac{19}{7}\eta}} - \frac{7dff}{12\eta e^3 z^{\frac{12}{7}\eta}} + \frac{7df^3}{5\eta e^4 z^{\frac{5}{7}\eta}} + \frac{7df^3}{\eta e^4}\ \textcircled{I}$
&c.	



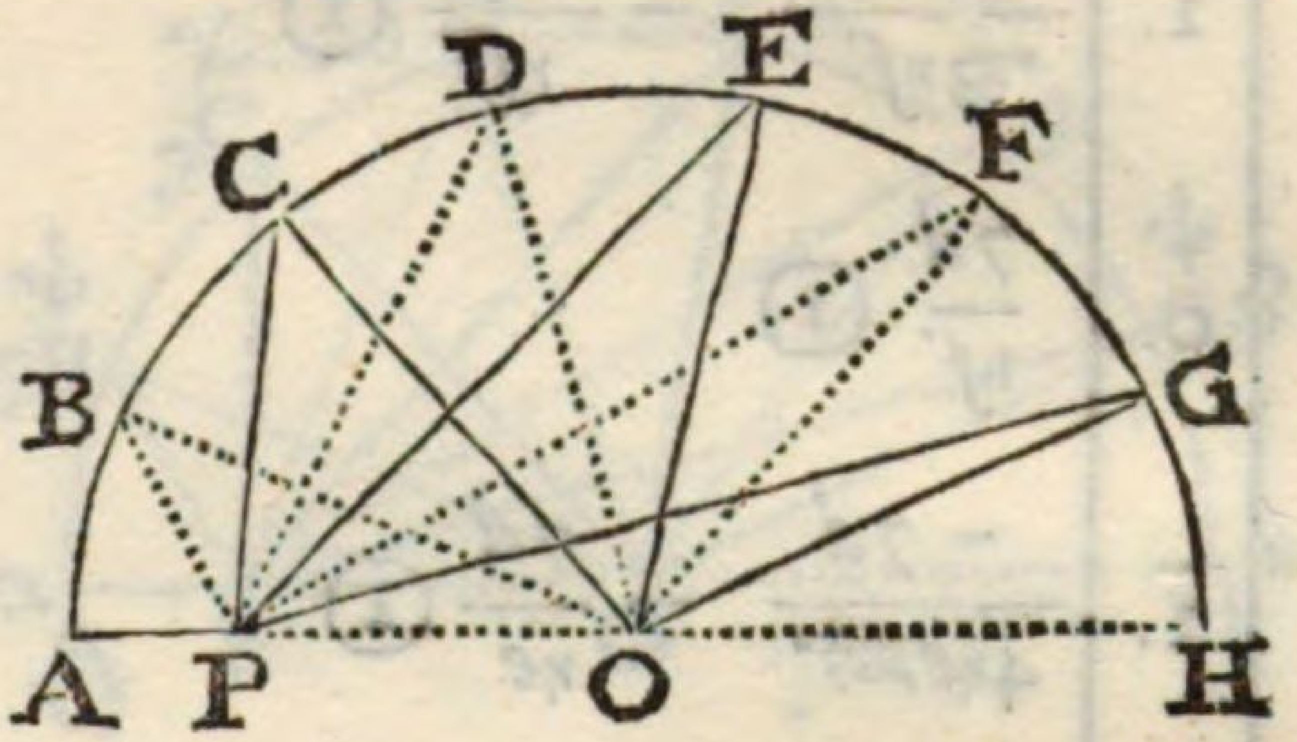
Cas. 1. $OP : OA :: \sqrt[7]{z^{\eta}} : \sqrt[7]{\frac{e}{f}}$;

$\textcircled{I} = \frac{1}{7}\sqrt[7]{\frac{f^5}{e^5}}$ in $-\gamma(BP:BO) + \beta(DP:DO) + \delta(FP:FO) - (HP:HO) + \chi(PBO) + \psi(PDO) - \phi(PFO)$

Cas. 2. $OP : OA :: \sqrt[7]{z^{\eta}} : \sqrt[7]{\frac{-e}{f}}$;

$\textcircled{I} = \frac{-1}{7}\sqrt[7]{\frac{-f^5}{e^5}}$ in $-(AP:AO) + \delta(CP:CO) + \beta(EP:EO) - \gamma(GP:GO) + \phi(PCO) - \psi(PEO) - \chi(PGO)$

θ	FORMA XVI. $\dfrac{dzz^{\theta n+\frac{5}{7}n-1}}{e+fz^{n}}$
&c.	
4.	$\dfrac{7dz^{\frac{26}{7}n}}{26nf}-\dfrac{7dez^{\frac{19}{7}n}}{19nff}+\dfrac{7deez^{\frac{12}{7}n}}{12nf^{3}}-\dfrac{7de^{3}z^{\frac{5}{7}n}}{5nf^{4}}+\dfrac{7de^{4}}{nf^{5}}\textcircled{I}$
3.	$\dfrac{7dz^{\frac{19}{7}n}}{19nf}-\dfrac{7dez^{\frac{12}{7}n}}{12nff}+\dfrac{7deez^{\frac{5}{7}n}}{5nf^{3}}-\dfrac{7de^{3}}{nf^{4}}\textcircled{I}$
2.	$\dfrac{7dz^{\frac{12}{7}n}}{12nf}-\dfrac{7dez^{\frac{5}{7}n}}{5nff}+\dfrac{7dee}{nf^{3}}\textcircled{I}$
1.	$\dfrac{7dz^{\frac{5}{7}n}}{5nf}-\dfrac{7de}{nff}\textcircled{I}$
0.	$\dfrac{7d}{nf}\textcircled{I}$
$\overline{1}$.	$\dfrac{-7d}{2nez^{\frac{2}{7}n}}-\dfrac{7d}{ne}\textcircled{I}$
$\overline{2}$.	$\dfrac{-7d}{9nez^{\frac{9}{7}n}}+\dfrac{7df}{2neez^{\frac{2}{7}n}}+\dfrac{7df}{nee}\textcircled{I}$
$\overline{3}$.	$\dfrac{-7d}{16nez^{\frac{16}{7}n}}+\dfrac{7df}{9neez^{\frac{9}{7}n}}-\dfrac{7dff}{2ne^{3}z^{\frac{2}{7}n}}-\dfrac{7dff}{ne^{3}}\textcircled{I}$
$\overline{4}$.	$\dfrac{-7d}{23nez^{\frac{23}{7}n}}+\dfrac{7df}{16neez^{\frac{16}{7}n}}-\dfrac{7dff}{9ne^{3}z^{\frac{9}{7}n}}+\dfrac{7df^{3}}{2ne^{4}z^{\frac{2}{7}n}}+\dfrac{7df^{3}}{ne^{4}}\textcircled{I}$
&c.	



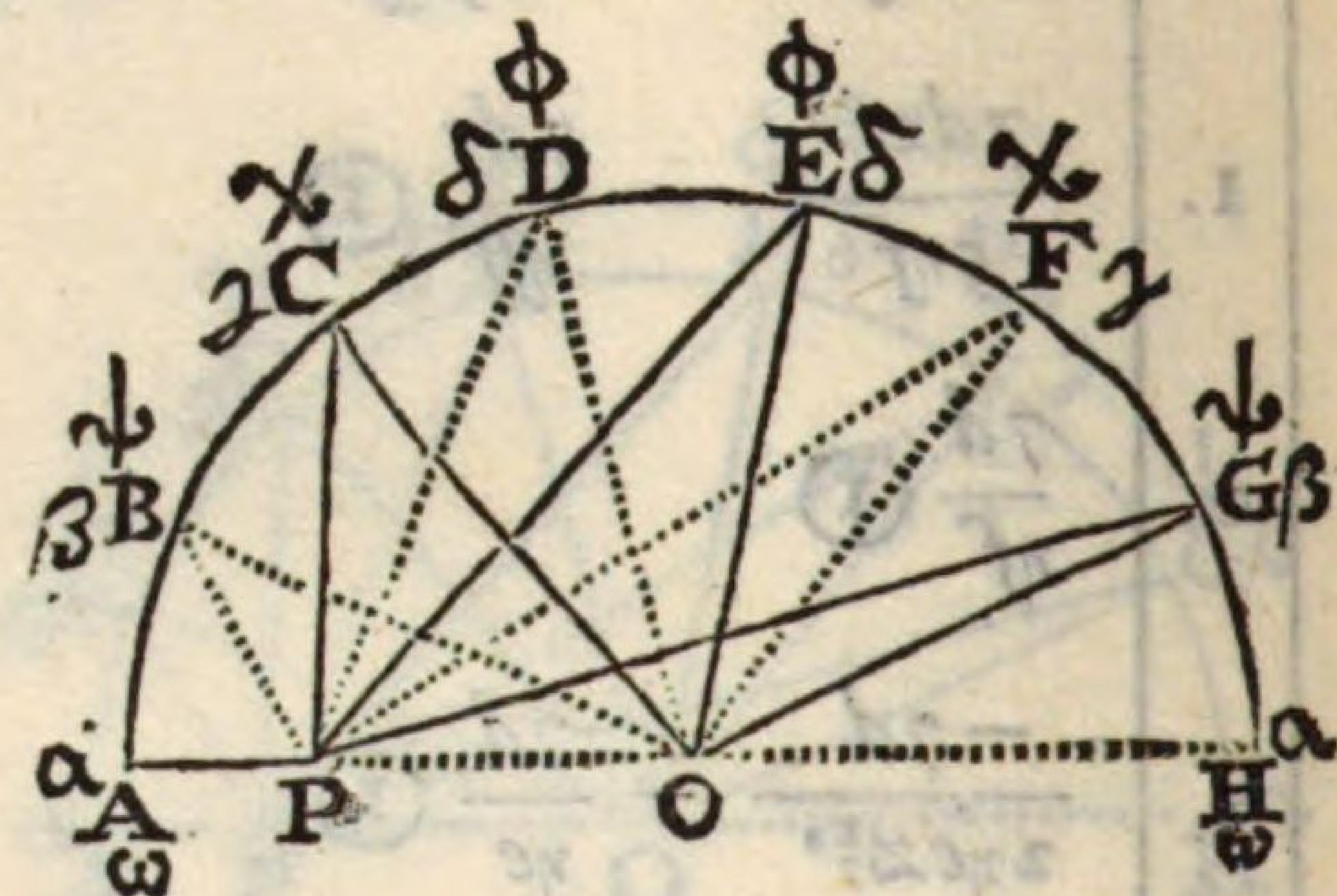
Caſ. 1. $OP:OA::\sqrt[7]{z^{n}}:\sqrt[7]{\dfrac{e}{f}}$;

$\textcircled{I}=\frac{1}{7}\sqrt[7]{\dfrac{ff}{ee}}$ in $+\gamma(BP:BO)-\beta(DP:DO)-\delta(FP:FO)$ $+(HP:HO)+\chi(PBO)+\psi(PDO)-\phi(PFO)$

Caſ. 2. $OP:OA::\sqrt[7]{z^{n}}:\sqrt[7]{\dfrac{-e}{f}}$;

$\textcircled{I}=-\frac{1}{7}\sqrt[7]{\dfrac{ff}{ee}}$ in $-(AP:AO)+\delta(CP:CO)+\beta(EP:EO)$ $-\gamma(GP:GO)-\phi(PCO)+\psi(PEO)+\chi(PGO)$

θ	FORMA XVII. $\dfrac{d\dot{z}z^{\theta\eta+\frac{3}{7}\eta-1}}{e+fz^{\eta}}$
&c.	
4.	$\dfrac{7dz^{\frac{24}{7}\eta}}{24\eta f} - \dfrac{7dez^{\frac{17}{7}\eta}}{17\eta ff} + \dfrac{7deez^{\frac{10}{7}\eta}}{10\eta f^3} - \dfrac{7de^3z^{\frac{3}{7}\eta}}{3\eta f^4} + \dfrac{7de^4}{\eta f^5}\textcircled{I}$
3.	$\dfrac{7dz^{\frac{17}{7}\eta}}{17\eta f} - \dfrac{7dez^{\frac{10}{7}\eta}}{10\eta ff} + \dfrac{7deez^{\frac{3}{7}\eta}}{3\eta f^3} - \dfrac{7de^3}{\eta f^4}\textcircled{I}$
2.	$\dfrac{7dz^{\frac{10}{7}\eta}}{10\eta f} - \dfrac{7dez^{\frac{3}{7}\eta}}{3\eta ff} + \dfrac{7dee}{\eta f^3}\textcircled{I}$
1.	$\dfrac{7dz^{\frac{3}{7}\eta}}{3\eta f} - \dfrac{7de}{\eta ff}\textcircled{I}$
0.	$\dfrac{7d}{\eta f}\textcircled{I}$
$\overline{1}$.	$\dfrac{-7d}{4\eta ez^{\frac{4}{7}\eta}} - \dfrac{7d}{\eta e}\textcircled{I}$
$\overline{2}$.	$\dfrac{-7d}{11\eta ez^{\frac{11}{7}\eta}} + \dfrac{7df}{4\eta eez^{\frac{4}{7}\eta}} + \dfrac{7df}{\eta ee}\textcircled{I}$
$\overline{3}$.	$\dfrac{-7d}{18\eta ez^{\frac{18}{7}\eta}} + \dfrac{7df}{11\eta eez^{\frac{11}{7}\eta}} - \dfrac{7dff}{4\eta e^3z^{\frac{4}{7}\eta}} - \dfrac{7dff}{\eta e^3}\textcircled{I}$
$\overline{4}$.	$\dfrac{-7d}{25\eta ez^{\frac{25}{7}\eta}} + \dfrac{7df}{18\eta eez^{\frac{18}{7}\eta}} - \dfrac{7dff}{11\eta e^3z^{\frac{11}{7}\eta}} + \dfrac{7df^3}{4\eta e^4z^{\frac{4}{7}\eta}} + \dfrac{7df^3}{\eta e^4}\textcircled{I}$
&c.	



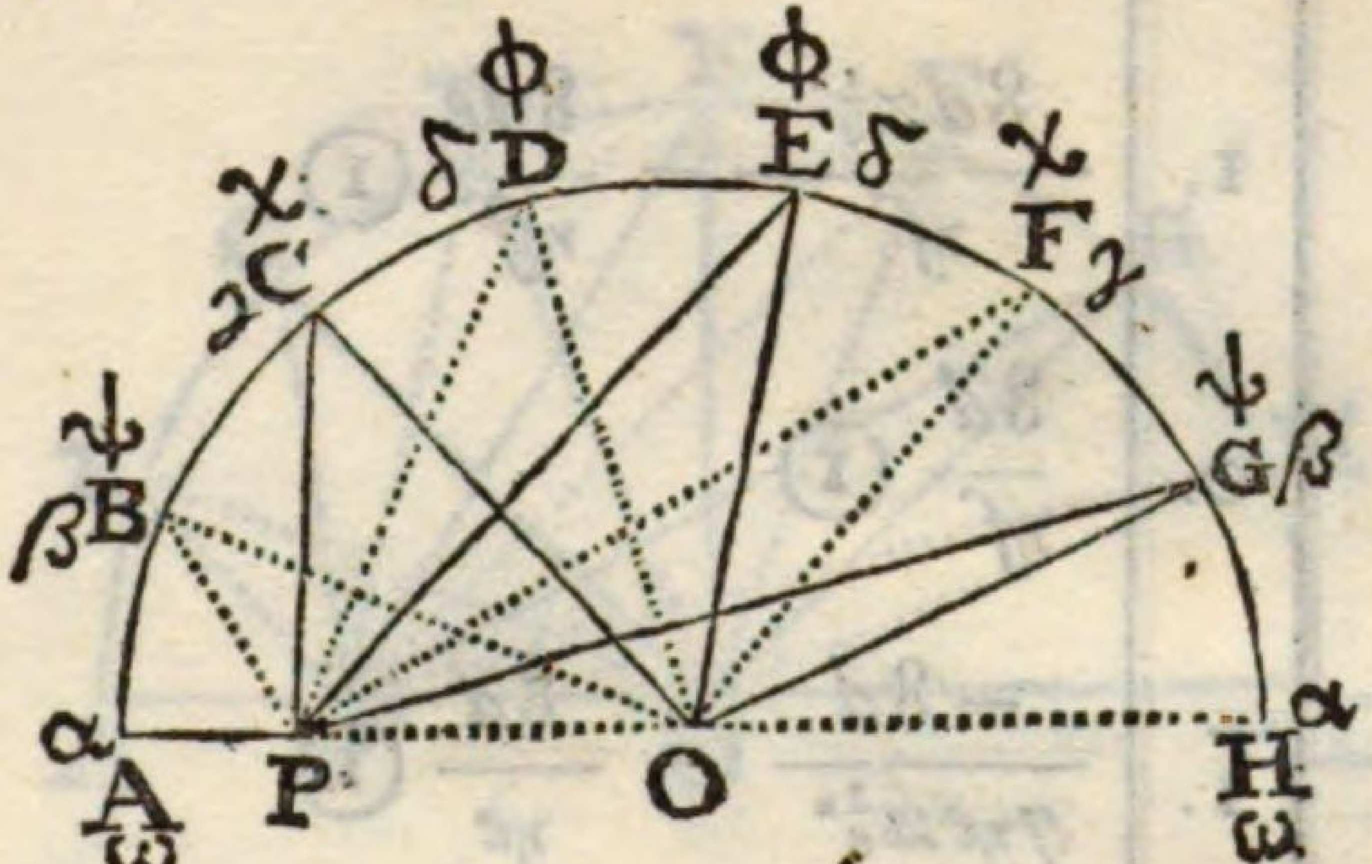
Caſ. 1. $OP:OA::\sqrt[7]{z^{\eta}}:\sqrt[7]{\dfrac{e}{f}}$;

$\textcircled{I} = \frac{1}{7}\sqrt[7]{\dfrac{f^4}{e^4}}$ in $-\delta(BP:BO) + \gamma(DP:DO) - \beta(FP:FO)$
$+(HP:HO) + \phi(PBO) - \chi(PDO) + \psi(PFO)$

Caſ. 2. $OP:OA::\sqrt[7]{z^{\eta}}:\sqrt[7]{\dfrac{-e}{f}}$;

$\textcircled{I} = -\frac{1}{7}\sqrt[7]{\dfrac{f^4}{e^4}}$ in $-(AP:AO) + \beta(CP:CO) - \gamma(EP:EO)$
$+\delta(GP:GO) + \psi(PCO) - \chi(PEO) + \phi(PGO)$

θ	FORMA XVIII. $\dfrac{dz z^{\theta n + \frac{4}{7}n - 1}}{e + f z^n}$
&c.	
4.	$\dfrac{7dz^{\frac{25}{7}n}}{25\eta f} - \dfrac{7dez^{\frac{18}{7}n}}{18\eta ff} + \dfrac{7deez^{\frac{11}{7}n}}{11\eta f^3} - \dfrac{7de^3 z^{\frac{4}{7}n}}{4\eta f^4} + \dfrac{7de^4}{\eta f^5} \textcircled{I}$
3.	$\dfrac{7dz^{\frac{18}{7}n}}{18\eta f} - \dfrac{7dez^{\frac{11}{7}n}}{11\eta ff} + \dfrac{7deez^{\frac{4}{7}n}}{4\eta f^3} - \dfrac{7de^3}{\eta f^4} \textcircled{I}$
2.	$\dfrac{7dz^{\frac{11}{7}n}}{11\eta f} - \dfrac{7dez^{\frac{4}{7}n}}{4\eta ff} + \dfrac{7dee}{\eta f^3} \textcircled{I}$
1.	$\dfrac{7dz^{\frac{4}{7}n}}{4\eta f} - \dfrac{7de}{\eta ff} \textcircled{I}$
0.	$\dfrac{7d}{\eta f} \textcircled{I}$
$\overline{1}$.	$\dfrac{-7d}{3\eta e z^{\frac{3}{7}n}} - \dfrac{7d}{\eta e} \textcircled{I}$
$\overline{2}$.	$\dfrac{-7d}{10\eta e z^{\frac{10}{7}n}} + \dfrac{7df}{3\eta ee z^{\frac{3}{7}n}} + \dfrac{7df}{\eta ee} \textcircled{I}$
$\overline{3}$.	$\dfrac{-7d}{17\eta e z^{\frac{17}{7}n}} + \dfrac{7df}{10\eta ee z^{\frac{10}{7}n}} - \dfrac{7dff}{3\eta e^3 z^{\frac{3}{7}n}} - \dfrac{7dff}{\eta e^3} \textcircled{I}$
$\overline{4}$.	$\dfrac{-7d}{24\eta e z^{\frac{24}{7}n}} + \dfrac{7df}{17\eta ee z^{\frac{17}{7}n}} - \dfrac{7dff}{10\eta e^3 z^{\frac{10}{7}n}} + \dfrac{7df^3}{3\eta e^4 z^{\frac{3}{7}n}} + \dfrac{7df^3}{\eta e^4} \textcircled{I}$
&c.	



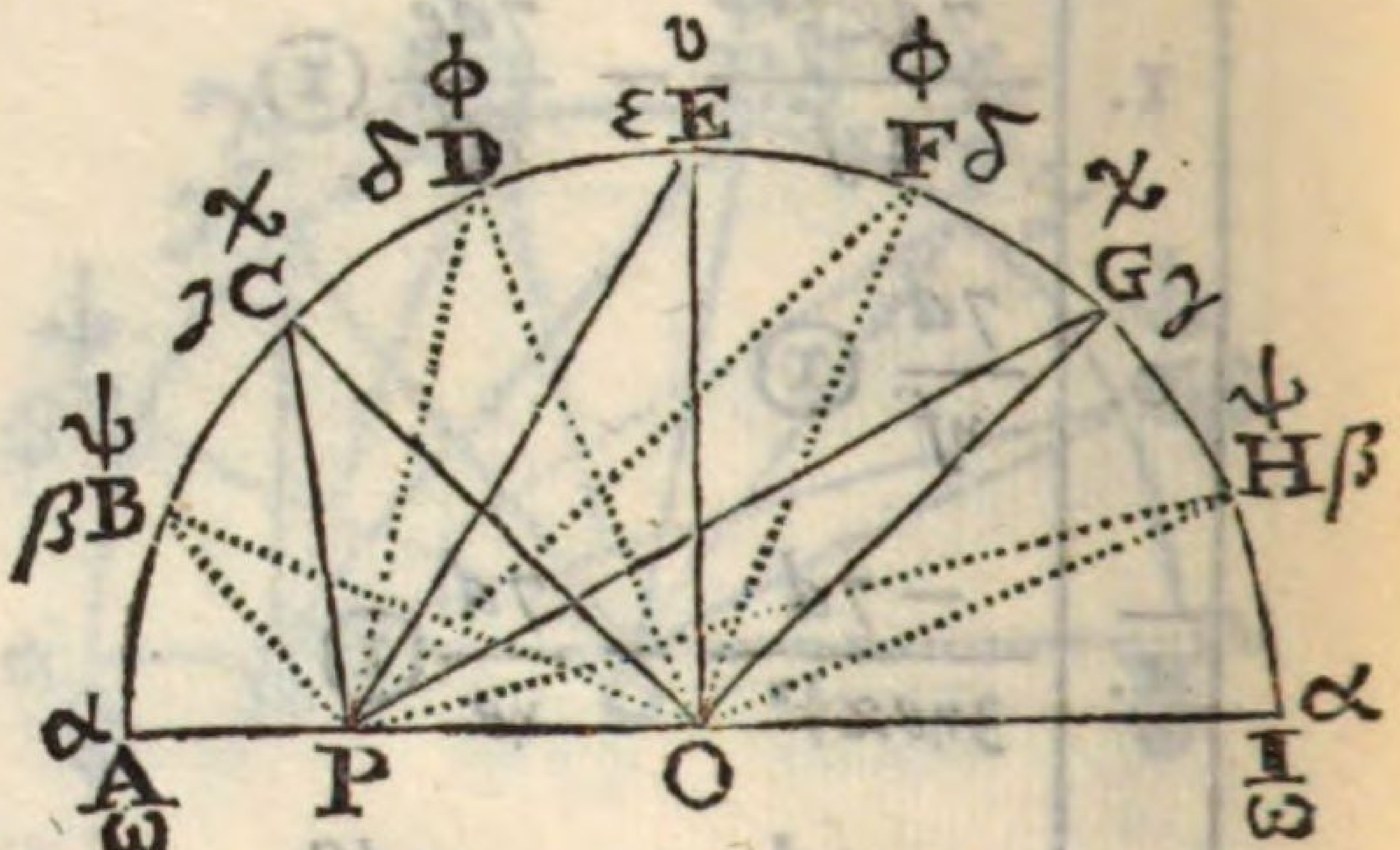
Caf. 1. $OP : OA :: \sqrt[7]{z^n} : \sqrt[7]{\frac{e}{f}}$;

$\textcircled{I} = \frac{1}{7}\sqrt[7]{\frac{f^3}{e^3}}$ in $+\delta(BP:BO) - \gamma(DP:DO) + \beta(FP:FO) - (HP:HO) + \phi(PBO) - \chi(PDO) + \psi(PFO)$.

Caf. 2. $OP : OA :: \sqrt[7]{z^n} : \sqrt[7]{\frac{-e}{f}}$;

$\textcircled{I} = \frac{-1}{7}\sqrt[7]{\frac{-f^3}{e^3}}$ in $-(AP:AO) + \beta(CP:CO) - \gamma(EP:EO) + \delta(GP:GO) - \psi(PCO) + \chi(PEO) - \phi(PGO)$

θ	FORMA XIX. $\dfrac{d\dot{z}z^{\theta n + \frac{1}{8}n - 1}}{e+fz^n}$
&c.	
4.	$\frac{8dz^{\frac{25}{8}n}}{25\eta f} - \frac{8dez^{\frac{17}{8}n}}{17\eta ff} + \frac{8deez^{\frac{9}{8}n}}{9\eta f^3} - \frac{8de^3 z^{\frac{1}{8}n}}{\eta f^4} + \frac{8de^4}{\eta f^5}$ (I)
3.	$\frac{8dz^{\frac{17}{8}n}}{17\eta f} - \frac{8dez^{\frac{9}{8}n}}{9\eta ff} + \frac{8deez^{\frac{1}{8}n}}{\eta f^3} - \frac{8de^3}{\eta f^4}$ (I)
2.	$\frac{8dz^{\frac{9}{8}n}}{9\eta f} - \frac{8dez^{\frac{1}{8}n}}{\eta ff} + \frac{8dee}{\eta f^3}$ (I)
1.	$\frac{8dz^{\frac{1}{8}n}}{\eta f} - \frac{8de}{\eta ff}$ (I)
0.	$\frac{8d}{\eta f}$ (I)
$\overline{1}$.	$\frac{-8d}{7\eta e z^{\frac{7}{8}n}} - \frac{8d}{\eta e}$ (I)
$\overline{2}$.	$\frac{-8d}{15\eta e z^{\frac{15}{8}n}} + \frac{8df}{7\eta ee z^{\frac{7}{8}n}} + \frac{8df}{\eta ee}$ (I)
$\overline{3}$.	$\frac{-8d}{23\eta e z^{\frac{23}{8}n}} + \frac{8df}{15\eta ee z^{\frac{15}{8}n}} - \frac{8dff}{7\eta e^3 z^{\frac{7}{8}n}} - \frac{8dff}{\eta e^3}$ (I)
$\overline{4}$.	$\frac{-8d}{31\eta e z^{\frac{31}{8}n}} + \frac{8df}{23\eta ee z^{\frac{23}{8}n}} - \frac{8dff}{15\eta e^3 z^{\frac{15}{8}n}} + \frac{8df^3}{7\eta e^4 z^{\frac{7}{8}n}} + \frac{8df^3}{\eta e^4}$ (I)
&c.	



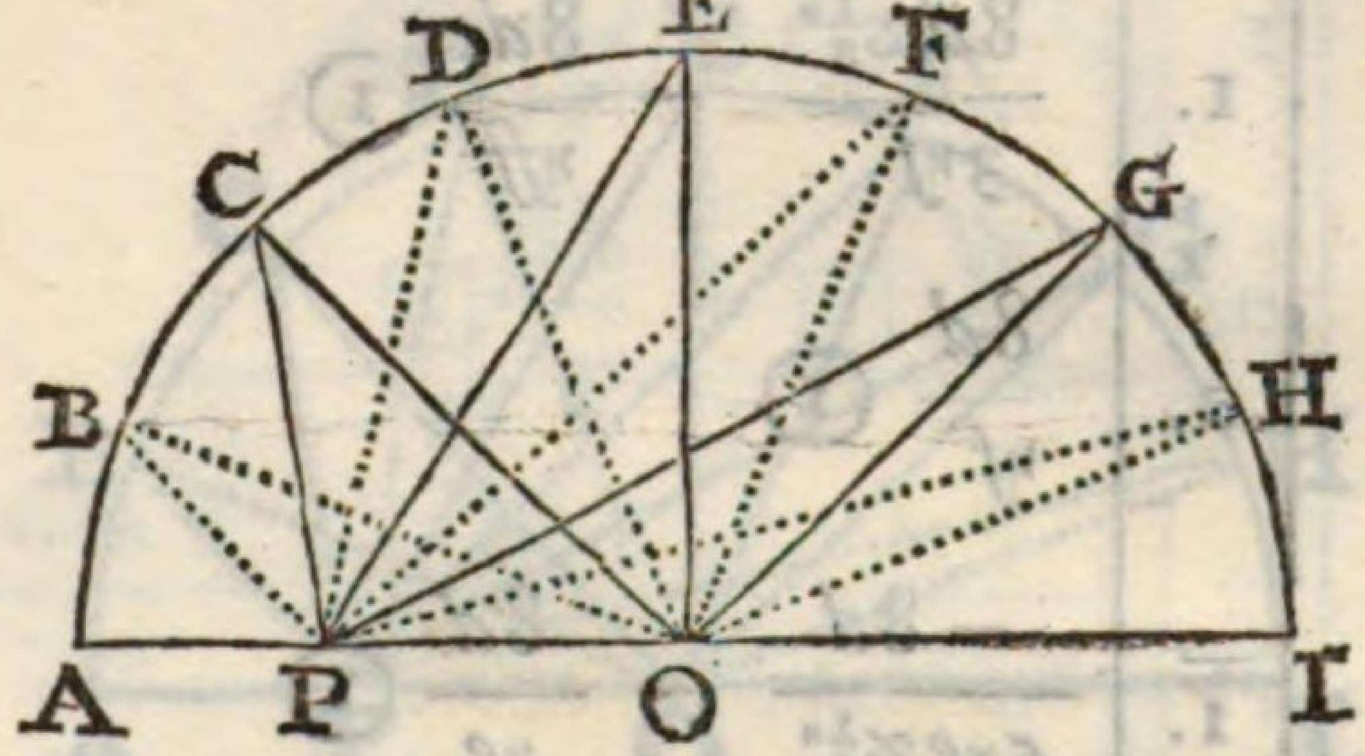
Cas. 1. $OP : OA :: \sqrt[8]{z^n} : \sqrt[8]{\frac{e}{f}}$;

(I) $= \frac{1}{8}\sqrt[8]{\frac{f^7}{e^7}}$ in $-\beta(BP:BO) - \delta(DP:DO) + \delta(FP:FO) + \beta(HP:HO) + \psi(PBO) + \phi(PDO) + \phi(PFO) + \psi(PHO)$

Cas. 2. $OP : OA :: \sqrt[8]{z^n} : \sqrt[8]{\frac{-e}{f}}$;

(I) $= -\frac{1}{8}\sqrt[8]{\frac{-f^7}{e^7}}$ in $-(AP:AO) - \gamma(CP:CO) + \gamma(GP:GO) + (IP:IO) + \chi(PCO) + \upsilon(PEO) + \chi(PGO)$

θ	FORMA XX. $\frac{d\dot{z}z^{\theta n+\frac{7}{8}n-1}}{e+fz^{n}}$
&c.	
4.	$\frac{8dz^{\frac{31}{8}n}}{31nf}-\frac{8dez^{\frac{23}{8}n}}{23nff}+\frac{8deez^{\frac{15}{8}n}}{15nf^{3}}-\frac{8de^{3}z^{\frac{7}{8}n}}{7nf^{4}}+\frac{8de^{4}}{nf^{5}}(\mathrm{I})$
3.	$\frac{8dz^{\frac{23}{8}n}}{23nf}-\frac{8dez^{\frac{15}{8}n}}{15nff}+\frac{8deez^{\frac{7}{8}n}}{7nf^{3}}-\frac{8de^{3}}{nf^{4}}(\mathrm{I})$
2.	$\frac{8dz^{\frac{15}{8}n}}{15nf}-\frac{8dez^{\frac{7}{8}n}}{7nff}+\frac{8dee}{nf^{3}}(\mathrm{I})$
1.	$\frac{8dz^{\frac{7}{8}n}}{7nf}-\frac{8de}{nff}(\mathrm{I})$
0.	$\frac{8d}{nf}(\mathrm{I})$
$\overline{1}$.	$\frac{-8d}{nez^{\frac{1}{8}n}}-\frac{8d}{ne}(\mathrm{I})$
$\overline{2}$.	$\frac{-8d}{9nez^{\frac{9}{8}n}}+\frac{8df}{neez^{\frac{1}{8}n}}+\frac{8df}{nee}(\mathrm{I})$
$\overline{3}$.	$\frac{-8d}{17nez^{\frac{17}{8}n}}+\frac{8df}{9neez^{\frac{9}{8}n}}-\frac{8dff}{ne^{3}z^{\frac{1}{8}n}}-\frac{8dff}{ne^{3}}(\mathrm{I})$
$\overline{4}$.	$\frac{-8d}{25nez^{\frac{25}{8}n}}+\frac{8df}{17neez^{\frac{17}{8}n}}-\frac{8dff}{9ne^{3}z^{\frac{9}{8}n}}+\frac{8df^{3}}{ne^{4}z^{\frac{1}{8}n}}+\frac{8df^{3}}{ne^{4}}(\mathrm{I})$
&c.	



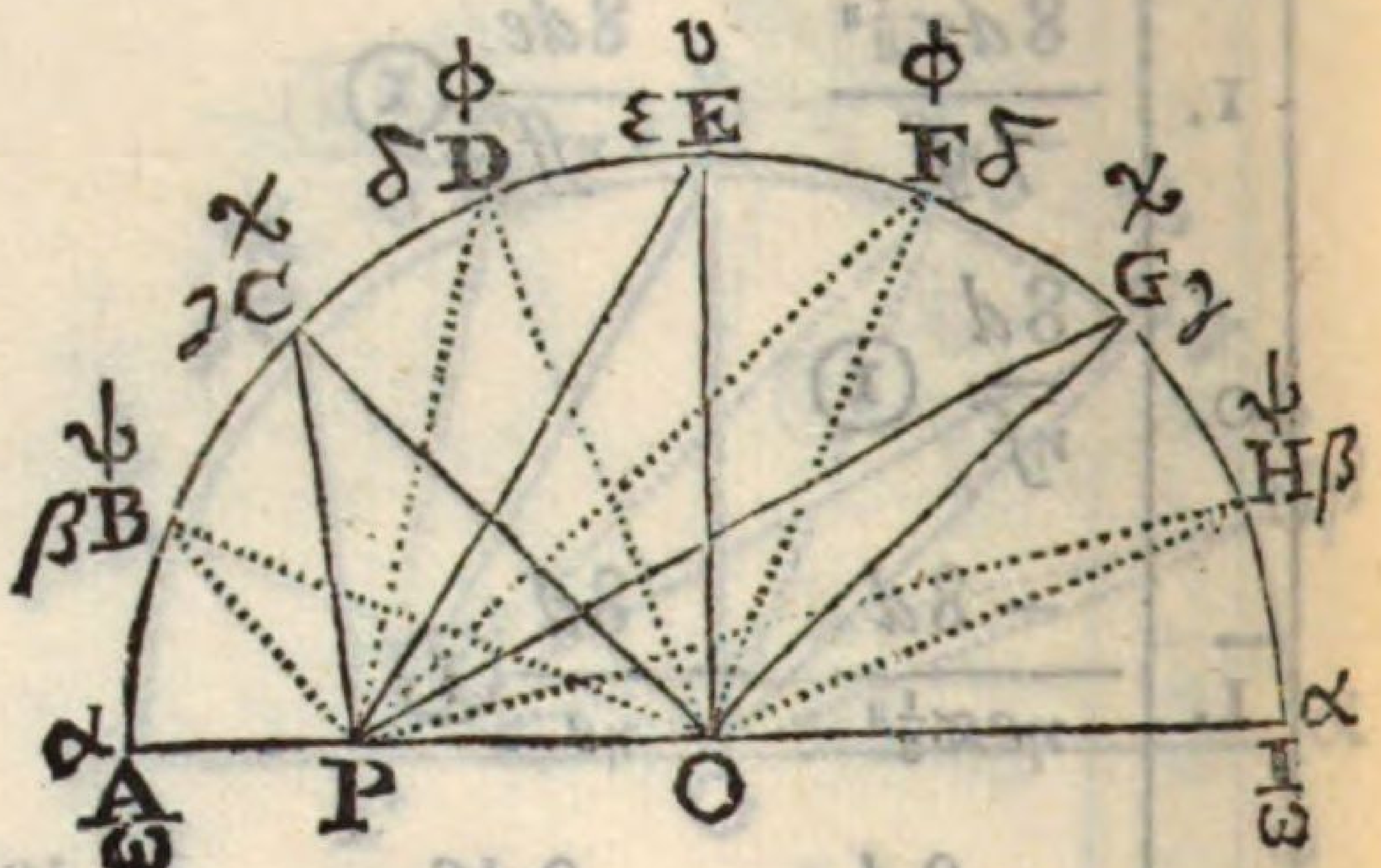
Cas. 1. $OP:OA::\sqrt[8]{z^{n}}:\sqrt[8]{\frac{e}{f}}$;

$(\mathrm{I})=\frac{1}{8}\sqrt[8]{\frac{f}{e}}$ in $+\beta(BP:BO)+\delta(DP:DO)-\delta(FP:FO)$ $-\beta(HP:HO)+\psi(PBO)+\phi(PDO)+\phi(PFO)+\psi(PHO)$

Cas. 2. $OP:OA::\sqrt[8]{z^{n}}:\sqrt[8]{\frac{-e}{f}}$;

$(\mathrm{I})=-\frac{1}{8}\sqrt[8]{\frac{-f}{e}}$ in $-(AP:AO)-\gamma(CP:CO)+\gamma(GP:GO)$ $+(IP:IO)-\chi(PCO)-\upsilon(PEO)-\chi(PGO)$

θ	FORMA XXI. $\frac{dz z^{\theta n + \frac{3}{8}n - 1}}{e + fz^n}$
&c.	
4.	$\frac{8dz^{\frac{27}{8}n}}{27\eta f} - \frac{8dez^{\frac{19}{8}n}}{19\eta ff} + \frac{8deez^{\frac{11}{8}n}}{11\eta f^3} - \frac{8de^3 z^{\frac{3}{8}n}}{3\eta f^4} + \frac{8de^4}{\eta f^5}$ $\textcircled{I}$
3.	$\frac{8dz^{\frac{19}{8}n}}{19\eta f} - \frac{8dez^{\frac{11}{8}n}}{11\eta ff} + \frac{8deez^{\frac{3}{8}n}}{3\eta f^3} - \frac{8de^3}{\eta f^4}$ $\textcircled{I}$
2.	$\frac{8dz^{\frac{11}{8}n}}{11\eta f} - \frac{8dez^{\frac{3}{8}n}}{3\eta ff} + \frac{8dee}{\eta f^3}$ $\textcircled{I}$
1.	$\frac{8dz^{\frac{3}{8}n}}{3\eta f} - \frac{8de}{\eta ff}$ $\textcircled{I}$
0.	$\frac{8d}{\eta f}$ $\textcircled{I}$
$\overline{1}$.	$\frac{-8d}{5\eta ez^{\frac{5}{8}n}} - \frac{8d}{\eta e}$ $\textcircled{I}$
$\overline{2}$.	$\frac{-8a}{13\eta ez^{\frac{13}{8}n}} + \frac{8df}{5\eta eez^{\frac{5}{8}n}} + \frac{8df}{\eta ee}$ $\textcircled{I}$
$\overline{3}$.	$\frac{-8d}{21\eta ez^{\frac{21}{8}n}} + \frac{8df}{13\eta eez^{\frac{13}{8}n}} - \frac{8dff}{5\eta e^3 z^{\frac{5}{8}n}} - \frac{8dff}{\eta e^3}$ $\textcircled{I}$
$\overline{4}$.	$\frac{-8d}{29\eta ez^{\frac{29}{8}n}} + \frac{8df}{21\eta eez^{\frac{21}{8}n}} - \frac{8dff}{13\eta e^3 z^{\frac{13}{8}n}} + \frac{8df^3}{5\eta e^4 z^{\frac{5}{8}n}} + \frac{8df^3}{\eta e^4}$ $\textcircled{I}$
&c.	



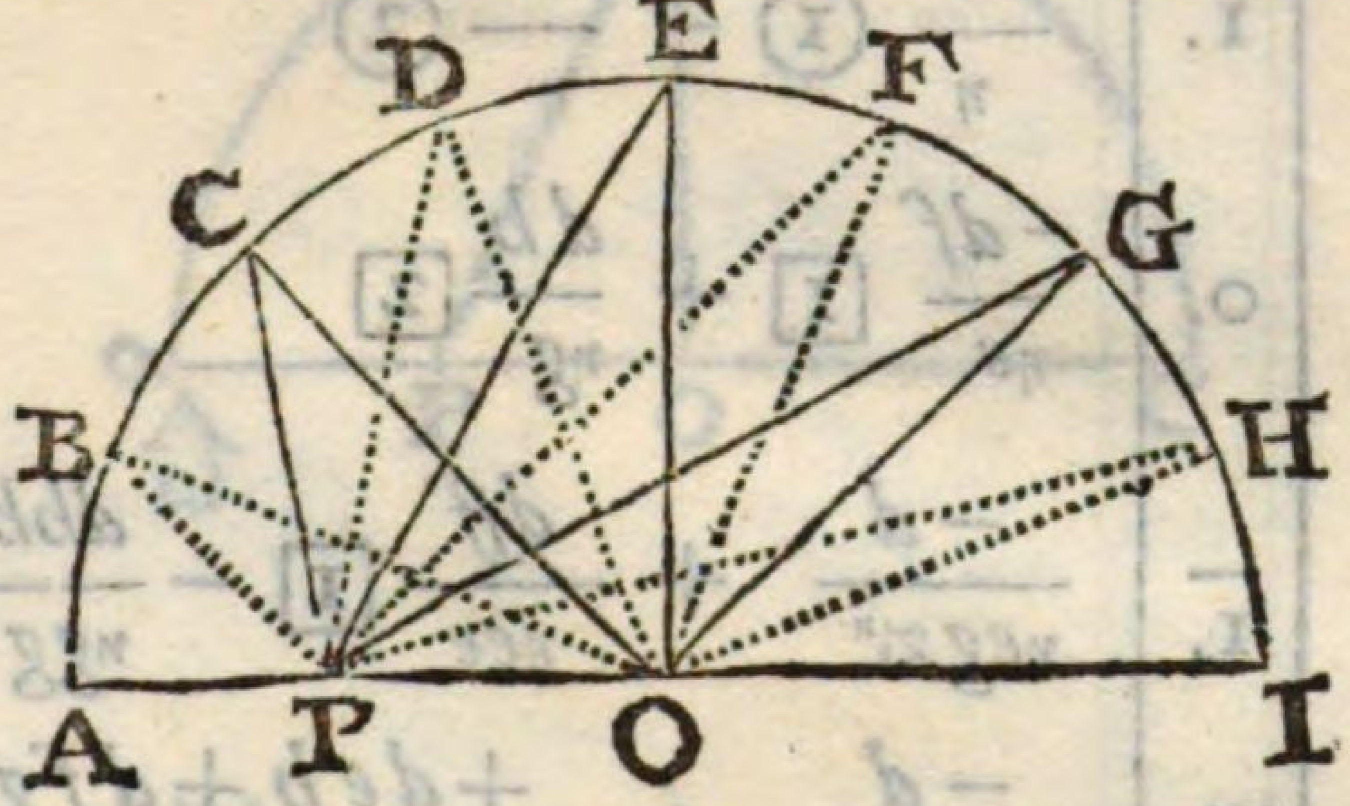
Cas. 1. $OP : OA :: \sqrt[8]{z^n} : \sqrt[8]{\frac{e}{f}}$;

$\textcircled{I} = \frac{1}{8}\sqrt[8]{\frac{f^5}{e^5}}$ in $-\delta(BP:BO) + \beta(DP:DO) - \beta(FP:FO) + \delta(HP:HO) + \phi(PBO) - \psi(PDO) - \psi(PFO) + \phi(PHO)$

Cas. 2. $OP : OA :: \sqrt[8]{z^n} : \sqrt[8]{\frac{-e}{f}}$;

$\textcircled{I} = -\frac{1}{8}\sqrt[8]{\frac{-f^5}{e^5}}$ in $-(AP:AO) + \gamma(CP:CO) - \gamma(GP:GO) + (IP:IO) + \chi(PCO) - \upsilon(PEO) + \chi(PGO)$

θ	FORMA XXII. $\dfrac{dz z^{\theta\eta+\frac{5}{8}\eta-1}}{e+fz^{\eta}}$
&c.	
4.	$\frac{8dz^{\frac{29}{8}\eta}}{29\eta f} - \frac{8dez^{\frac{21}{8}\eta}}{21\eta ff} + \frac{8deez^{\frac{13}{8}\eta}}{13\eta f^{3}} - \frac{8de^{3}z^{\frac{5}{8}\eta}}{5\eta f^{4}} + \frac{8de^{4}}{\eta f^{5}}$ (I)
3.	$\frac{8dz^{\frac{21}{8}\eta}}{21\eta f} - \frac{8dez^{\frac{13}{8}\eta}}{13\eta ff} + \frac{8deez^{\frac{5}{8}\eta}}{5\eta f^{3}} - \frac{8de^{3}}{\eta f^{4}}$ (I)
2.	$\frac{8dz^{\frac{13}{8}\eta}}{13\eta f} - \frac{8dez^{\frac{5}{8}\eta}}{5\eta ff} + \frac{8dee}{\eta f^{3}}$ (I)
1.	$\frac{8dz^{\frac{5}{8}\eta}}{5\eta f} - \frac{8de}{\eta ff}$ (I)
0.	$\frac{8d}{\eta f}$ (I)
$\overline{1}$.	$\frac{-8d}{3\eta ez^{\frac{3}{8}\eta}} - \frac{8d}{\eta e}$ (I)
$\overline{2}$.	$\frac{-8d}{11\eta ez^{\frac{11}{8}\eta}} + \frac{8df}{3\eta eez^{\frac{3}{8}\eta}} + \frac{8df}{\eta ee}$ (I)
$\overline{3}$.	$\frac{-8d}{19\eta ez^{\frac{19}{8}\eta}} + \frac{8df}{11\eta eez^{\frac{11}{8}\eta}} - \frac{8dff}{3\eta e^{3}z^{\frac{3}{8}\eta}} - \frac{8dff}{\eta e^{3}}$ (I)
$\overline{4}$.	$\frac{-8d}{27\eta ez^{\frac{27}{8}\eta}} + \frac{8df}{19\eta eez^{\frac{19}{8}\eta}} - \frac{8dff}{11\eta e^{3}z^{\frac{11}{8}\eta}} + \frac{8df^{3}}{3\eta e^{4}z^{\frac{3}{8}\eta}} + \frac{8df^{3}}{\eta e^{4}}$ (I)
&c.	



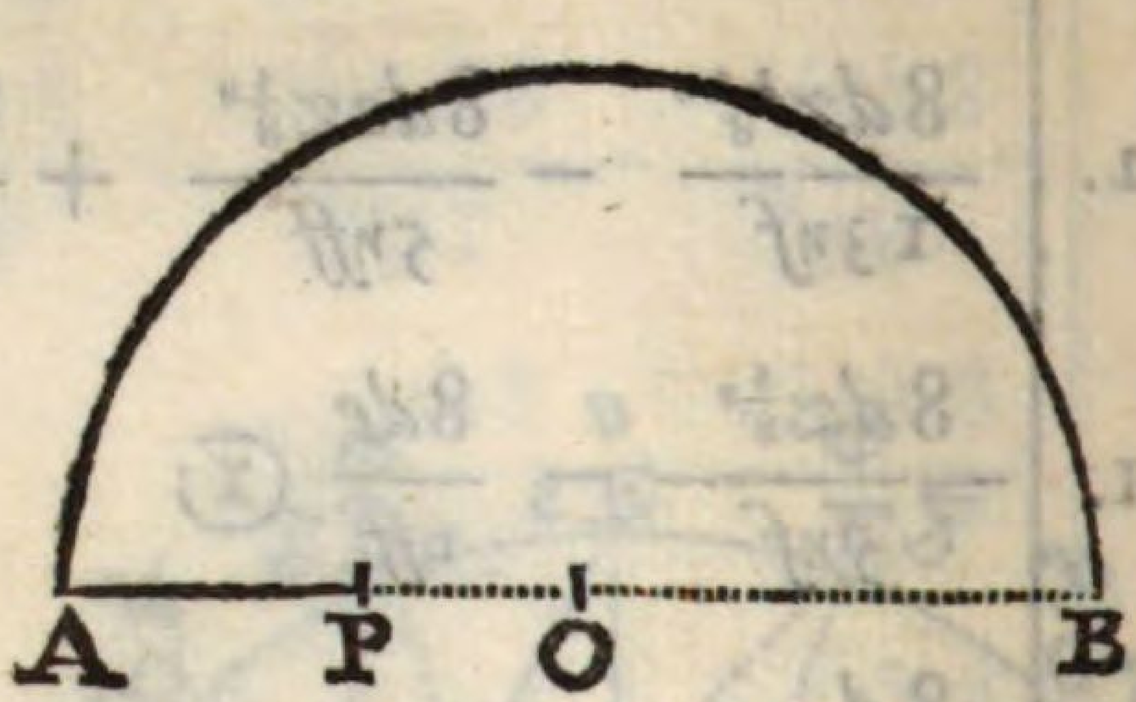
Caf. 1. $OP:OA::\sqrt[8]{z^{\eta}}:\sqrt[8]{\frac{e}{f}}$;

(I) $= \frac{1}{8}\sqrt[8]{\frac{f^{3}}{e^{3}}}$ in $+\delta(BP:BO) - \beta(DP:DO) + \beta(FP:FO) - \delta(HP:HO) + \phi(PBO) - \psi(PDO) - \psi(PFO) + \phi(PHO)$

Caf. 2. $OP:OA::\sqrt[8]{z^{\eta}}:\sqrt[8]{\frac{-e}{f}}$;

(I) $= \frac{-1}{8}\sqrt[8]{\frac{-f^{3}}{e^{3}}}$ in $-(AP:AO) + \gamma(CP:CO) - \gamma(GP:GO) + (IP:IO) - \chi(PCO) + \upsilon(PEO) - \chi(PGO)$

θ	FORMA XXIII. $\frac{dz\,z^{\theta\eta-1}}{\overline{e+fz^{\eta}}\times\overline{g+hz^{\eta}}}$
&c.	
4.	$\frac{dz^{2\eta}}{2\eta fh}+\frac{-dehz^{\eta}-dfgz^{\eta}}{\eta ffhh}-\frac{de^{3}}{\eta f^{3}}$ ① $+\frac{dg^{3}}{\eta h^{3}}$ ②
3.	$\frac{dz^{\eta}}{\eta fh}+\frac{dee}{\eta ff}$ ① $-\frac{dgg}{\eta hh}$ ②
2.	$\frac{-de}{\eta f}$ ① $+\frac{dg}{\eta h}$ ②
1.	$\frac{d}{\eta}$ ① $-\frac{d}{\eta}$ ②
0.	$\frac{-df}{\eta e}\boxed{1}+\frac{dh}{\eta g}\boxed{2}$
$\overline{1}$.	$\frac{-d}{\eta egz^{\eta}}+\frac{dff}{\eta ee}\boxed{1}-\frac{dhh}{\eta gg}\boxed{2}$
$\overline{2}$.	$\frac{-d}{2\eta egz^{2\eta}}+\frac{+deh+dfg}{\eta eeggz^{\eta}}-\frac{df^{3}}{\eta e^{3}}\boxed{1}+\frac{dh^{3}}{\eta g^{3}}\boxed{2}$
&c.	



$p=fg-eh$

Cas. 1.

$OP:OA::z^{\eta}:\frac{e}{f}$; ① $=\frac{1}{p}(BP:BO)$

$OP:OA::z^{\eta}:\frac{g}{h}$; ② $=\frac{1}{p}(BP:BO)$

$OP:OA::\frac{1}{z^{\eta}}:\frac{f}{e}$; $\boxed{1}=\frac{1}{p}(BP:BO)$

$OP:OA::\frac{1}{z^{\eta}}:\frac{h}{g}$; $\boxed{2}=\frac{1}{p}(BP:BO)$

Cas. 2.

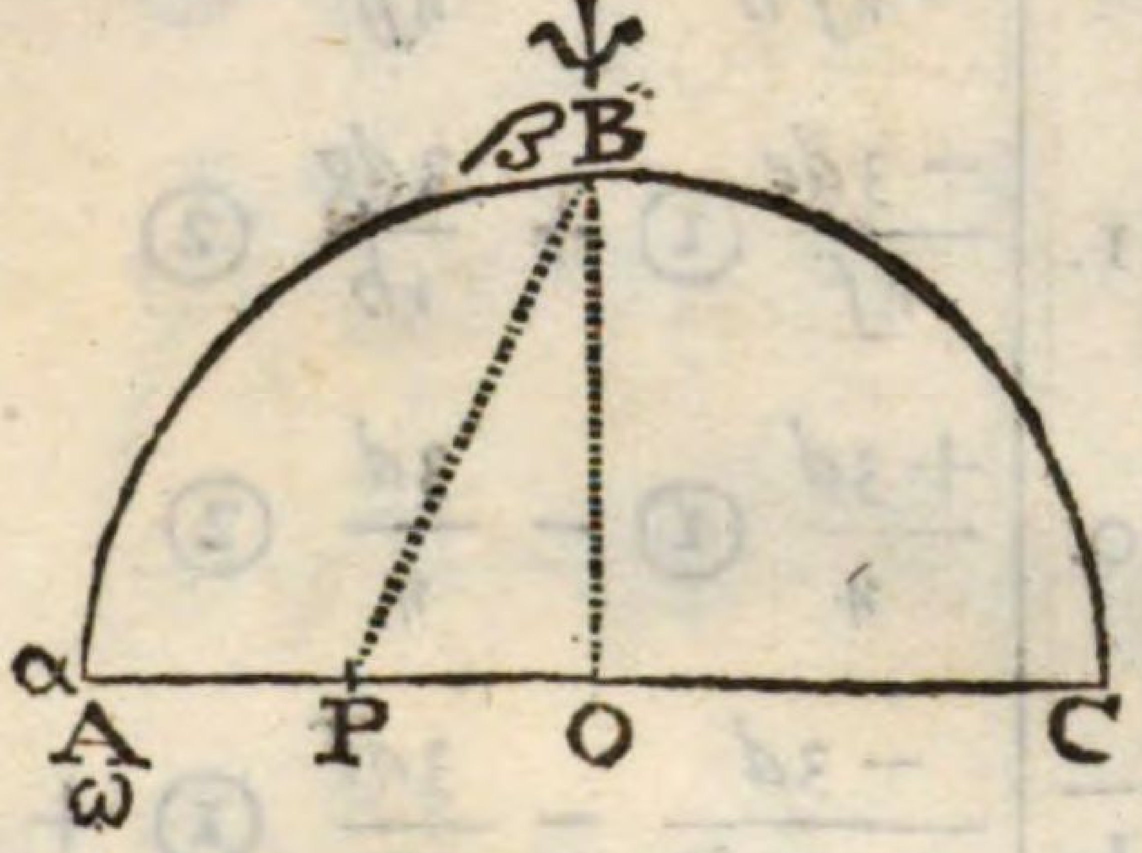
$OP:OA::z_{\eta}:\frac{-e}{f}$; ① $=\frac{1}{p}(AP:AO)$

$OP:OA::z^{\eta}:\frac{-g}{h}$; ② $=\frac{1}{p}(AP:AO)$

$OP::OA::\frac{1}{z^{\eta}}:\frac{-f}{e}$; $\boxed{1}=\frac{1}{p}(AP:AO)$

$OP::OA::\frac{1}{z^{\eta}}:\frac{-h}{g}$; $\boxed{2}=\frac{1}{p}(AP:AO)$

θ	FORMA XXIV. $\dfrac{dzz^{\theta n+\frac{1}{2}n-1}}{\overline{e+fz^n}\times\overline{g+hz^n}}$
&c.	
3.	$\dfrac{2dz^{\frac{3}{2}n}}{3\eta fh}-\dfrac{+2dfgz^{\frac{1}{2}n}+2dehz^{\frac{1}{2}n}}{\eta ffhh}-\dfrac{2de^3}{\eta f^3}$ ① $+\dfrac{2dg^3}{\eta h^3}$ ②
2.	$\dfrac{2dz^{\frac{1}{2}n}}{\eta fh}+\dfrac{2dee}{\eta ff}$ ① $-\dfrac{2dgg}{\eta hh}$ ②
1.	$\dfrac{-2de}{\eta f}$ ① $+\dfrac{2dg}{\eta h}$ ②
0.	$+\dfrac{2d}{\eta}$ ① $-\dfrac{2d}{\eta}$ ②
$\overline{1}$.	$\dfrac{-2d}{\eta egz^{\frac{1}{2}n}}-\dfrac{2df}{\eta e}$ ① $+\dfrac{2dh}{\eta g}$ ②
$\overline{2}$.	$\dfrac{-2d}{3\eta egz^{\frac{3}{2}n}}+\dfrac{+2dfg+2deh}{\eta eeggz^{\frac{1}{2}n}}+\dfrac{2dff}{\eta ee}$ ① $-\dfrac{2dhh}{\eta gg}$ ②
&c.	

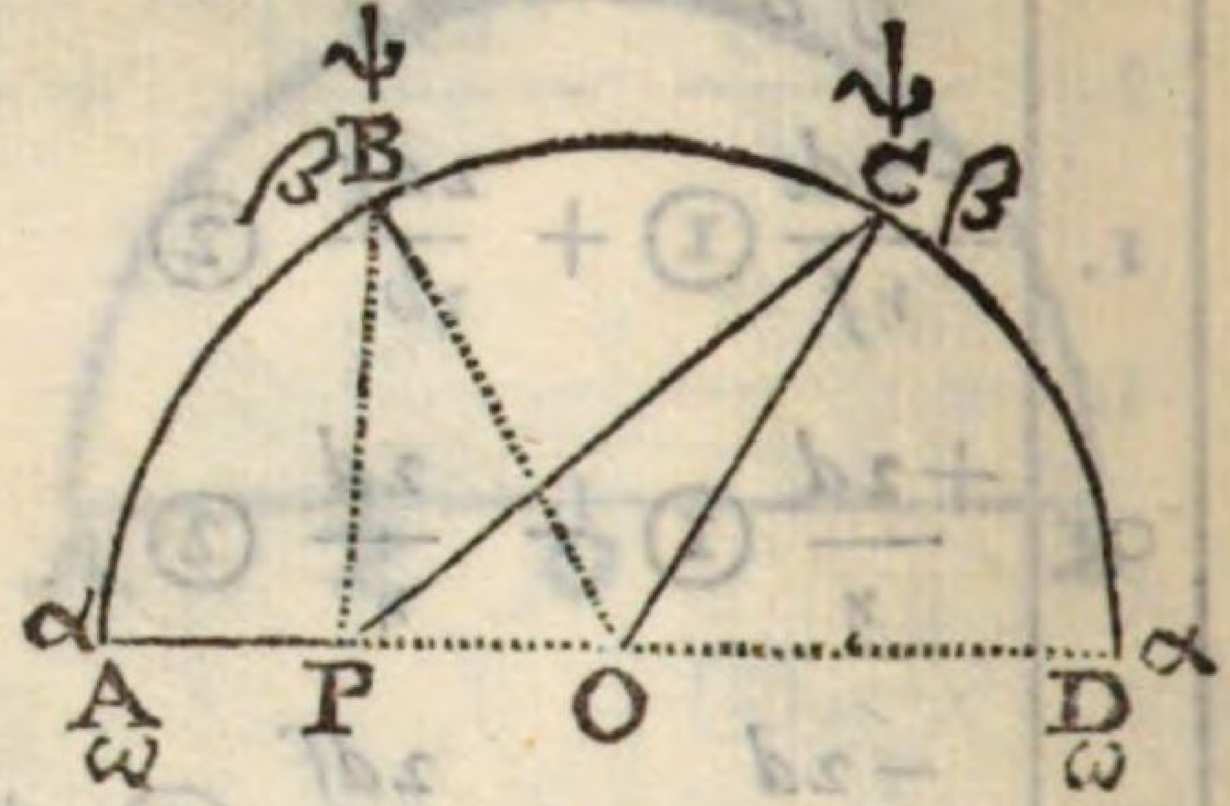


$$p=fg-eh$$

Caſ. 1.
$OP:OA::\sqrt[2]{z^n}:\sqrt[2]{\dfrac{e}{f}}$; ① $=\dfrac{1}{2p}\sqrt[2]{\dfrac{f}{e}}$ in $\psi\,(PBO)$.
$OP:OA::\sqrt[2]{z^n}:\sqrt[2]{\dfrac{g}{h}}$; ② $=\dfrac{1}{2p}\sqrt[2]{\dfrac{h}{g}}$ in $\psi\,(PBO)$.

Caſ. 2.
$OP:OA::\sqrt[2]{z^n}:\sqrt[2]{\dfrac{-e}{f}}$; ① $=\dfrac{-1}{2p}\sqrt[2]{\dfrac{-f}{e}}$ in $-(AP:AO)+(CP:CO)$.
$OP:OA::\sqrt[2]{z^n}:\sqrt[2]{\dfrac{-g}{h}}$; ② $=\dfrac{-1}{2p}\sqrt[2]{\dfrac{-h}{g}}$ in $-(AP:AO)+(CP:CO)$.

θ	FORMA XXV. $\qquad \dfrac{dzz^{\theta n+\frac{1}{3}n-1}}{\overline{e+fz^n}\times\overline{g+hz^n}}$
&c.	
3.	$\dfrac{3dz^{\frac{4}{3}n}}{4\eta fh}-\dfrac{+3dehz^{\frac{1}{3}n}+3dfgz^{\frac{1}{3}n}}{\eta ffhh}-\dfrac{3de^3}{\eta f^3}\textcircled{1}+\dfrac{3dg^3}{\eta h^3}\textcircled{2}$
2.	$\dfrac{3dz^{\frac{1}{3}n}}{\eta fh}+\dfrac{3dee}{\eta ff}\textcircled{1}-\dfrac{3dgg}{\eta hh}\textcircled{2}$
1.	$\dfrac{-3de}{\eta f}\textcircled{1}+\dfrac{3dg}{\eta h}\textcircled{2}$
0.	$\dfrac{+3d}{\eta}\textcircled{1}-\dfrac{3d}{\eta}\textcircled{2}$
$\overline{1}$.	$\dfrac{-3d}{2\eta egz^{\frac{2}{3}n}}-\dfrac{3df}{\eta e}\textcircled{1}+\dfrac{3dh}{\eta g}\textcircled{2}$
$\overline{2}$.	$\dfrac{-3d}{5\eta egz^{\frac{5}{3}n}}+\dfrac{+3deh+3dfg}{2\eta eeggz^{\frac{2}{3}n}}+\dfrac{3dff}{\eta ee}\textcircled{1}-\dfrac{3dhh}{\eta gg}\textcircled{2}$
&c.	



$$p=fg-eh$$

Caf. 1.

$OP:OA::\sqrt[3]{z^n}:\sqrt[3]{\frac{e}{f}}$; $\textcircled{1}=\frac{1}{3p}\sqrt[3]{\frac{ff}{ee}}$ in $-\beta(BP:BO)$ $+(DP:DO)$ $+\psi(PBO)$.

$OP:OA::\sqrt[3]{z^n}:\sqrt[3]{\frac{g}{h}}$; $\textcircled{2}=-\frac{1}{3p}\sqrt[3]{\frac{hh}{gg}}$ in $-\beta(BP:BO)$ $+(DP:DO)$ $+\psi(PBO)$.

Caf. 2.

$OP:OA::\sqrt[3]{z^n}:\sqrt[3]{\frac{-e}{f}}$; $\textcircled{1}=\frac{-1}{3p}\sqrt[3]{\frac{ff}{ee}}$ in $-(AP:AO)$ $+\beta(CP:CO)$ $+\psi(PCO)$.

$OP:OA::\sqrt[3]{z^n}:\sqrt[3]{\frac{-g}{h}}$; $\textcircled{2}=\frac{-1}{3p}\sqrt[3]{\frac{hh}{gg}}$ in $-(AP:AO)$ $+\beta(CP:CO)$ $+\psi(PCO)$.

θ	FORMA XXVI. $\dfrac{dz z^{\theta n + \frac{2}{3}n - 1}}{\overline{e+fz^n} \times \overline{g+hz^n}}$
&c.	
3.	$\dfrac{3dz^{\frac{5}{3}n}}{5\eta fh} - \dfrac{+3dfgz^{\frac{2}{3}n} + 3dehz^{\frac{2}{3}n}}{2\eta ffhh} - \dfrac{3de^3}{\eta f^3}\textcircled{1} + \dfrac{3dg^3}{\eta h^3}\textcircled{2}$
2.	$\dfrac{3dz^{\frac{2}{3}n}}{2\eta fh} + \dfrac{3dee}{\eta ff}\textcircled{1} - \dfrac{3dgg}{\eta hh}\textcircled{2}$
1.	$\dfrac{-3de}{\eta f}\textcircled{1} + \dfrac{3dg}{\eta h}\textcircled{2}$
0.	$\dfrac{+3d}{\eta}\textcircled{1} - \dfrac{3d}{\eta}\textcircled{2}$
$\overline{1}$.	$\dfrac{-3d}{\eta egz^{\frac{1}{3}n}} - \dfrac{3df}{\eta e}\textcircled{1} + \dfrac{3dh}{\eta g}\textcircled{2}$
$\overline{2}$.	$\dfrac{-3d}{4\eta egz^{\frac{4}{3}n}} + \dfrac{+3dfg + 3deh}{\eta eeggz^{\frac{1}{3}n}} + \dfrac{3dff}{\eta ee}\textcircled{1} - \dfrac{3dhh}{\eta gg}\textcircled{2}$
&c.	

$$p = fg - eh$$

Caf. 1.

$$OP : OA :: \sqrt[3]{z^n} : \sqrt[3]{\frac{e}{f}}; \quad \textcircled{1} = \frac{1}{3p}\sqrt[3]{\frac{f}{e}} \text{ in } \beta(BP:BO) - (DP:DO) + \psi(PBO).$$

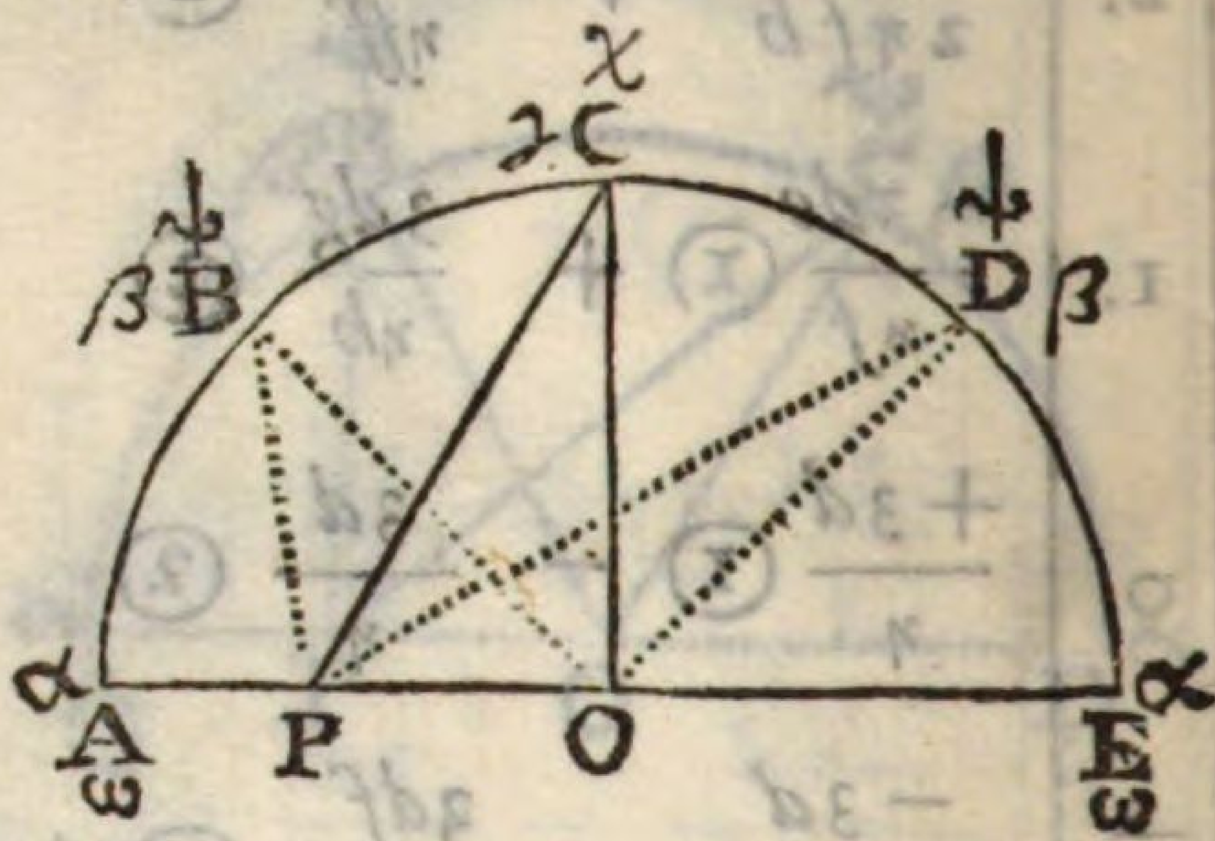
$$OP :: OA :: \sqrt[3]{z^n} : \sqrt[3]{\frac{g}{h}}; \quad \textcircled{2} = \frac{1}{3p}\sqrt[3]{\frac{h}{g}} \text{ in } \beta(BP:BO) - (DP:DO) + \psi(PBO).$$

Caf. 2.

$$OP : OA :: \sqrt[3]{z^n} : \sqrt[3]{\frac{-e}{f}}; \quad \textcircled{1} = \frac{-1}{3p}\sqrt[3]{\frac{-f}{e}} \text{ in } -(AP:AO) + \beta(CP:CO) - \psi(PCO).$$

$$OP : OA :: \sqrt[3]{z^n} : \sqrt[3]{\frac{-g}{h}}; \quad \textcircled{2} = \frac{-1}{3p}\sqrt[3]{\frac{-h}{g}} \text{ in } -(AP:AO) + \beta(CP:CO) - \psi(PCO).$$

θ	FORMA XXVII. $\dfrac{d\dot{z}z^{\theta\eta+\frac{1}{4}\eta-1}}{\overline{e+fz^{\eta}}\times\overline{g+hz^{\eta}}}$
&c.	
3.	$\dfrac{4dz^{\frac{5}{4}\eta}}{5\eta fh}-\dfrac{+4dehz^{\frac{1}{4}\eta}+4dfgz^{\frac{1}{4}\eta}}{\eta ffhh}-\dfrac{4de^3}{\eta f^4}\textcircled{1}+\dfrac{4dg^3}{\eta h^3}\textcircled{2}$
2.	$\dfrac{4dz^{\frac{1}{4}\eta}}{\eta fh}+\dfrac{4dee}{\eta ff}\textcircled{1}-\dfrac{4dgg}{\eta hh}\textcircled{2}$
1.	$\dfrac{-4de}{\eta f}\textcircled{1}+\dfrac{4dg}{\eta h}\textcircled{2}$
0.	$\dfrac{+4d}{\eta}\textcircled{1}-\dfrac{4d}{\eta}\textcircled{2}$
$\overline{1}$.	$\dfrac{-4d}{3\eta egz^{\frac{3}{4}\eta}}-\dfrac{4df}{\eta e}\textcircled{1}+\dfrac{4dh}{\eta g}\textcircled{2}$
$\overline{2}$.	$\dfrac{-4d}{7\eta egz^{\frac{7}{4}\eta}}+\dfrac{+4deh+4dfg}{3\eta eeggz^{\frac{3}{4}\eta}}+\dfrac{4dff}{\eta ee}\textcircled{1}-\dfrac{4dhh}{\eta gg}\textcircled{2}$
&c.	

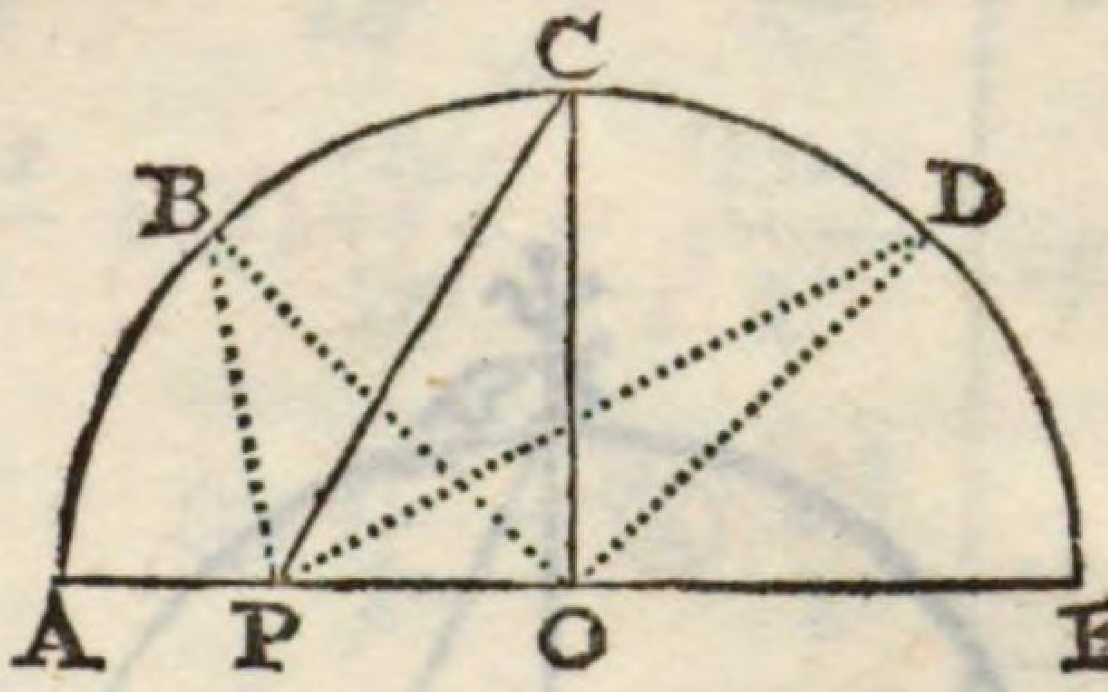


$$p=fg-eh$$

Caf. 1.
$$OP:OA::\sqrt[4]{z^{\eta}}:\sqrt[4]{\frac{e}{f}};\quad \textcircled{1}=\frac{1}{4p}\sqrt[4]{\frac{f^3}{e^3}}\text{ in }-\beta(BP:BO)+\beta(DP:DO)+\psi(PBO)+\psi(PDO).$$
$$OP:OA::\sqrt[4]{z^{\eta}}:\sqrt[4]{\frac{g}{h}};\quad \textcircled{2}=\frac{1}{4p}\sqrt[4]{\frac{h^3}{g^3}}\text{ in }-\beta(BP:BO)+\beta(DP:DO)+\psi(PBO)+\psi(PDO).$$

Caf. 2.
$$OP:OA::\sqrt[4]{z^{\eta}}:\sqrt[4]{\frac{-e}{f}};\quad \textcircled{1}=\frac{-1}{4p}\sqrt[4]{\frac{-f^3}{e^3}}\text{ in }-(AP:AO)+(EP:EO)+\chi(PCO).$$
$$OP:OA::\sqrt[4]{z^{\eta}}:\sqrt[4]{\frac{-g}{h}};\quad \textcircled{2}=\frac{-1}{4p}\sqrt[4]{\frac{-h^3}{g^3}}\text{ in }-(AP:AO)+(EP:EO)+\chi(PCO).$$

θ	FORMA XXVIII. $\dfrac{d\dot{z}z^{\theta\eta+\frac{3}{4}\eta-1}}{\overline{e+fz^{\eta}}\times\overline{g+hz^{\eta}}}$
&c.	
3.	$\dfrac{4dz^{\frac{7}{4}\eta}}{7\eta fh}-\dfrac{+4dehz^{\frac{3}{4}\eta}+4dfgz^{\frac{3}{4}\eta}}{3\eta ffhh}-\dfrac{4de^{3}}{\eta f^{3}}\textcircled{1}+\dfrac{4dg^{3}}{\eta h^{3}}\textcircled{2}$
2.	$\dfrac{4dz^{\frac{3}{4}\eta}}{3\eta fh}+\dfrac{4dee}{\eta ff}\textcircled{1}-\dfrac{4dgg}{\eta hh}\textcircled{2}$
1.	$\dfrac{-4de}{\eta f}\textcircled{1}+\dfrac{4dg}{\eta h}\textcircled{2}$
0.	$\dfrac{+4d}{\eta}\textcircled{1}-\dfrac{4d}{\eta}\textcircled{2}$
- 1.	$-\dfrac{-4d}{\eta egz}-\dfrac{4df}{\eta e}\textcircled{1}+\dfrac{4dh}{\eta g}\textcircled{2}$
- 2.	$\dfrac{-4d}{5\eta egz^{\frac{5}{4}\eta}}+\dfrac{+4dfg+4deh}{\eta eeggz^{\frac{1}{4}\eta}}+\dfrac{4dff}{\eta ee}\textcircled{1}-\dfrac{4dhh}{\eta gg}\textcircled{2}$
&c.	



$$p=fg-eh$$

Cas. 1.

$$OP:OA::\sqrt[4]{z^{\eta}}:\sqrt[4]{\frac{e}{f}};\quad \textcircled{1}=\frac{1}{4p}\sqrt[4]{\frac{f}{e}}\text{ in }\beta(BP:BO)-\beta(DP:DO)+\psi(PBO)+\psi(PDO).$$

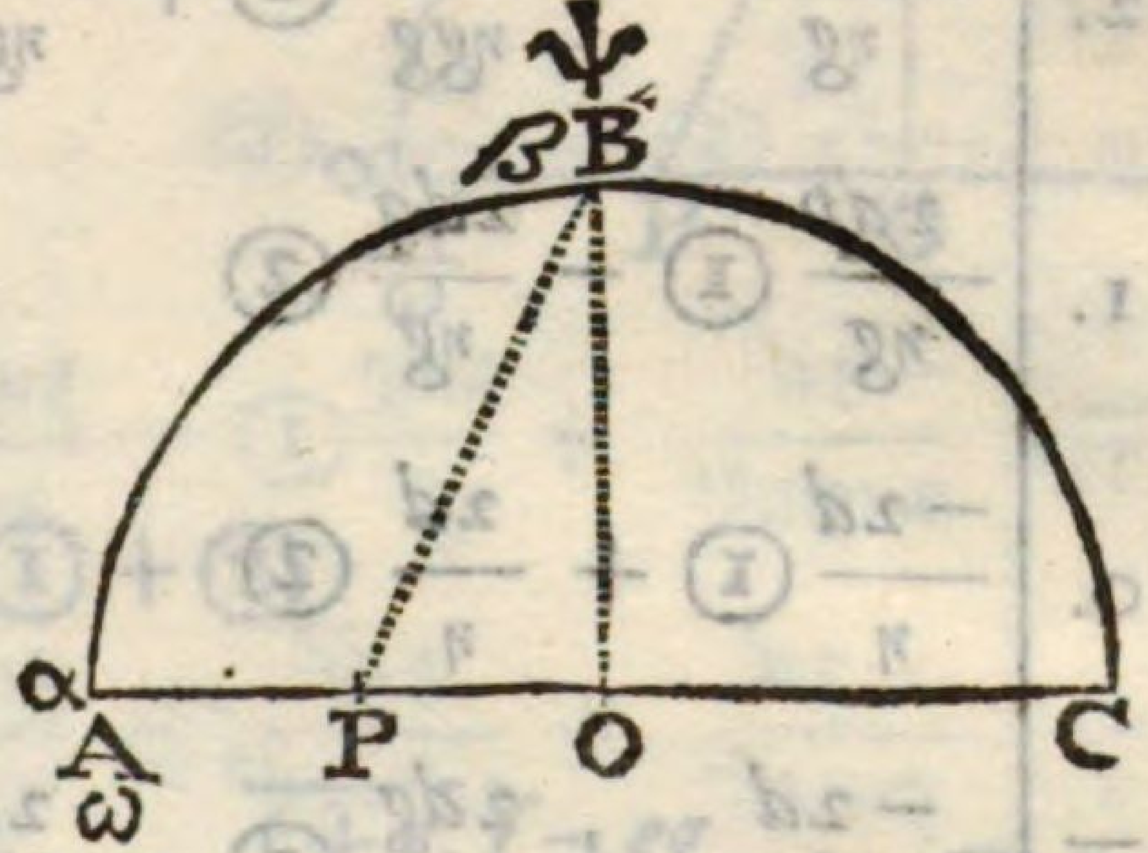
$$OP:OA::\sqrt[4]{z^{\eta}}:\sqrt[4]{\frac{g}{h}};\quad \textcircled{2}=\frac{1}{4p}\sqrt[4]{\frac{h}{g}}\text{ in }\beta(BP:BO)-\beta(DP:DO)+\psi(PBO)+\psi(PDO).$$

Cas. 2.

$$OP:OA::\sqrt[4]{z^{\eta}}:\sqrt[4]{\frac{-e}{f}};\quad \textcircled{1}=\frac{-1}{4p}\sqrt[4]{\frac{-f}{e}}\text{ in }-(AP:AO)+(EP:EO)-\chi(PCO).$$

$$OP:OA::\sqrt[4]{z^{\eta}}:\sqrt[4]{\frac{-g}{h}};\quad \textcircled{2}=\frac{-1}{4p}\sqrt[4]{\frac{-h}{g}}\text{ in }-(AP:AO)+(EP:EO)-\chi(PCO).$$

θ	FORMA XXIX. $\frac{dz z^{\theta n-1}}{e+fz^{n}+gz^{2n}}$
&c.	
4.	$\frac{dz^{2n}}{2\eta g} - \frac{dfz^{n}}{\eta gg} + \frac{+dff \atop -deg}{2\eta g^{3}}\mathrm{M} + \frac{-df^{3} \atop +3defg}{2\eta g^{3}}$ ①
3.	$\frac{dz^{n}}{\eta g} - \frac{df}{2\eta gg}\mathrm{M} + \frac{+dff \atop -2deg}{2\eta gg}$ ①
2.	$\frac{d}{2\eta g}\mathrm{M} - \frac{df}{2\eta g}$ ①
1.	$\frac{d}{\eta}$ ①
0.	$\frac{-d}{2\eta e}\mathrm{m} - \frac{df}{2\eta e}$ ①
$\overline{1}$.	$\frac{-d}{\eta e z^{n}} + \frac{df}{2\eta ee}\mathrm{m} + \frac{+dff \atop -2deg}{2\eta ee}$ ①
$\overline{2}$.	$\frac{-d}{2\eta e z^{2n}} + \frac{df}{\eta ee z^{n}} + \frac{+dff \atop -deg}{2\eta e^{3}}\mathrm{m} + \frac{-df^{3} \atop +3defg}{2\eta e^{3}}$ ①
&c.	



$\mathrm{M} = 1\,(e+fz^{n}+gz^{2n} : e)$. $\mathrm{m} = 1\,(e+fz^{n}+gz^{2n} : gz^{2n})$.

Cas. 1. $OP : OA :: \frac{1}{2}f+gz^{n} : \sqrt{\frac{1}{4}ff-eg}$;

① $= \pm\frac{1}{2}\sqrt{\frac{1}{\frac{1}{4}ff-eg}}$ in $(CP : CA)$.

Cas. 2. $OP : OA :: \frac{1}{2}f+gz^{n} : \sqrt{eg-\frac{1}{4}ff}$;

① $= +\frac{1}{2}\sqrt{\frac{1}{eg-\frac{1}{4}ff}}$ in $\angle\,(PBO)$.

θ	FORMÆ XXX. $\dfrac{d\dot{z}z^{\theta\eta+\frac{1}{2}\eta-1}}{e+fz^{\eta}+gz^{2\eta}}$ *Cas.* 1.
&c.	
4.	$\dfrac{2dz^{\frac{5}{2}\eta}}{5\eta g}-\dfrac{2dfz^{\frac{3}{2}\eta}}{3\eta gg}+\dfrac{\begin{matrix}+dff\\-2deg\end{matrix}}{\eta g^{3}}z^{\frac{1}{2}\eta}-\dfrac{2dp^{4}}{\eta g^{4}}$ ① $+\dfrac{2dq^{4}}{\eta g^{4}}$ ②
3.	$\dfrac{2dz^{\frac{3}{2}\eta}}{3\eta g}-\dfrac{2dfz^{\frac{1}{2}\eta}}{\eta gg}+\dfrac{2dp^{3}}{\eta g^{3}}$ ① $-\dfrac{2dq^{3}}{\eta g^{3}}$ ②
2.	$\dfrac{2dz^{\frac{1}{2}\eta}}{\eta g}-\dfrac{2dpp}{\eta gg}$ ① $+\dfrac{2dqq}{\eta gg}$ ②
1.	$\dfrac{2dp}{\eta g}$ ① $-\dfrac{2dq}{\eta g}$ ②
0.	$\dfrac{-2d}{\eta}$ ① $+\dfrac{2d}{\eta}$ ②
$\overline{1}$.	$\dfrac{-2d}{\eta ez^{\frac{1}{2}\eta}}+\dfrac{2dg}{\eta p}$ ① $-\dfrac{2dg}{\eta q}$ ②
$\overline{2}$.	$\dfrac{-2d}{3\eta ez^{\frac{3}{2}\eta}}+\dfrac{2df}{\eta eez^{\frac{1}{2}\eta}}-\dfrac{2dgg}{\eta pp}$ ① $+\dfrac{2dgg}{\eta qq}$ ②
$\overline{3}$.	$\dfrac{-2d}{5\eta ez^{\frac{5}{2}\eta}}+\dfrac{2df}{3\eta eez^{\frac{3}{2}\eta}}-\dfrac{\begin{matrix}-2deg\\+2dff\end{matrix}}{\eta e^{3}z^{\frac{1}{2}\eta}}+\dfrac{2dg^{3}}{\eta p^{3}}$ ① $-\dfrac{2dg^{3}}{\eta q^{3}}$ ②
&c.	

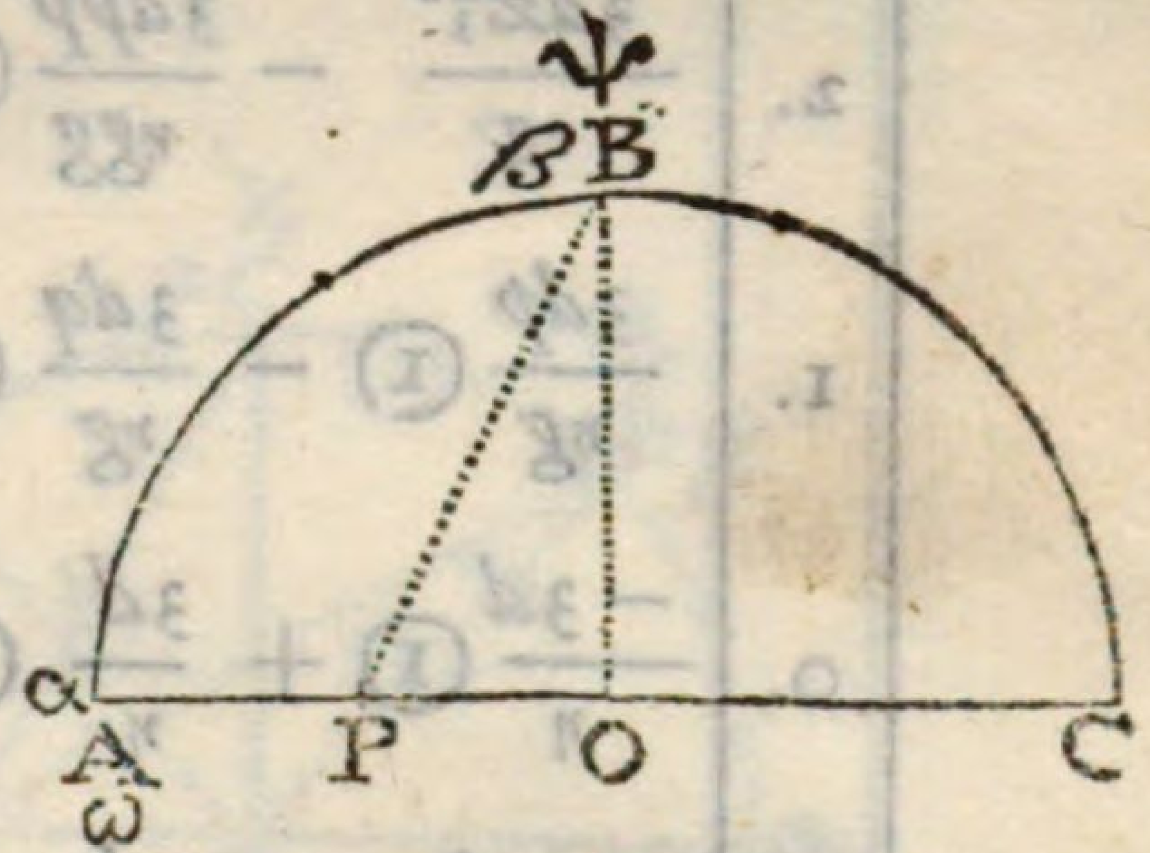
Fig. Pag. subseq.

$$a=\sqrt{ff-4eg};\quad p=\tfrac{1}{2}f+\tfrac{1}{2}a;\quad q=\tfrac{1}{2}f-\tfrac{1}{2}a.$$

Cas. 1. { $OP:OA::\sqrt[2]{z^{\eta}}:\sqrt[2]{\dfrac{p}{g}}$; ① $=\dfrac{1}{2a}\sqrt[2]{\dfrac{g}{p}}$ in ∠ (PBO).
$OP:OA::\sqrt[2]{z^{\eta}}:\sqrt[2]{\dfrac{q}{g}}$; ② $=\dfrac{1}{2a}\sqrt[2]{\dfrac{g}{q}}$ in ∠ (PBO).

Cas. 2. { $OP:OA::\sqrt[2]{z^{\eta}}:\sqrt[2]{\dfrac{-p}{g}}$; ① $=\dfrac{-1}{2a}\sqrt[2]{\dfrac{-g}{p}}$ in $(CP:AP)$
$OP:OA::\sqrt[2]{z^{\eta}}:\sqrt[2]{\dfrac{-q}{g}}$; ② $=\dfrac{-1}{2a}\sqrt[2]{\dfrac{-g}{q}}$ in $(CP:AP)$

θ	FORMÆ XXX. $\frac{d\dot{z}z^{\theta\eta+\frac{1}{2}\eta-1}}{e+fz^{\eta}+gz^{2\eta}}$ *Caſ.* 2.
&c.	
3.	$\frac{2dz^{\frac{3}{2}\eta}}{3\eta g}-\frac{2dfz^{\frac{1}{2}\eta}}{\eta gg}+\frac{\overline{r-f},\,f+eg}{2\eta gg}d\mathrm{M}+\frac{\overline{r+f},\,f-eg}{2\eta gg}d\ \mathrm{in}\ \textcircled{1}+\textcircled{2}$
2.	$\frac{2dz^{\frac{1}{2}\eta}}{\eta g}+\frac{f-r}{2\eta g}d\mathrm{M}-\frac{+f+r}{2\eta g}d\ \mathrm{in}\ \textcircled{1}+\textcircled{2}$
1.	$\frac{-1}{2\eta}d\mathrm{M}+\frac{1}{2\eta}d\ \mathrm{in}\ \textcircled{1}+\textcircled{2}$
0.	$\frac{g}{2\eta r}d\mathrm{M}+\frac{g}{2\eta r}d\ \mathrm{in}\ \textcircled{1}+\textcircled{2}$
$\overline{1}$.	$\frac{-2d}{\eta ez^{\frac{1}{2}\eta}}+\frac{r-f}{2\eta er}gd\mathrm{M}-\frac{+r+f}{2\eta er}gd\ \mathrm{in}\ \textcircled{1}+\textcircled{2}$
$\overline{2}$.	$\frac{-2d}{3\eta ez^{\frac{3}{2}\eta}}+\frac{2df}{\eta eez^{\frac{1}{2}\eta}}+\frac{\overline{f-r},\,f-eg}{2\eta eer}gd\mathrm{M}+\frac{\overline{f+r},\,f-eg}{2\eta eer}gd\ \mathrm{in}\ \textcircled{1}+\textcircled{2}$
&c.	

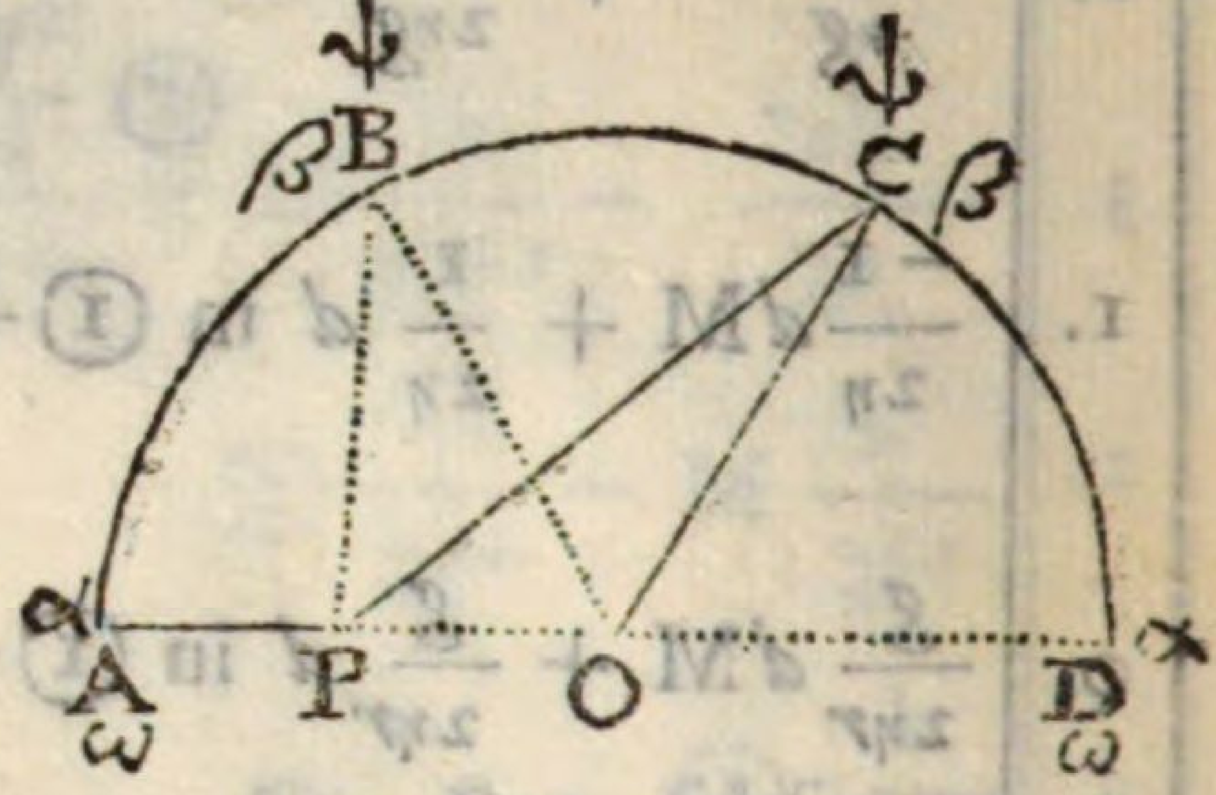


$$r=\sqrt{eg},\quad s=\sqrt{-fg+2\sqrt{eg^3}},\quad \mathrm{M}=\tfrac{1}{s}(r+sz^{\frac{1}{2}\eta}+gz^{\eta}:r-sz^{\frac{1}{2}\eta}+gz^{\eta}).$$

Caſ. 1. $\begin{cases} OP:OA::{}^{+}\tfrac{1}{2}s+gz^{\frac{1}{2}\eta}:\sqrt{\tfrac{1}{4}ss-gr};\ \textcircled{1}={}^{-}\tfrac{1}{2}\sqrt{\frac{1}{\frac{1}{4}ss-gr}}\ \mathrm{in}\,(CP:AP). \\ OP:OA::{}^{-}\tfrac{1}{2}s+gz^{\frac{1}{2}\eta}:\sqrt{\tfrac{1}{4}ss-gr};\ \textcircled{2}={}^{-}\tfrac{1}{2}\sqrt{\frac{1}{\frac{1}{4}ss-rg}}\ \mathrm{in}\,(CP:AP). \end{cases}$

Caſ. 2. $\begin{cases} OP:OA::{}^{+}\tfrac{1}{2}s+gz^{\frac{1}{2}\eta}:\sqrt{gr-\tfrac{1}{4}ss};\ \textcircled{1}={}^{+}\tfrac{1}{2}\sqrt{\frac{1}{gr-\frac{1}{4}ss}}\ \mathrm{in}\ \psi(PBO). \\ OP:OA::{}^{-}\tfrac{1}{2}s+gz^{\frac{1}{2}\eta}:\sqrt{gr-\tfrac{1}{4}ss};\ \textcircled{2}={}^{+}\tfrac{1}{2}\sqrt{\frac{1}{gr-\frac{1}{4}ss}}\ \mathrm{in}\ \psi(PBO). \end{cases}$

θ	FORMA XXXI. $\dfrac{d\dot{z}z^{\theta\eta+\frac{1}{3}\eta-1}}{e+fz^{\eta}+gz^{\eta}}$
&c.	
3.	$\dfrac{3dz^{\frac{4}{3}\eta}}{4\eta g} - \dfrac{3dfz^{\frac{1}{3}\eta}}{\eta gg} + \dfrac{3dp^3}{\eta g^3}\,①\, - \dfrac{3dq^3}{\eta g^3}\,②$
2.	$\dfrac{3dz^{\frac{1}{3}\eta}}{\eta g} - \dfrac{3dpp}{\eta gg}\,①\, + \dfrac{3dqq}{\eta gg}\,②$
1.	$\dfrac{3dp}{\eta g}\,①\, - \dfrac{3dq}{\eta g}\,②$
0.	$\dfrac{-3d}{\eta}\,①\, + \dfrac{3d}{\eta}\,②$
$\overline{1}$.	$\dfrac{-3d}{2\eta ez^{\frac{2}{3}\eta}} + \dfrac{3dg}{\eta p}\,①\, - \dfrac{3dg}{\eta q}\,②$
$\overline{2}$.	$\dfrac{-3d}{5\eta ez^{\frac{5}{3}\eta}} + \dfrac{3df}{2\eta eez^{\frac{2}{3}\eta}} - \dfrac{3dgg}{\eta pp}\,①\, + \dfrac{3dgg}{\eta qq}\,②$
&c.	



$$a = \sqrt{ff - 4eg};\quad p = \tfrac{1}{2}f + \tfrac{1}{2}a;\quad q = \tfrac{1}{2}f - \tfrac{1}{2}a.$$

Caf. 1.

$$OP : OA :: \sqrt[3]{z^{\eta}} : \sqrt[3]{\frac{p}{g}};\quad ① = \frac{1}{3a}\sqrt[3]{\frac{gg}{pp}} \text{ in } -\beta(BP:BO) + (DP:DO) + \psi(PBO).$$

$$OP :: OA :: \sqrt[3]{z^{\eta}} : \sqrt[3]{\frac{q}{g}};\quad ② = \frac{1}{3a}\sqrt[3]{\frac{gg}{qq}} \text{ in } -\beta(BP:BO) + (DP:DO) + \psi(PBO).$$

Caf. 2.

$$OP : OA :: \sqrt[3]{z^{\eta}} : \sqrt[3]{\frac{-p}{g}};\quad ① = \frac{-1}{3a}\sqrt[3]{\frac{gg}{pp}} \text{ in } -(AP:AO) + \beta(CP:CO) + \psi(PCO).$$

$$OP : OA :: \sqrt[3]{z^{\eta}} : \sqrt[3]{\frac{-q}{g}};\quad ② = \frac{-1}{3a}\sqrt[3]{\frac{gg}{qq}} \text{ in } -(AP:AO) + \beta(CP:CO) + \psi(PCO).$$

θ	FORMA XXXII. $\dfrac{d\dot{z}z^{\theta n+\frac{2}{3}n-1}}{e+fz^{n}+gz^{2n}}$
&c.	
3.	$\dfrac{3dz^{\frac{5}{3}n}}{5ng} - \dfrac{3dfz^{\frac{2}{3}n}}{2ngg} + \dfrac{3dp^3}{ng^3}\,①\,-\,\dfrac{3dq^3}{ng^3}\,②$
2.	$\dfrac{3dz^{3n}}{2ng} - \dfrac{3dpp}{ngg}\,①\,+\,\dfrac{3dqq}{ngg}\,②$
1.	$\dfrac{3dp}{ng}\,①\,-\,\dfrac{3dq}{ng}\,②$
0.	$\dfrac{-3d}{n}\,①\,+\,\dfrac{3d}{n}\,②$
$\overline{1}$.	$\dfrac{-3d}{nez^{\frac{1}{3}n}} + \dfrac{3dg}{np}\,①\,-\,\dfrac{3dg}{nq}\,②$
$\overline{2}$.	$-\dfrac{-3d}{4nez^{\frac{4}{3}n}} + \dfrac{3df}{neez^{\frac{1}{3}n}} - \dfrac{3dgg}{npp}\,①\,+\,\dfrac{3dgg}{nqq}\,②$
&c.	

$$a = \sqrt{ff-4eg};\quad p = \tfrac{1}{2}f + \tfrac{1}{2}a;\quad q = \tfrac{1}{2}f - \tfrac{1}{2}a.$$

Cas. 1.

$$OP:OA::\sqrt[3]{z^n}:\sqrt[3]{\frac{p}{g}};\quad ① = \frac{1}{3a}\sqrt[3]{\frac{g}{p}} \text{ in } \beta(BP:BO) - (DP:DO) + \downarrow(PBO).$$

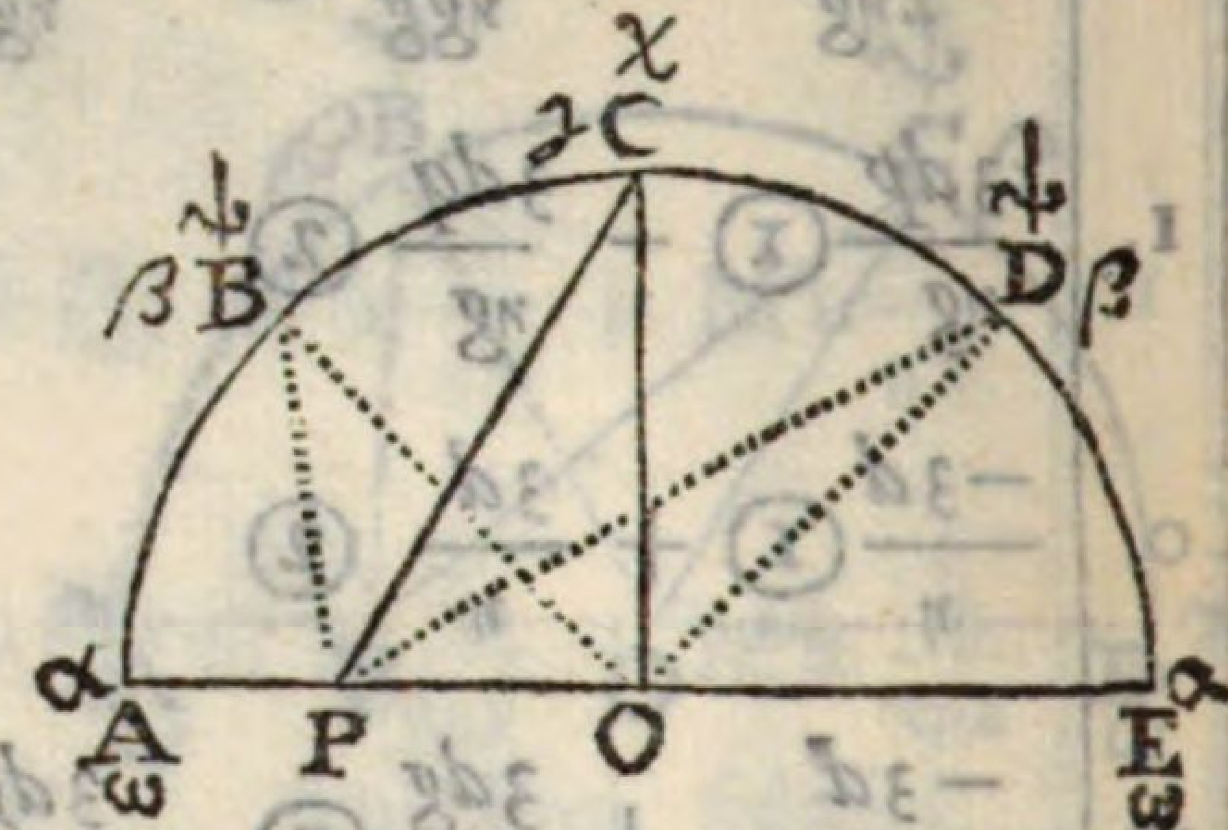
$$OP:OA::\sqrt[3]{z^n}:\sqrt[3]{\frac{q}{g}};\quad ② = \frac{1}{3a}\sqrt[3]{\frac{g}{q}} \text{ in } \beta(BP:BO) - (DP:DO) + \downarrow(PBO).$$

Cas. 2.

$$OP:OA::\sqrt[3]{z^n}:\sqrt[3]{\frac{-p}{g}};\quad ① = \frac{-1}{3a}\sqrt[3]{\frac{-g}{p}} \text{ in } \neg(AP:AO) + \beta(CP:CO) - \downarrow(PCO).$$

$$QO:OA::\sqrt[3]{z^n}:\sqrt[3]{\frac{-q}{g}};\quad ② = \frac{-1}{3a}\sqrt[3]{\frac{-g}{q}} \text{ in } -(AP:AO) + \beta(CP:CO) - \downarrow(PCO).$$

θ	FORMÆ XXXIII. $\dfrac{d\dot{z}z^{\theta n+\frac{1}{4}n-1}}{e+fz^{n}+gz^{2n}}$ *Cas.* 1.
&c.	
3.	$\dfrac{4dz^{\frac{5}{4}n}}{5\eta g} - \dfrac{4dfz^{\frac{1}{4}n}}{\eta gg} + \dfrac{4dp^{3}}{\eta g^{3}}\textcircled{1} - \dfrac{4dq^{3}}{\eta g^{3}}\textcircled{2}$
2.	$\dfrac{4dz^{\frac{1}{4}n}}{\eta g} - \dfrac{4dpp}{\eta gg}\textcircled{1} + \dfrac{4dqq}{\eta gg}\textcircled{2}$
1.	$\dfrac{4dp}{\eta g}\textcircled{1} - \dfrac{4dq}{\eta g}\textcircled{2}$
0.	$\dfrac{-4d}{\eta}\textcircled{1} + \dfrac{4d}{\eta}\textcircled{2}$
$\overline{1}$.	$\dfrac{-4d}{3\eta egz^{\frac{3}{4}n}} + \dfrac{4dg}{\eta p}\textcircled{1} - \dfrac{4dg}{\eta q}\textcircled{2}$
$\overline{2}$.	$\dfrac{-4d}{7\eta ez^{\frac{7}{4}n}} + \dfrac{4df}{3\eta eez^{\frac{3}{4}n}} - \dfrac{4dgg}{\eta pp}\textcircled{1} + \dfrac{4dgg}{\eta qq}\textcircled{2}$
&c.	



$$a=\sqrt{ff-4eg};\quad p=\tfrac{1}{2}f+\tfrac{1}{2}a;\quad q=\tfrac{1}{2}f-\tfrac{1}{2}a.$$

Cas. 1.

$$OP:OA::\sqrt[4]{z^{n}}:\sqrt[4]{\frac{p}{g}};\quad \textcircled{1}=\frac{1}{4a}\sqrt[4]{\frac{g^{3}}{p^{3}}}\ \text{in}\ -\beta(BP:BO)+\beta(DP:DO)+\psi(PBO)+\psi(PDO).$$

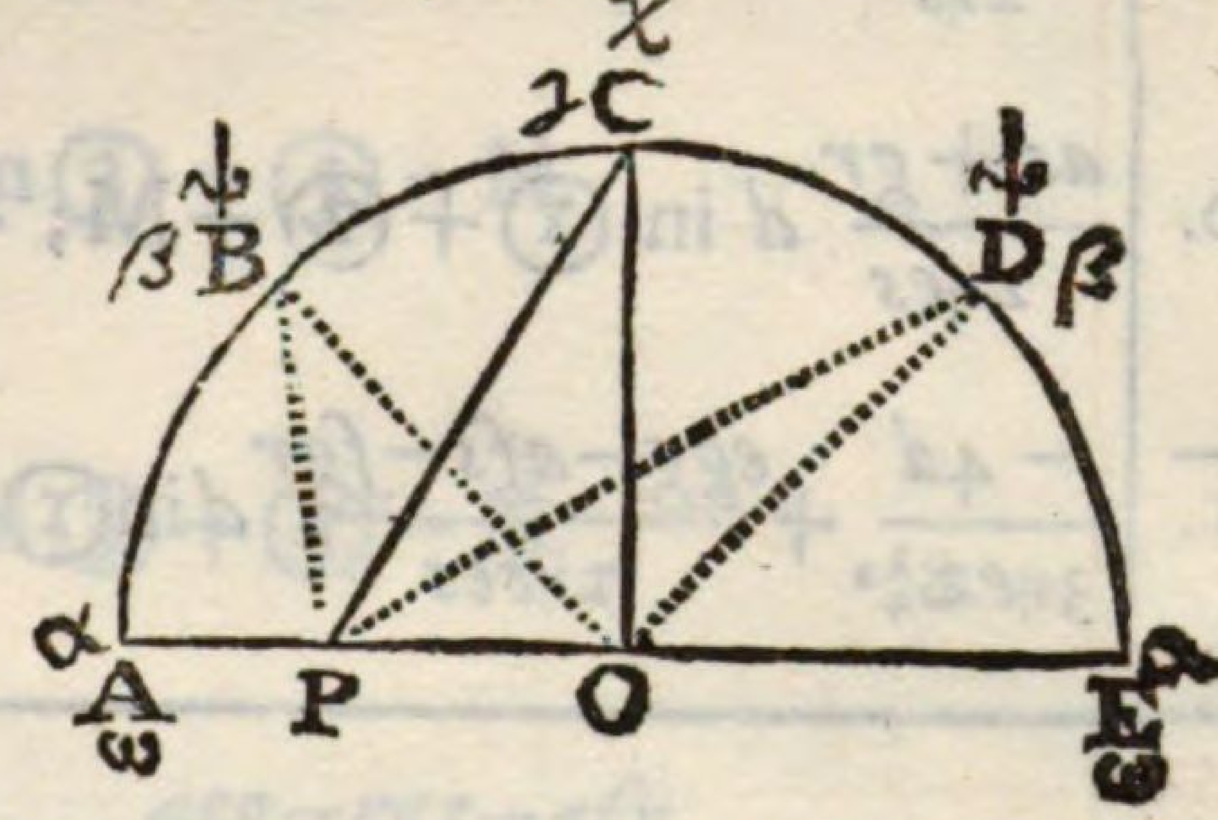
$$OP:OA::\sqrt[4]{z^{n}}:\sqrt[4]{\frac{q}{g}};\quad \textcircled{2}=\frac{1}{4a}\sqrt[4]{\frac{g^{3}}{q^{3}}}\ \text{in}\ -\beta(BP:BO)+\beta(DP:DO)+\psi(PBO)+\psi(PDO).$$

Cas. 2.

$$OP:OA::\sqrt[4]{z^{n}}:\sqrt[4]{\frac{-p}{g}};\quad \textcircled{1}=\frac{-1}{4a}\sqrt[4]{\frac{-g^{3}}{p^{3}}}\ \text{in}\ -(AP:AO)+(EP:EO)+\chi(PCO).$$

$$OP:OA::\sqrt[4]{z^{n}}:\sqrt[4]{\frac{-q}{g}};\quad \textcircled{2}=\frac{-1}{4a}\sqrt[4]{\frac{-g^{3}}{q^{3}}}\ \text{in}\ -(AP:AO)+(EP:EO)+\chi(PCO).$$

θ	FORMÆ XXXIV. $\frac{dz\,z^{\theta\eta+\frac{3}{4}\eta-1}}{e+fz^{\eta}+gz^{2\eta}}$ *Caſ.* 1.
&c.	
3.	$\frac{4dz^{\frac{7}{4}\eta}}{7\eta g}-\frac{4dfz^{\frac{3}{4}\eta}}{3\eta gg}+\frac{4dp^{3}}{\eta g^{3}}$ ① $-\frac{4dq^{3}}{\eta g^{3}}$ ②
2.	$\frac{4dz^{\frac{3}{4}\eta}}{3\eta g}-\frac{4dpp}{\eta gg}$ ① $+\frac{4dqq}{\eta gg}$ ②
1.	$\frac{4dp}{\eta g}$ ① $-\frac{4dq}{\eta g}$ ②
0.	$\frac{-4d}{\eta}$ ① $+\frac{4d}{\eta}$ ②
$\overline{1}$.	$\frac{-4d}{\eta ez^{\frac{1}{4}\eta}}+\frac{4dg}{\eta p}$ ① $-\frac{4dg}{\eta q}$ ②
$\overline{2}$.	$\frac{-4d}{5\eta ez^{\frac{5}{4}\eta}}+\frac{4df}{\eta eez^{\frac{1}{4}\eta}}-\frac{4dgg}{\eta pp}$ ① $+\frac{4dgg}{\eta qq}$ ②
&c.	



$$a=\sqrt{ff-4eg};\quad p=\tfrac{1}{2}f+\tfrac{1}{2}a;\quad q=\tfrac{1}{2}f-\tfrac{1}{2}a.$$

Caſ. 1.

$OP:OA::\sqrt[4]{z^{\eta}}:\sqrt[4]{\frac{p}{g}}$; ① $=\frac{1}{4a}\sqrt[4]{\frac{g}{p}}$ in $+\beta(BP:BO)-\beta(DP:DO)+\psi(PBO)+\psi(PDO)$.

$OP:OA::\sqrt[4]{z^{\eta}}:\sqrt[4]{\frac{q}{g}}$; ② $=\frac{1}{4a}\sqrt[4]{\frac{g}{q}}$ in $+\beta(BP:BO)-\beta(DP:DO)+\psi(PBO)+\psi(PDO)$.

Caſ. 2.

$OP:OA::\sqrt[4]{z^{\eta}}:\sqrt[4]{\frac{-p}{g}}$; ① $=\frac{-1}{4a}\sqrt[4]{\frac{-g}{p}}$ in $-(AP:AO)+(EP:EO)-\chi(PCO)$.

$OP:OA::\sqrt[4]{z^{\eta}}:\sqrt[4]{\frac{-q}{g}}$; ② $=\frac{-1}{4a}\sqrt[4]{\frac{-g}{q}}$ in $-(AP:AO)+(EP:EO)-\chi(PCO)$.

θ	FORMÆ XXXIII. $\frac{d\dot{z}z^{\theta\eta+\frac{1}{4}\eta-1}}{e+fz^{\eta}+gz^{2\eta}}$ *Caf.* 2.
&c.	
2.	$\frac{4dz^{\frac{1}{4}\eta}}{\eta g}+\frac{fg-as-gr}{2\eta gs}d$ in ①+②+M; $+\frac{gr-as-fg}{2\eta gs}d$ in ③+④+N.
1.	$\frac{-g}{2\eta s}d$ in ①+②+M; $+\frac{g}{2\eta s}d$ in ③+④+N.
0.	$\frac{as+gr}{2\eta es}d$ in ①+②+M; $+\frac{as-gr}{2\eta es}d$ in ③+④+N.
$\overline{1}$.	$\frac{-4d}{3\eta ez^{\frac{3}{4}\eta}}+\frac{egg-afs-fgr}{2\eta ees}d$ in ①+②+M; $+\frac{fgr-afs-egg}{2\eta ees}d$ in ③+④+N.
&c.	

$$r=\sqrt{eg};\quad s=\sqrt{-fg+2\sqrt{eg^3}};\quad a=\sqrt[4]{eg^3};$$

$$b=\sqrt{+\sqrt{-fg^3+2\sqrt{eg^7}}+2\sqrt[4]{eg^7}};\quad c=\sqrt{-\sqrt{-fg^3+2\sqrt{eg^7}}+2\sqrt[4]{eg^7}}.$$

Caf. 1.

$$OP:OA::\mp\tfrac{1}{2}c+gz^{\frac{1}{4}\eta}:\sqrt{\tfrac{1}{4}cc-ag};\ ①=-\tfrac{1}{2}\sqrt{\frac{1}{\frac{1}{4}cc-ag}}\text{ in }(CP:AP).$$

$$OP:OA::-\tfrac{1}{2}c+gz^{\frac{1}{4}\eta}:\sqrt{\tfrac{1}{4}cc-ag};\ ②=-\tfrac{1}{2}\sqrt{\frac{1}{\frac{1}{4}cc-ag}}\text{ in }(CP:AP).$$

$$OP:OA::\mp\tfrac{1}{2}b+gz^{\frac{1}{4}\eta}:\sqrt{\tfrac{1}{4}bb-ag};\ ③=-\tfrac{1}{2}\sqrt{\frac{1}{\frac{1}{4}bb-ag}}\text{ in }(CP:AP).$$

$$OP:OA::-\tfrac{1}{2}b+gz^{\frac{1}{4}\eta}:\sqrt{\tfrac{1}{4}bb-ag};\ ④=-\tfrac{1}{2}\sqrt{\frac{1}{\frac{1}{4}bb-ag}}\text{ in }(CP:AP).$$

Caf. 2.

$$OP:OA::\mp\tfrac{1}{2}c+gz^{\frac{1}{4}\eta}:\sqrt{ag-\tfrac{1}{4}cc};\ ①=+\tfrac{1}{2}\sqrt{\frac{1}{ag-\frac{1}{4}cc}}\text{ in }\psi(PBO).$$

$$OP:OA::-\tfrac{1}{2}c+gz^{\frac{1}{4}\eta}:\sqrt{ag-\tfrac{1}{4}cc};\ ②=+\tfrac{1}{2}\sqrt{\frac{1}{ag-\frac{1}{4}cc}}\text{ in }\psi(PBO).$$

$$OP:OA::\mp\tfrac{1}{2}b+gz^{\frac{1}{4}\eta}:\sqrt{ag-\tfrac{1}{4}bb};\ ③=+\tfrac{1}{2}\sqrt{\frac{1}{ag-\frac{1}{4}bb}}\text{ in }\psi(PBO).$$

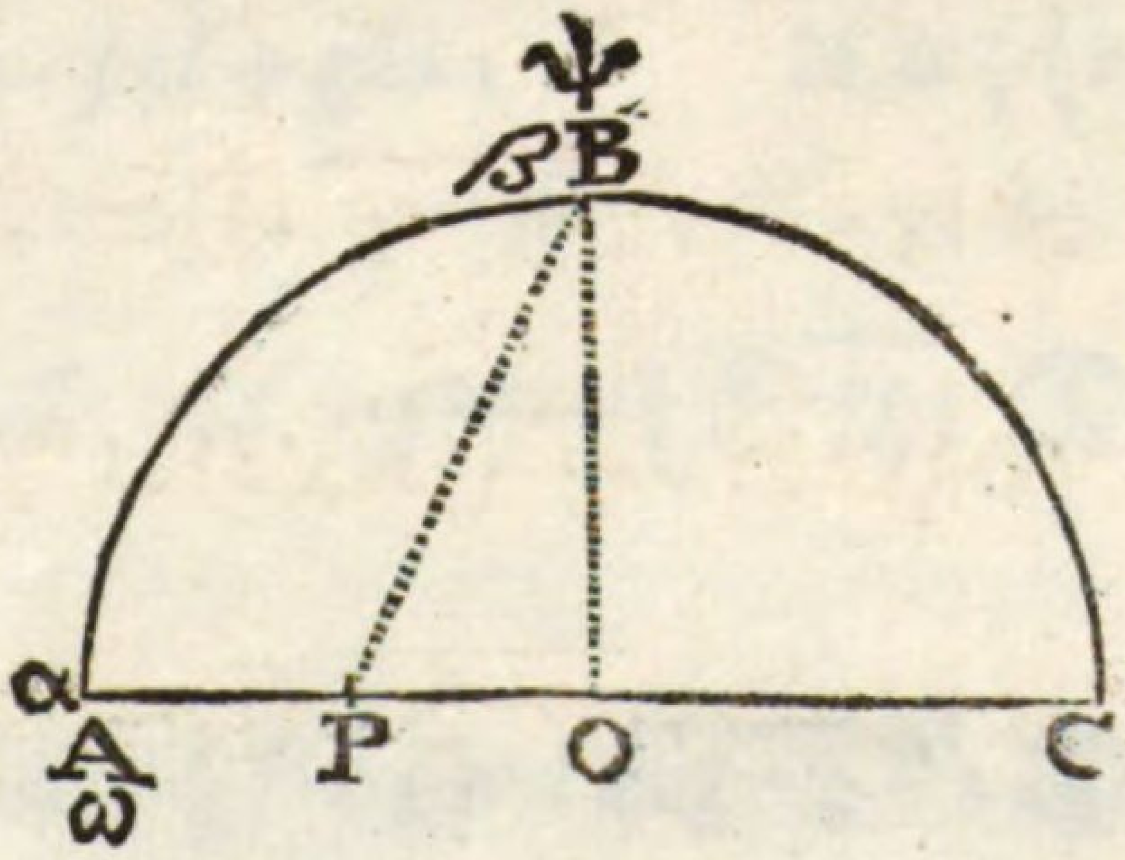
$$OP:OA::-\tfrac{1}{2}b+gz^{\frac{1}{4}\eta}:\sqrt{ag-\tfrac{1}{4}bb};\ ④=+\tfrac{1}{2}\sqrt{\frac{1}{ag-\frac{1}{4}bb}}\text{ in }\psi(PBO).$$

$$M=\tfrac{1}{b}(a+bz^{\frac{1}{4}\eta}+gz^{\frac{1}{2}\eta}:a-bz^{\frac{1}{4}\eta}+gz^{\frac{1}{2}\eta}).$$

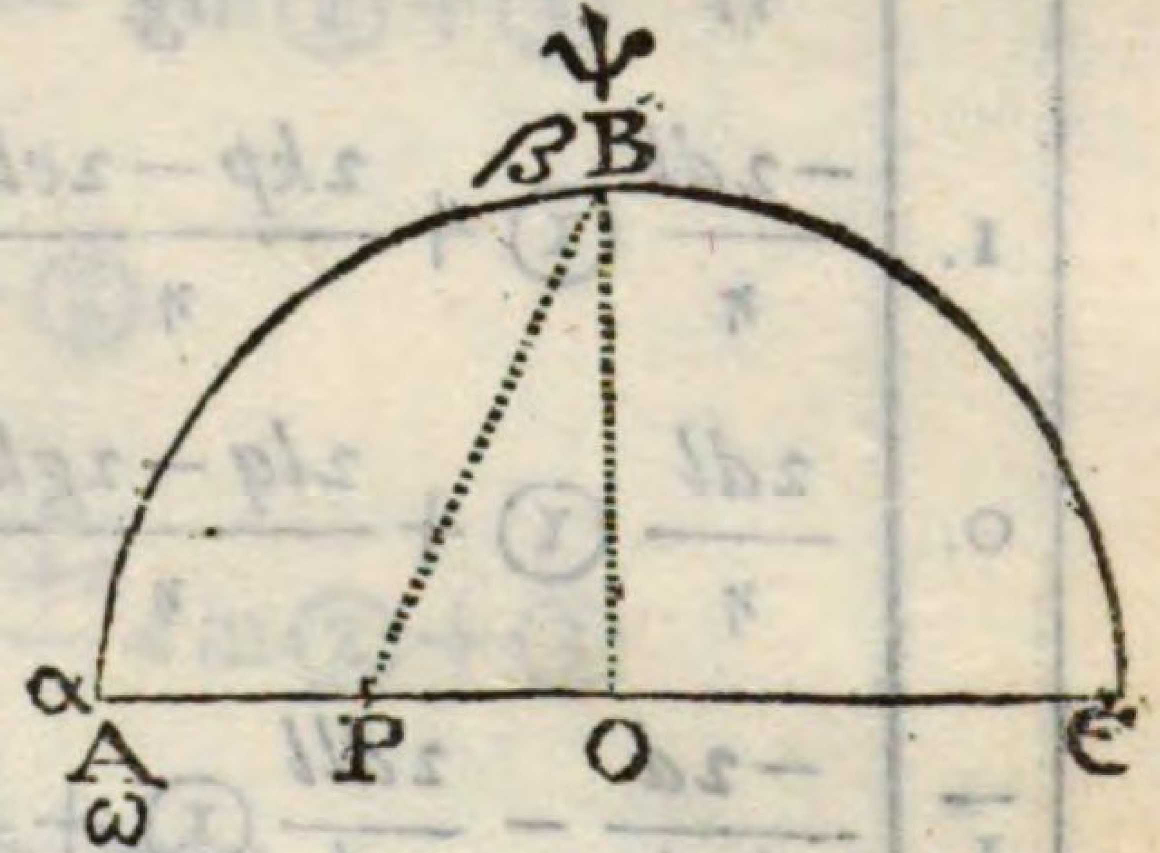
$$N=\tfrac{1}{c}(a+cz^{\frac{1}{4}\eta}+gz^{\frac{1}{2}\eta}:a-cz^{\frac{1}{4}\eta}+gz^{\frac{1}{2}\eta}).$$

θ	FORMÆ XXXIV. $\dfrac{dzz^{\theta n+\frac{3}{4}n-1}}{e+fz^{n}+gz^{2n}}$ *Cas.* 2.
&c.	
2.	$\frac{4dz^{\frac{3}{4}n}}{3\eta g}+\frac{ar-fs-af}{2\eta gs}d$ in ①+②−M; $+\frac{af-fs-ar}{2\eta gs}d$ in ③+④−N.
1.	$\frac{s+a}{2\eta s}d$ in ①+②−M; $+\frac{s-a}{2\eta s}d$ in ③+④−N.
0.	$\frac{-ar}{2\eta es}d$ in ①+②−M; $+\frac{ar}{2\eta es}d$ in ③+④−N.
$\bar{1}$.	$\frac{-4d}{\eta ez^{\frac{1}{4}n}}+\frac{afr-egs-aeg}{2\eta ees}d$ in ①+②−M; $+\frac{aeg-egs-afr}{2\eta ees}d$ in ③+④−N.
&c.	

Quantitates r, s, a, b, c, ①, ②, ③, ④, M, N eosdem habent valores ac in pagina præcedente.



θ	FORMA XXXV. $\dfrac{dzz^{\theta\eta-1}}{\overline{e+fz^\eta+gz^{2\eta}}\times\overline{k+lz^\eta}}$
&c.	
4.	$\dfrac{dz^\eta}{\eta gl} - \dfrac{\begin{matrix}+fl\\+gk\end{matrix}}{2\eta ggll}dM + \dfrac{dk^3}{2\eta ll}N + \dfrac{\begin{matrix}+effl-f^3k\\-2eegl+3efgk\end{matrix}}{2\eta gg}d\textcircled{1}$
3.	$\dfrac{d}{2\eta gl}M - \dfrac{dkk}{2\eta l}N + \dfrac{ffk-2egk-efl}{2\eta g}d\textcircled{1}$
2.	$\dfrac{dk}{2\eta}N + \dfrac{2el-fk}{2\eta}d\textcircled{1}$
1.	$\dfrac{-dl}{2\eta}N + \dfrac{2gk-fl}{2\eta}d\textcircled{1}$
0.	$\dfrac{-d}{2\eta ek}m + \dfrac{dll}{2\eta k}N + \dfrac{ffl-2egl-fgk}{2\eta e}d\textcircled{1}$
$\overline{1}$.	$\dfrac{-d}{\eta ekz^\eta} + \dfrac{\begin{matrix}+el\\+fk\end{matrix}}{2\eta eekk}dm - \dfrac{dl^3}{2\eta kk}N + \dfrac{\begin{matrix}+ffgk-f^3l\\-2eggk+3efgl\end{matrix}}{2\eta ee}d\textcircled{1}$
&c.	



$$M = 1(e+fz^\eta+gz^{2\eta} : e); \quad m = 1(e+fz^\eta+gz^{2\eta} : gz^{2\eta})$$

$$t = ell - fkl + gkk; \quad N = \tfrac{1}{t}(e+fz^\eta+gz^{2\eta} : \overline{k+lz^\eta}|^2).$$

Cas. 1. $OP : OA :: \tfrac{1}{2}f+gz^\eta : \sqrt{\tfrac{1}{4}ff-eg}$; $\textcircled{1} = \dfrac{-1}{2t}\sqrt{\dfrac{1}{\frac{1}{4}ff-eg}}$ in $(CP : AP)$.

Cas. 2. $OP : OA :: \tfrac{1}{2}f+gz^\eta : \sqrt{eg-\tfrac{1}{4}ff}$; $\textcircled{1} = \dfrac{+1}{2t}\sqrt{\dfrac{1}{eg-\frac{1}{4}ff}}$ in $\angle(PBO)$.

θ	FORMÆ XXXVI. $\dfrac{d\dot{z}z^{\theta n+\frac{1}{2}n-1}}{\overline{e+fz^{n}+gz^{2n}},\overline{k+lz^{n}}}$ Cas. 1.
&c.	
3.	$\frac{2dz^{\frac{1}{2}n}}{ngl}-\frac{2dk^{3}}{nll}$ ① $+\frac{2kp^{3}-2elpp}{ngg}d$ ② $+\frac{2elqq-2kq^{3}}{ngg}d$ ③
2.	$\frac{2dkk}{nl}$ ① $+\frac{2elp-2kpp}{ng}d$ ② $+\frac{2kqq-2elq}{ng}d$ ③
1.	$\frac{-2dk}{n}$ ① $+\frac{2kp-2el}{n}d$ ② $+\frac{2el-2kq}{n}d$ ③
0.	$\frac{2dl}{n}$ ① $+\frac{2lq-2gk}{n}d$ ② $+\frac{2gk-2lp}{n}d$ ③
$\overline{1}$.	$\frac{-2d}{nekz^{\frac{1}{2}n}}-\frac{2dll}{nk}$ ① $+\frac{2ggk-2glq}{np}d$ ② $+\frac{2glp-2ggk}{nq}d$ ③
&c.	

$a=\sqrt{ff-4eg}$; $p=\frac{1}{2}f+\frac{1}{2}a$; $q=\frac{1}{2}f-\frac{1}{2}a$; $t=ell-fkl+gkk$.

Cas. 1.

$OP:OA::\sqrt[2]{z^{n}}:\sqrt[2]{\frac{k}{l}}$; ① $=\frac{1}{2t}\sqrt[2]{\frac{l}{k}}$ in $\not\downarrow(PBO)$.

$OP:OA::\sqrt[2]{z^{n}}:\sqrt[2]{\frac{p}{g}}$; ② $=\frac{1}{2at}\sqrt[2]{\frac{g}{p}}$ in $\not\downarrow(PBO)$.

$OP:OA::\sqrt[2]{z^{n}}:\sqrt[2]{\frac{q}{g}}$; ③ $=\frac{1}{2at}\sqrt[2]{\frac{g}{q}}$ in $\not\downarrow(PBO)$.

Cas. 2.

$OP:OA::\sqrt[2]{z^{n}}:\sqrt[2]{\frac{-k}{l}}$; ① $=\frac{-1}{2t}\sqrt[2]{-\frac{l}{k}}$ in $(CP:AP)$.

$OP:OA::\sqrt[2]{z^{n}}:\sqrt[2]{\frac{-p}{g}}$; ② $=\frac{-1}{2at}\sqrt[2]{\frac{-g}{p}}$ in $(CP:AP)$.

$OP:OA::\sqrt[2]{z^{n}}:\sqrt[2]{\frac{-q}{g}}$; ③ $=\frac{-1}{2at}\sqrt[2]{\frac{-g}{q}}$ in $(CP:AP)$.

Vide Fig. Pag. præcedentis

θ	FORMÆ XXXVI. $\dfrac{dzz^{\theta\eta+\frac{1}{2}\eta-1}}{\overline{e+fz^{\eta}+gz^{2\eta}}\times\overline{k+lz^{\eta}}}$ *Cas.* 2.
&c.	
3.	$\frac{2dz^{\frac{1}{2}\eta}}{\eta gl}-\frac{2k^{3}}{\eta ll}d\textcircled{1}+\frac{\overline{el-fk},\overline{f-r}+egk}{2\eta g}d\mathrm{M}+\frac{\overline{el-fk},\overline{f+r}+egk}{2\eta g}d\text{ in }\textcircled{2}+\textcircled{3}$
2.	$\frac{2kk}{\eta l}d\textcircled{1}+\frac{-el+k,\overline{f-r}}{2\eta}d\mathrm{M}+\frac{-el+k,\overline{f+r}}{2\eta}d\text{ in }\textcircled{2}+\textcircled{3}$
1.	$\frac{-2k}{\eta}d\textcircled{1}+\frac{rl-gk}{2\eta}d\mathrm{M}-\frac{+rl+gk}{2\eta}d\text{ in }\textcircled{2}+\textcircled{3}.$
0.	$\frac{2l}{\eta}d\textcircled{1}+\frac{ggk-gl,\overline{f-r}}{2\eta r}d\mathrm{M}+\frac{-ggk+gl,\overline{f+r}}{2\eta r}d\text{ in }\textcircled{2}+\textcircled{3}$
$\overline{1}$.	$\frac{-2d}{\eta ekz^{\frac{1}{2}\eta}}-\frac{2ll}{\eta k}d\textcircled{1}+\frac{\overline{fgl-ggk},\overline{f-r}-eggl}{2\eta er}d\mathrm{M}+\frac{\overline{ggk-fgl},\overline{f+r}+eggl}{2\eta er}d\text{ in }\textcircled{2}+\textcircled{3}$
&c.	

$$r=\sqrt{eg}.\quad s=\sqrt{-fg+2\sqrt{eg^{3}}}.\quad t=ell-fkl+gkk.$$

$$\mathrm{M}=\tfrac{1}{st}(r+sz^{\frac{1}{2}\eta}+gz^{\eta}:r-sz^{\frac{1}{2}\eta}+gz^{\eta}).$$

Cas. 1.

$$OP:OA::\sqrt[2]{z^{\eta}}:\sqrt[2]{\frac{k}{l}};\;\ldots\;\textcircled{1}=\frac{+1}{2t}\sqrt[2]{\frac{l}{k}}\text{ in }\psi(PBO).$$

$$OP:OA::{+\tfrac{1}{2}s}+gz^{\frac{1}{2}\eta}:\sqrt{\tfrac{1}{4}ss-gr};\;\textcircled{2}=\frac{+1}{2t}\sqrt{\frac{1}{\frac{1}{4}ss-gr}}\text{ in }(CP:AP).$$

$$OP:OA::{-\tfrac{1}{2}s}+gz^{\frac{1}{2}\eta}:\sqrt{\tfrac{1}{4}ss-gr};\;\textcircled{3}=\frac{+1}{2t}\sqrt{\frac{1}{\frac{1}{4}ss-gr}}\text{ in }(CP:AP).$$

Cas. 2.

$$OP:OA::\sqrt[2]{z^{\eta}}:\sqrt[2]{\frac{-k}{l}};\;\ldots\;\textcircled{1}=\frac{-1}{2t}\sqrt[2]{\frac{-l}{k}}\text{ in }(CP:AP).$$

$$OP:OA::{+\tfrac{1}{2}s}+gz^{\frac{1}{2}\eta}:\sqrt{gr-\tfrac{1}{4}ss};\;\textcircled{2}=\frac{-1}{2t}\sqrt{\frac{1}{gr-\frac{1}{4}ss}}\text{ in }\psi(PBO).$$

$$OP:OA::{-\tfrac{1}{2}s}+gz^{\frac{1}{2}\eta}:\sqrt{gr-\tfrac{1}{4}ss};\;\textcircled{3}=\frac{-1}{2t}\sqrt{\frac{1}{gr-\frac{1}{4}ss}}\text{ in }\psi(PBO).$$

θ	FORMA XXXVII. $\qquad \dfrac{d\dot{z}z^{\theta\eta+\frac{1}{3}\eta-1}}{\overline{e+fz^{\eta}+gz^{2\eta}},\ \overline{k+lz^{\eta}}}$
&c.	
3.	$\dfrac{3dz^{\frac{1}{3}\eta}}{\eta gl} - \dfrac{3dk^3}{\eta ll}\textcircled{1} + \dfrac{3kp^3-3elpp}{\eta gg}d\textcircled{2} + \dfrac{3elqq-3kq^3}{\eta gg}d\textcircled{3}$
2.	$\dfrac{3dkk}{\eta l}\textcircled{1} + \dfrac{3elp-3kpp}{\eta g}d\textcircled{2} + \dfrac{3kqq-3elq}{\eta g}d\textcircled{3}$
1.	$\dfrac{-3dk}{\eta}\textcircled{1} + \dfrac{3kp-3el}{\eta}d\textcircled{2} + \dfrac{3el-3kq}{\eta}d\textcircled{3}$
0.	$\dfrac{3dl}{\eta}\textcircled{1} + \dfrac{3lq-3gk}{\eta}d\textcircled{2} + \dfrac{3gk-3lp}{\eta}d\textcircled{3}$
$\overline{1}$.	$\dfrac{-3d}{2\eta ekz^{\frac{2}{3}\eta}} - \dfrac{3dll}{\eta k}\textcircled{1} + \dfrac{3ggk-3glq}{\eta p}d\textcircled{2} + \dfrac{3glp-3ggk}{\eta q}d\textcircled{3}$
&c.	

$a=\sqrt{ff-4eg}$; $p=\frac{1}{2}f+\frac{1}{2}a$; $q=\frac{1}{2}f-\frac{1}{2}a$; $t=ell-fkl+gkk$;

Cas. 1.

$OP:OA::\sqrt[3]{z^{\eta}}:\sqrt[3]{\dfrac{k}{l}}$; $\textcircled{1}=\dfrac{1}{3t}\sqrt[3]{\dfrac{ll}{kk}}$ in $-\beta(BP:BO)+(DP:DO)+\downarrow(PBO)$.

$OP:OA::\sqrt[3]{z^{\eta}}:\sqrt[3]{\dfrac{p}{g}}$; $\textcircled{2}=\dfrac{1}{3at}\sqrt[3]{\dfrac{gg}{pp}}$ in $-\beta(BP:BO)+(DP:DO)+\downarrow(PBO)$.

$OP:OA::\sqrt[3]{z^{\eta}}:\sqrt[3]{\dfrac{q}{g}}$; $\textcircled{3}=\dfrac{1}{3at}\sqrt[3]{\dfrac{gg}{qq}}$ in $-\beta(BP:BO)+(DP:DO)+\downarrow(PBO)$

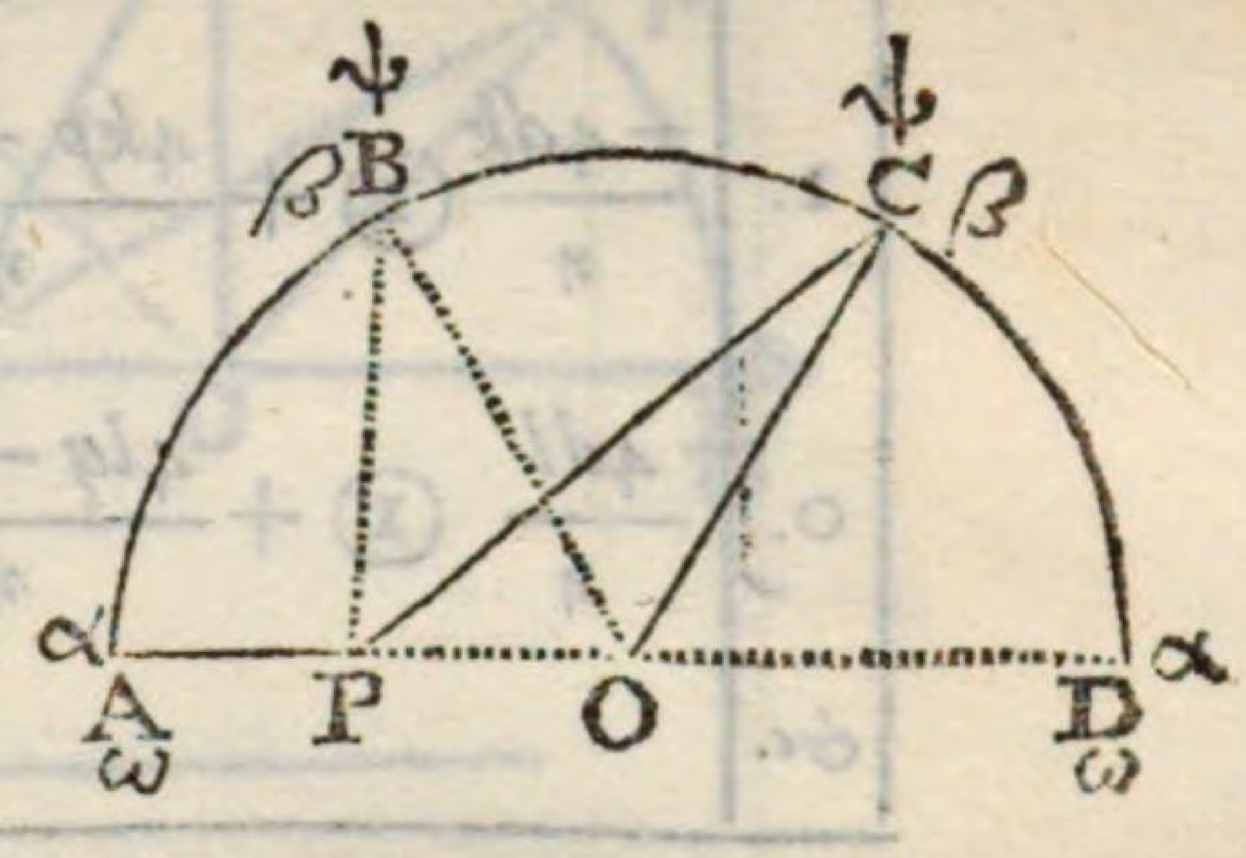
Cas. 2.

$OP:OA::\sqrt[3]{z^{\eta}}:\sqrt[3]{\dfrac{-k}{l}}$; $\textcircled{1}=\dfrac{-1}{3t}\sqrt[3]{\dfrac{ll}{kk}}$ in $-(AP:AO)+\beta(CP:CO)+\downarrow(PCO)$.

$OP:OA::\sqrt[3]{z^{\eta}}:\sqrt[3]{\dfrac{-p}{g}}$; $\textcircled{2}=\dfrac{-1}{3at}\sqrt[3]{\dfrac{gg}{pp}}$ in $-(AP:AO)+\beta(CP:CO)+\downarrow(PCO)$.

$OP:OA::\sqrt[3]{z^{\eta}}:\sqrt[3]{\dfrac{-q}{g}}$; $\textcircled{3}=\dfrac{-1}{3at}\sqrt[3]{\dfrac{gg}{qq}}$ in $-(AP:AO)+\beta(CP:CO)+\downarrow(PCO)$.

θ	FORMÆ XXXVIII. $\frac{d\dot{z}z^{\theta\eta+\frac{2}{3}\eta-1}}{\overline{e+fz^{\eta}+gz^{2\eta}}\times\overline{k+lz^{\eta}}}$
&c.	
3.	$\frac{3dz^{\frac{2}{3}\eta}}{2\eta gl}-\frac{3dk^3}{\eta ll}\textcircled{1}+\frac{3kp^3-3elpp}{\eta gg}d\textcircled{2}+\frac{3elqq-3kq^3}{\eta gg}d\textcircled{3}$
2.	$\frac{3dkk}{\eta l}\textcircled{1}+\frac{3elp-3kpp}{\eta g}d\textcircled{2}+\frac{3kqq-3elq}{\eta g}d\textcircled{3}$
1.	$\frac{-3dk}{\eta}\textcircled{1}+\frac{3kp-3el}{\eta}d\textcircled{2}+\frac{3el-3kq}{\eta}d\textcircled{3}$
0.	$\frac{3dl}{\eta}\textcircled{1}+\frac{3lq-3gk}{\eta}d\textcircled{2}+\frac{3gk-3lp}{\eta}d\textcircled{3}$
$\overline{1}$.	$\frac{-3d}{\eta ekz^{\frac{1}{3}\eta}}-\frac{3dll}{\eta k}\textcircled{1}+\frac{3ggk-3glq}{\eta p}d\textcircled{2}+\frac{3glp-3ggk}{\eta q}d\textcircled{3}$
&c.	



$a=\sqrt{\overline{ff-4eg}}$; $p=\frac{1}{2}f+\frac{1}{2}a$; $q=\frac{1}{2}f-\frac{1}{2}a$; $t=ell-fkl+gkk$;

Cas. 1.

$OP:OA::\sqrt[3]{z^{\eta}}:\sqrt[3]{\frac{k}{l}}$; $\textcircled{1}=\frac{1}{3t}\sqrt[3]{\frac{l}{k}}$ in $\beta(BP:BO)-(DP:DO)+\psi(PBO)$.

$OP:OA::\sqrt[3]{z^{\eta}}:\sqrt[3]{\frac{p}{g}}$; $\textcircled{2}=\frac{1}{3at}\sqrt[3]{\frac{g}{p}}$ in $\beta(BP:BO)-(DP:DO)+\psi(PBO)$.

$OP:OA::\sqrt[3]{z^{\eta}}:\sqrt[3]{\frac{q}{g}}$; $\textcircled{3}=\frac{1}{3at}\sqrt[3]{\frac{g}{q}}$ in $\beta(BP:BO)-(DP:DO)+\psi(PBO)$.

Cas. 2.

$OP:OA::\sqrt[3]{z^{\eta}}:\sqrt[3]{\frac{-k}{l}}$; $\textcircled{1}=\frac{-1}{3t}\sqrt[3]{\frac{-l}{k}}$ in $-(AP:AO)+\beta(CP:CO)-\psi(PCO)$.

$OP:OA::\sqrt[3]{z^{\eta}}:\sqrt[3]{\frac{-p}{g}}$; $\textcircled{2}=\frac{-1}{3at}\sqrt[3]{\frac{-g}{p}}$ in $-(AP:AO)+\beta(CP:CO)-\psi(PCO)$.

$OP:OA::\sqrt[3]{z^{\eta}}:\sqrt[3]{\frac{-q}{g}}$; $\textcircled{3}=\frac{-1}{3at}\sqrt[3]{\frac{-g}{q}}$ in $-(AP:AO)+\beta(CP:CO)-\psi(PCO)$.

θ	FORMÆ XXXIX. $\dfrac{d\dot{z}z^{\theta\eta+\frac{1}{4}\eta-1}}{e+fz^{\eta}+gz^{2\eta}\times k+lz^{\eta}}$ Cas. 1.
&c.	
2.	$\frac{4dkk}{\eta l}\textcircled{1}+\frac{4elp-4kpp}{\eta g}d\textcircled{2}+\frac{4kqq-4elq}{\eta g}d\textcircled{3}$
1.	$\frac{-4dk}{\eta}\textcircled{1}+\frac{4kp-4el}{\eta}d\textcircled{2}+\frac{4el-4kq}{\eta}d\textcircled{3}$
0.	$\frac{4dl}{\eta}\textcircled{1}+\frac{4lq-4gk}{\eta}d\textcircled{2}+\frac{4gk-4lp}{\eta}d\textcircled{3}$
&c.	

$a=\sqrt{ff-4eg},\quad p=\frac{1}{2}f+\frac{1}{2}a,\quad q=\frac{1}{2}f-\frac{1}{2}a,\quad t=ell-fkl+gkk.$

Cas. 1.

$OP:OA::\sqrt[4]{z^{\eta}}:\sqrt[4]{\frac{k}{l}};\quad \textcircled{1}=\frac{1}{4t}\sqrt[4]{\frac{l^3}{k^3}}$ in $-\beta(BP:BO)$
$+\beta(DP:DO)+\psi(PBO)+\psi(PDO).$

$OP:OA::\sqrt[4]{z^{\eta}}:\sqrt[4]{\frac{p}{g}};\quad \textcircled{2}=\frac{1}{4at}\sqrt[4]{\frac{g^3}{p^3}}$ in $-\beta(BP:BO)$
$+\beta(DP:DO)+\psi(PBO)+\psi(PDO).$

$OP:OA::\sqrt[4]{z^{\eta}}:\sqrt[4]{\frac{q}{g}};\quad \textcircled{3}=\frac{1}{4at}\sqrt[4]{\frac{g^3}{q^3}}$ in $-\beta(BP:BO)$
$+\beta(DP:DO)+\psi(PBO)+\psi(PDO).$

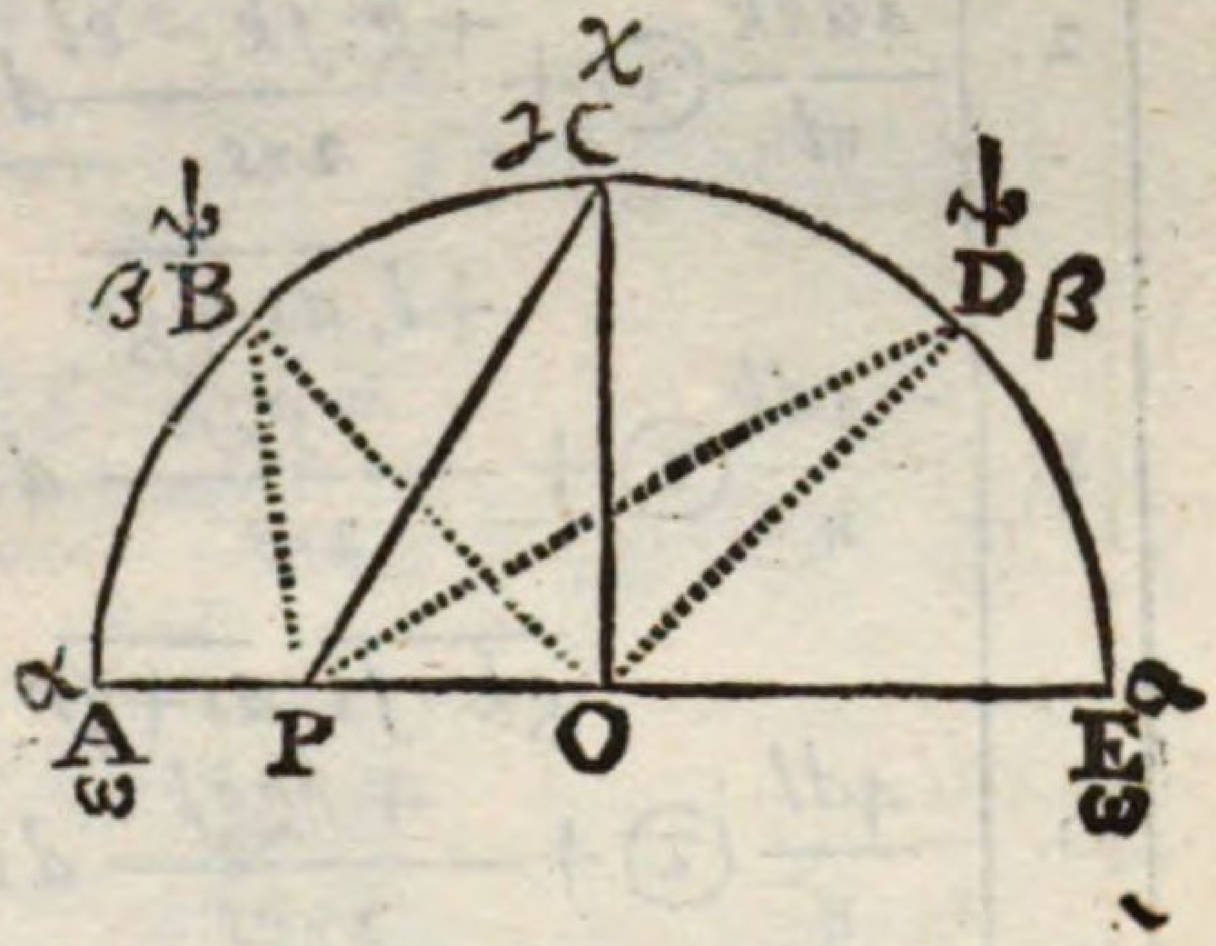
Cas. 2.

$OP:OA::\sqrt[4]{z^{\eta}}:\sqrt[4]{\frac{-k}{l}};\quad \textcircled{1}=\frac{-1}{4t}\sqrt[4]{\frac{-l^3}{k^3}}$ in $-(AP:AO)$
$+(EP:EO)+\chi(PCO).$

$OP:OA::\sqrt[4]{z^{\eta}}:\sqrt[4]{\frac{-p}{g}};\quad \textcircled{2}=\frac{-1}{4at}\sqrt[4]{\frac{-g^3}{p^3}}$ in $-(AP:AO)$
$+(EP:EO)+\chi(PCO).$

$OP:OA::\sqrt[4]{z^{\eta}}:\sqrt[4]{\frac{-q}{g}};\quad \textcircled{3}=\frac{-1}{4at}\sqrt[4]{\frac{-g^3}{q^3}}$ in $-(AP:AO)$
$+(EP:EO)+\chi(PCO).$

θ	FORMÆ XL. $\quad \dfrac{dz z^{\theta\eta+\frac{3}{4}\eta-1}}{e+fz^{\eta}+gz^{2\eta}\times \overline{k+lz^{\eta}}}$ Caſ. 1.
&c.	
2.	$\frac{4dkk}{\eta l}\textcircled{1}+\frac{4elp-4kpp}{\eta g}d\textcircled{2}+\frac{4kqq-4elq}{\eta g}d\textcircled{3}$
1.	$\frac{-4dk}{\eta}\textcircled{1}+\frac{4kp-4el}{\eta}d\textcircled{2}+\frac{4el-4kq}{\eta}d\textcircled{3}$
0.	$\frac{4dl}{\eta}\textcircled{1}+\frac{4ql-4gk}{\eta}d\textcircled{2}+\frac{4gk-4lp}{\eta}d\textcircled{3}$
&c.	



$a=\sqrt{ff-4eg},\quad p=\frac{1}{2}f+\frac{1}{2}a,\quad q=\frac{1}{2}f-\frac{1}{2}a,\quad t=ell-fkl+gkk.$

Caſ. 1.

$$OP:OA::\sqrt[4]{z^{\eta}}:\sqrt[4]{\frac{k}{l}};\ \textcircled{1}=\frac{1}{4t}\sqrt[4]{\frac{l}{k}}\text{ in }+\beta(BP:BO)-\beta(DP:DO)+\psi(PBO)+\psi(PDO).$$

$$OP:OA::\sqrt[4]{z^{\eta}}:\sqrt[4]{\frac{p}{g}};\ \textcircled{2}=\frac{1}{4at}\sqrt[4]{\frac{g}{p}}\text{ in }+\beta(BP:BO)-\beta(DP:DO)+\psi(PBO)+\psi(PDO).$$

$$OP:OA::\sqrt[4]{z^{\eta}}:\sqrt[4]{\frac{q}{g}};\ \textcircled{3}=\frac{1}{4at}\sqrt[4]{\frac{g}{q}}\text{ in }+\beta(BP:BO)-\beta(DP:DO)+\psi(PBO)+\psi(PDO).$$

Caſ. 2.

$$OP:OA::\sqrt[4]{z^{\eta}}:\sqrt[4]{\frac{-k}{l}};\ \textcircled{1}=\frac{-1}{4t}\sqrt[4]{\frac{-l}{k}}\text{ in }-(AP:AO)+(EP:EO)-\chi(PCO).$$

$$OP:OA::\sqrt[4]{z^{\eta}}:\sqrt[4]{\frac{-p}{g}};\ \textcircled{2}=\frac{-1}{4at}\sqrt[4]{\frac{-g}{p}}\text{ in }-(AP:AO)+(EP:EO)-\chi(PCO).$$

$$OP:OA::\sqrt[4]{z^{\eta}}:\sqrt[4]{\frac{-q}{g}};\ \textcircled{3}=\frac{-1}{4at}\sqrt[4]{\frac{-g}{q}}\text{ in }-(AP:AO)+(EP:EO)-\chi(PCO).$$

θ	FORMÆ XXXIX. $\dfrac{dz z^{\theta n + \frac{1}{4}\eta - 1}}{\overline{e+fz^n+gz^{2n}} \times \overline{k+lz^n}}$ Cas. 2.
&c.	
2.	$\dfrac{4dkk}{\eta l}$ ① $+\dfrac{-k,\overline{as+gr}\;+g,\overline{fk-el}}{2\eta s}d$ in ②+③+M; $+\dfrac{-k,\overline{as-gr}\;-g\,\overline{fk-el}}{2\eta s}d$ in ④+⑤+N.
1.	$\dfrac{-4dk}{\eta}$ ① $+\dfrac{+l,\overline{as+gr}\;-ggk}{2\eta s}d$ in ②+③+M; $+\dfrac{+l,\overline{as-gr}\;+ggk}{2\eta s}d$ in ④+⑤+N.
0.	$\dfrac{4dl}{\eta}$ ① $+\dfrac{\overline{gk-fl},\overline{as+gr}\;+eggl}{2\eta es}d$ in ②+③+M; $+\dfrac{\overline{gk-fl},\overline{as-gr}\;-eggl}{2\eta es}d$ in ④+⑤+N.
&c.	

Valores quantitatum r, s, t, a, b, c, M, N petendi ſunt a pag. ſubſequente.

Caſ. 1.

$$OP:OA::\sqrt[4]{z^n}:\sqrt[4]{\frac{k}{l}};\quad ① = \frac{-1}{4t}\sqrt[4]{\frac{l^3}{k^3}} \text{ in } -\beta(BP:BO) + \beta(DP:DO) + \psi(PBO) + \psi(PDO).$$

$$OP:OA::+\tfrac{1}{2}c+gz^{\frac{1}{4}n}:\sqrt{\tfrac{1}{4}cc-ag};\quad ② = \frac{-1}{2t}\sqrt{\frac{1}{\frac{1}{4}cc-ag}} \text{ in}(EP:AP).$$

$$OP:OA::-\tfrac{1}{2}c+gz^{\frac{1}{4}n}:\sqrt{\tfrac{1}{4}cc-ag};\quad ③ = \frac{-1}{2t}\sqrt{\frac{1}{\frac{1}{4}cc-ag}} \text{ in }(EP:AP)$$

$$OP:OA::+\tfrac{1}{2}b+gz^{\frac{1}{4}n}:\sqrt{\tfrac{1}{4}bb-ag};\quad ④ = \frac{-1}{2t}\sqrt{\frac{1}{\frac{1}{4}bb-ag}} \text{ in }(EP:AP).$$

$$OP:OA::-\tfrac{1}{2}b+gz^{\frac{1}{4}n}:\sqrt{\tfrac{1}{4}bb-ag};\quad ⑤ = \frac{-1}{2t}\sqrt{\frac{1}{\frac{1}{4}bb-ag}} \text{ in }(EP:AP).$$

Caſ. 2.

$$OP:OA::\sqrt[4]{z^n}:\sqrt[4]{\frac{-k}{l}};\quad ① = \frac{-1}{4t}\sqrt[4]{\frac{-l^3}{k^3}} \text{ in } -(AP:AO) + (EP:EO) + \chi(PCO).$$

$$OP:OA::+\tfrac{1}{2}c+gz^{\frac{1}{4}n}:\sqrt{ag-\tfrac{1}{4}cc};\quad ② = \frac{+1}{2t}\sqrt{\frac{1}{ag-\frac{1}{4}cc}} \text{ in } \chi(PCO).$$

$$OP:OA::-\tfrac{1}{2}c+gz^{\frac{1}{4}n}:\sqrt{ag-{}^{1}cc};\quad ③ = \frac{+1}{2t}\sqrt{\frac{1}{ag-\frac{1}{4}cc}} \text{ in } \chi(PCO).$$

$$OP:OA::+\tfrac{1}{2}b+gz^{\frac{1}{4}n}:\sqrt{ag-\tfrac{1}{4}bb};\quad ④ = \frac{+1}{2t}\sqrt{\frac{1}{ag-\frac{1}{4}bb}} \text{ in } \chi(PCO).$$

$$OP:OA::-\tfrac{1}{2}b+gz^{\frac{1}{4}n}:\sqrt{ag-\tfrac{1}{4}bb};\quad ⑤ = \frac{+1}{2t}\sqrt{\frac{1}{ag-\frac{1}{4}bb}} \text{ in } \chi(PCO).$$

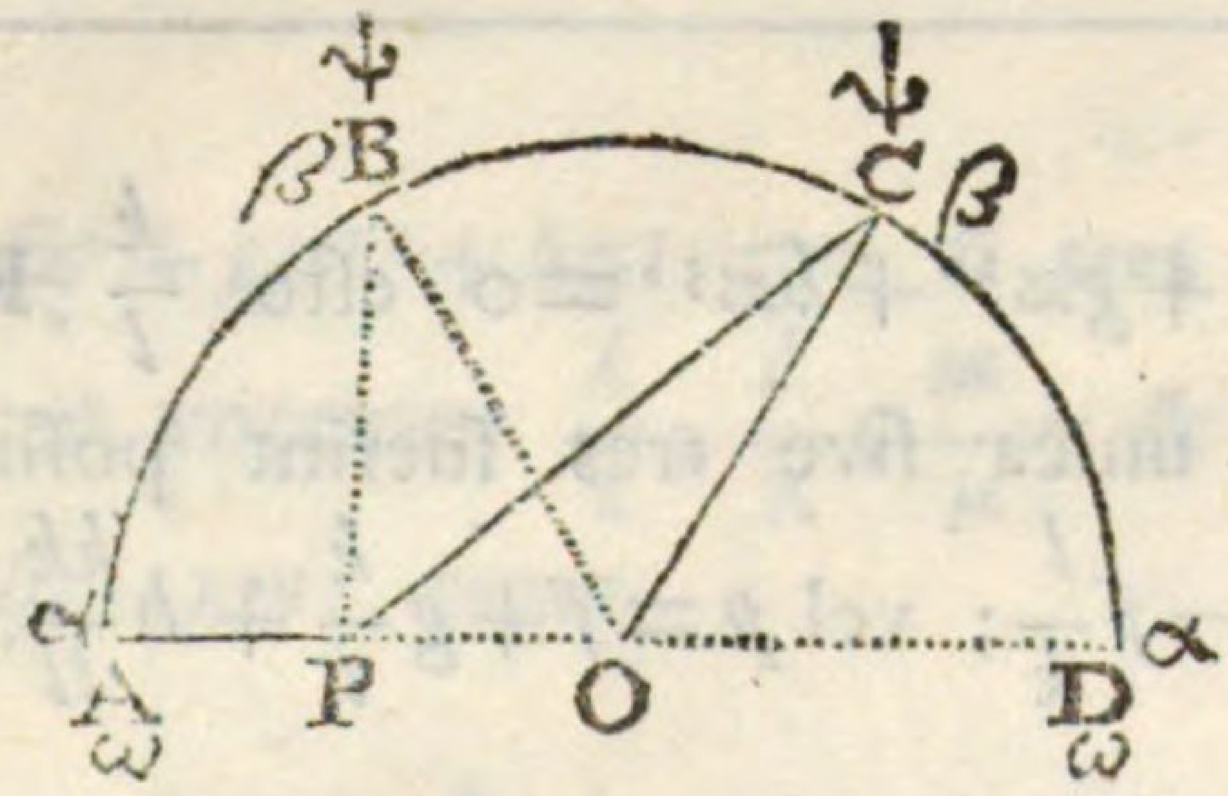
θ	FORMÆ XL. $\frac{d\dot{z}z^{\theta n+\frac{3}{4}n-1}}{\overline{e+fz^{n}+gz^{2n}}\times\overline{k+lz^{n}}}$ *Cas.* 2.
&c.	
2.	$\frac{4dkk}{\eta l}$ ① $+\frac{\overline{el-fk},\overline{s+a}\;+akr}{2\eta s}d$ in ②+③−M; $+\frac{\overline{el-fk},\overline{s-a}\;-akr}{2\eta s}d$ in ④+⑤−N.
1.	$\frac{-4dk}{\eta}$ ① $+\frac{+gk,\overline{s+a}\;-alr}{2\eta s}d$ in ②+③−M; $+\frac{+gk,\overline{s-a}\;+alr}{2\eta s}d$ in ④+⑤−N.
0.	$\frac{4dl}{\eta}$ ① $+\frac{-egl,\overline{s+a}\;-ar,\overline{gk-fl}}{2\eta es}d$ in ②+③−M; $+\frac{-egl,\overline{s-a}\;+ar,\overline{gk-fl}}{2\eta es}d$ in ④+⑤−N.
&c.	

$r=\sqrt{eg}$; $s=\sqrt{-fg+2\sqrt{eg^{3}}}$, $t=ell-fkl+gkk$; $a=\sqrt[4]{eg^{3}}$,

$b=\sqrt{+\sqrt{-fg^{3}+2\sqrt{eg^{7}}}+2\sqrt[4]{eg^{7}}}$; $c=\sqrt{-\sqrt{-fg^{3}+2\sqrt{eg^{7}}}+2\sqrt[4]{eg^{7}}}$.

$\mathrm{M}=\frac{1}{bt}(a+bz^{\frac{1}{4}n}+gz^{\frac{1}{2}n}:a-bz^{\frac{1}{4}n}+gz^{\frac{1}{2}n})$.

$\mathrm{N}=\frac{1}{ct}(a+cz^{\frac{1}{4}n}+gz^{\frac{1}{2}n}:a-cz^{\frac{1}{4}n}+gz^{\frac{1}{2}n})$.

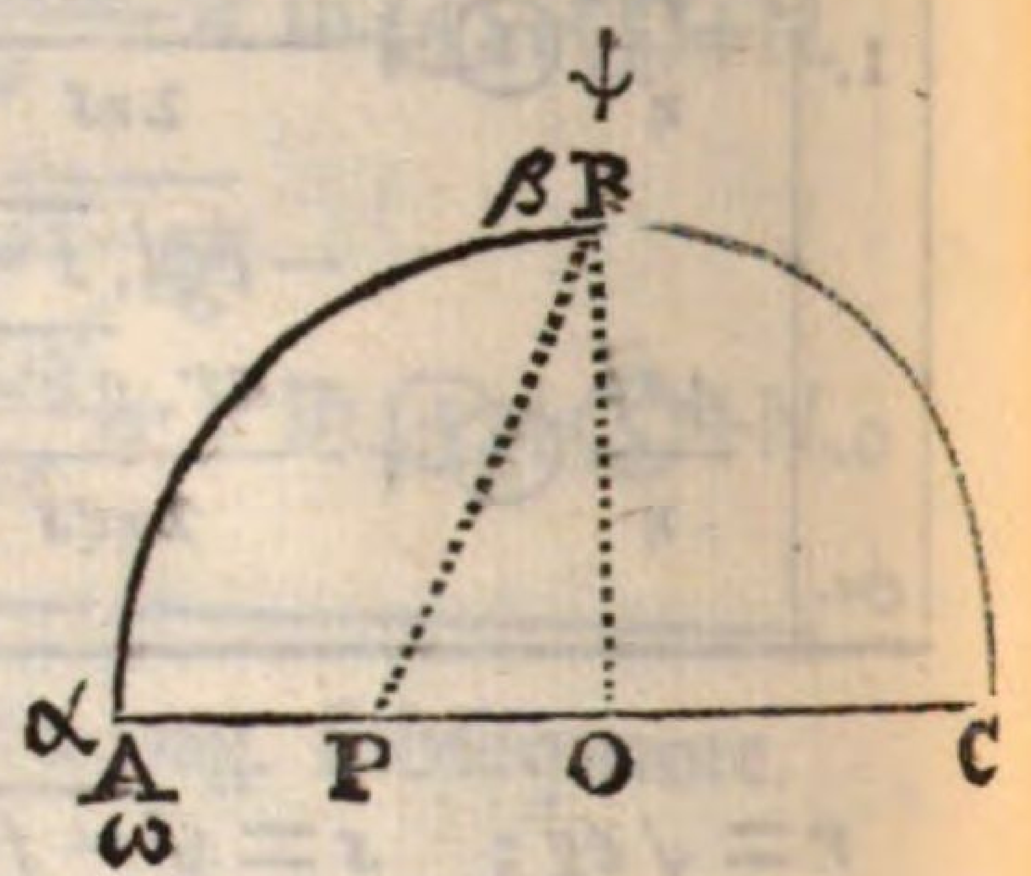


Cas. 1. $OP:OA::\sqrt[4]{\sqrt{z^{n}}}:\sqrt[4]{\frac{k}{l}}$; ① $=\frac{-+1}{4t}\sqrt[4]{\frac{l}{k}}$ in $+\beta(BP:BO)$ $-\beta(DP:DO)+\psi(PBO)+\psi(PDO)$.

Cas. 2. $OP:OA::\sqrt[4]{\sqrt{z^{n}}}:\sqrt[4]{\frac{-k}{l}}$; ① $=\frac{-1}{4t}\sqrt[4]{\frac{-l}{k}}$ in $-(AP:AO)$ $+(EP:EO)-\chi(PCO)$.

Valores quantitatum ②, ③, ④, ⑤ petendi sunt a pag. præcedente.

θ	FORMA XLI. $\dfrac{d\dot{z}z^{\theta n-1}}{e+fz^n+gz^{2n}+hz^{3n}}$
—	
&c.	
4.	$\dfrac{dz^n}{\eta h} + \dfrac{\begin{matrix}+hk\\-lq\end{matrix}}{2\eta hhl} d\mathrm{M} - \dfrac{dk^3}{2\eta l}\mathrm{N} + \dfrac{\begin{matrix}+llpqq+klq^3\\-2hllpp+3ehllq\end{matrix}}{2\eta hh} d\textcircled{I}$
3.	$\dfrac{d}{2\eta h}\mathrm{M} - \dfrac{dkk}{2\eta}\mathrm{N} - \dfrac{+klqq+2ehll+llpq}{2\eta h} d\textcircled{I}$
2.	$\dfrac{-dkl}{2\eta}\mathrm{N} + \dfrac{2llp+klq}{2\eta} d\textcircled{I}$
1.	$\dfrac{-dll}{2\eta}\mathrm{N} + \dfrac{-2hkl-llq}{2\eta} d\textcircled{I}$
0.	$\dfrac{-d}{2\eta e}\mathrm{m} - \dfrac{dl^3}{\eta k}\mathrm{N} + \dfrac{llqq-2hllp+hklq}{2\eta p} d\textcircled{I}$
$\overline{1}$.	$\dfrac{-d}{\eta e z^n} + \dfrac{\begin{matrix}+lp\\-kq\end{matrix}}{2\eta eel} d\mathrm{m} - \dfrac{dl^4}{2\eta kk}\mathrm{N} - \dfrac{\begin{matrix}+hklqq+llq^3\\+2ehhll-3hllpq\end{matrix}}{2\eta pp} d\textcircled{I}$
&c.	



Æquationis $e+fz^n+gz^{2n}+hz^{3n}=0$ esto $\dfrac{k}{l}$ Radix possibilis seu valor ipsius z^n, sive unica sive tres fuerint possibiles, & ponantur $p=-e\dfrac{l}{k}$, $q=-e\dfrac{ll}{kk}-f\dfrac{l}{k}$; vel $p=f+g\dfrac{k}{l}+h\dfrac{kk}{ll}$, $q=g+h\dfrac{k}{l}$;

$$\mathrm{M}=1\,(p+qz^n+hz^{2n}:p);\quad \mathrm{m}=1\,(p+qz^n+hz^{2n}:hz^{2n});$$

$$t=pll+qkl+hkk,\quad \mathrm{N}=\tfrac{1}{t}\,(p+qz^n+hz^{2n}:\overline{lz^n-k}|^2);$$

Cas. 1. $OP:OA::\tfrac{1}{2}q+hz^n:\sqrt[2]{\tfrac{1}{4}qq-hp}$; $\textcircled{I}=\dfrac{-1}{2t}\sqrt[2]{\dfrac{1}{\tfrac{1}{4}qq-hp}}$ in $(CP:AP)$.

Cas. 2. $OP:OA::\tfrac{1}{2}q+hz^n:\sqrt[2]{hp-\tfrac{1}{4}qq}$; $\textcircled{I}=\dfrac{+1}{2t}\sqrt[2]{\dfrac{1}{hp-\tfrac{1}{4}qq}}$ in $\downarrow(PBO)$.

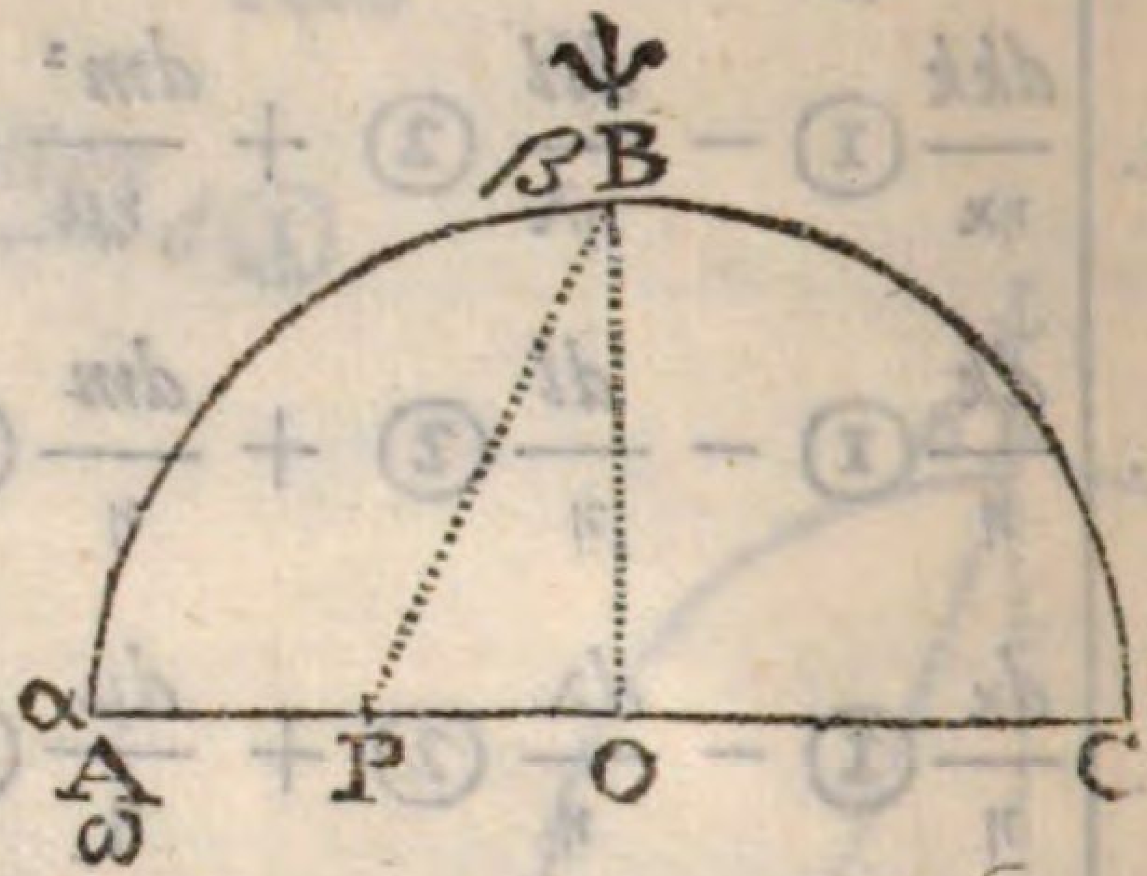
θ	FORMÆ XLI. $\frac{dz z^{\theta n-1}}{e+fz^n+gz^{2n}+hz^{3n}}$ *Casus* Aliter.
&c.	
4.	$\frac{dz^n}{\eta h} + \frac{dk^3}{\eta\kappa\kappa}\textcircled{1} - \frac{dl^3}{\eta\lambda\lambda}\textcircled{2} + \frac{dm^3}{\eta\mu\mu}\textcircled{3}$
3.	$\frac{dkk}{\eta\kappa}\textcircled{1} - \frac{dll}{\eta\lambda}\textcircled{2} + \frac{dm^2}{\eta\mu}\textcircled{3}$
2.	$\frac{dk}{\eta}\textcircled{1} - \frac{dl}{\eta}\textcircled{2} + \frac{dm}{\eta}\textcircled{3}$
1.	$\frac{d\kappa}{\eta}\textcircled{1} - \frac{d\lambda}{\eta}\textcircled{2} + \frac{d\mu}{\eta}\textcircled{3}$
0.	$\frac{d\kappa\kappa}{\eta k}\boxed{1} - \frac{d\lambda\lambda}{\eta l}\boxed{2} + \frac{d\mu\mu}{\eta m}\boxed{3}$
$\overline{1}$.	$\frac{-d}{\eta e z^n} + \frac{d\kappa^3}{\eta kk}\boxed{1} - \frac{d\lambda^3}{\eta ll}\boxed{2} + \frac{d\mu^3}{\eta m^2}\boxed{3}$
&c.	

Æquationis $e+fz^n+gz^{2n}+hz^{3n}=0$ si tres sunt Radices possibiles seu valores ipsius z^n vocentur $\frac{k}{\kappa}$; $\frac{l}{\lambda}$; $\frac{m}{\mu}$; & ponantur $p=k\lambda-\kappa l$; $q=k\mu-\kappa m$; $r=l\mu-\lambda m$; erunt

$$\textcircled{1}=\frac{1}{pq}(\kappa z^n-k:k);\quad \textcircled{2}=\frac{1}{pr}(\lambda z^n-l:l);\quad \textcircled{3}=\frac{1}{qr}(\mu z^n-m:m);$$

$$\boxed{1}=\frac{1}{pq}(\kappa z^n-k:\kappa z^n);\quad \boxed{2}=\frac{1}{pr}(\lambda z^n-l:\lambda z^n);\quad \boxed{3}=\frac{1}{qr}(\mu z^n-m:\mu z^n).$$

θ	FORMÆ XLII. $\dfrac{dzz^{\theta n+\frac{1}{2}n-1}}{e+fz^n+gz^{2n}+hz^{3n}}$ *Cas.* 1.
&c.	
3.	$\dfrac{2dz^{\frac{1}{2}n}}{\eta h}+\dfrac{2dk^3}{\eta\varkappa\varkappa}\textcircled{1}-\dfrac{2dl^3}{\eta\lambda\lambda}\textcircled{2}+\dfrac{2dm^3}{\eta\mu\mu}\textcircled{3}$
2.	$\dfrac{2dkk}{\eta\varkappa}\textcircled{1}-\dfrac{2dll}{\eta\lambda}\textcircled{2}+\dfrac{2dmm}{\eta\mu}\textcircled{3}$
1.	$\dfrac{2dk}{\eta}\textcircled{1}-\dfrac{2dl}{\eta}\textcircled{2}+\dfrac{2dm}{\eta}\textcircled{3}$
0.	$\dfrac{2d\varkappa}{\eta}\textcircled{1}-\dfrac{2d\lambda}{\eta}\textcircled{2}+\dfrac{2d\mu}{\eta}\textcircled{3}$
$\overline{1}$.	$\dfrac{-2d}{\eta ez^{\frac{1}{2}n}}+\dfrac{2d\varkappa\varkappa}{\eta k}\textcircled{1}-\dfrac{2d\lambda\lambda}{\eta l}\textcircled{2}+\dfrac{2d\mu\mu}{\eta m}\textcircled{3}$
&c.	



Æquationis $e+fz^n+gz^{2n}+hz^{3n}=0$ si tres sunt Radices possibiles seu valores ipsius z^n vocentur $\frac{k}{\varkappa}$; $\frac{l}{\lambda}$; $\frac{m}{\mu}$; & ponantur $p=k\lambda-\varkappa l$; $q=k\mu-\varkappa m$; $r=l\mu-\lambda m$;

Cas. 1.
$$\begin{cases} OP:OA::\sqrt[2]{z^n}:\sqrt[2]{\dfrac{k}{\varkappa}}; & \textcircled{1}=\dfrac{-1}{2pq}\sqrt[2]{\dfrac{\varkappa}{k}}\ \text{in}\ (CP:AP) \\ OP:OA::\sqrt[2]{z^n}:\sqrt[2]{\dfrac{l}{\lambda}}; & \textcircled{2}=\dfrac{-1}{2pr}\sqrt[2]{\dfrac{\lambda}{l}}\ \text{in}\ (CP:AP) \\ OP:OA::\sqrt[2]{z^n}:\sqrt[2]{\dfrac{m}{\mu}}; & \textcircled{3}=\dfrac{-1}{2qr}\sqrt[2]{\dfrac{\mu}{m}}\ \text{in}\ (CP:AP) \end{cases}$$

Cas. 2.
$$\begin{cases} OP:OA::\sqrt[2]{z^n}:\sqrt[2]{\dfrac{-k}{\varkappa}}; & \textcircled{1}=\dfrac{+1}{2pq}\sqrt[2]{\dfrac{-\varkappa}{k}}\ \text{in}\ \psi\,(PBO) \\ OP:OA::\sqrt[2]{z^n}:\sqrt[2]{\dfrac{-l}{\lambda}}; & \textcircled{2}=\dfrac{+1}{2pr}\sqrt[2]{\dfrac{-\lambda}{l}}\ \text{in}\ \psi\,(PBO) \\ OP:OA::\sqrt[2]{z^n}:\sqrt[2]{\dfrac{-m}{\mu}}; & \textcircled{3}=\dfrac{+1}{2qr}\sqrt[2]{\dfrac{-\mu}{m}}\ \text{in}\ \psi\,(PBO) \end{cases}$$

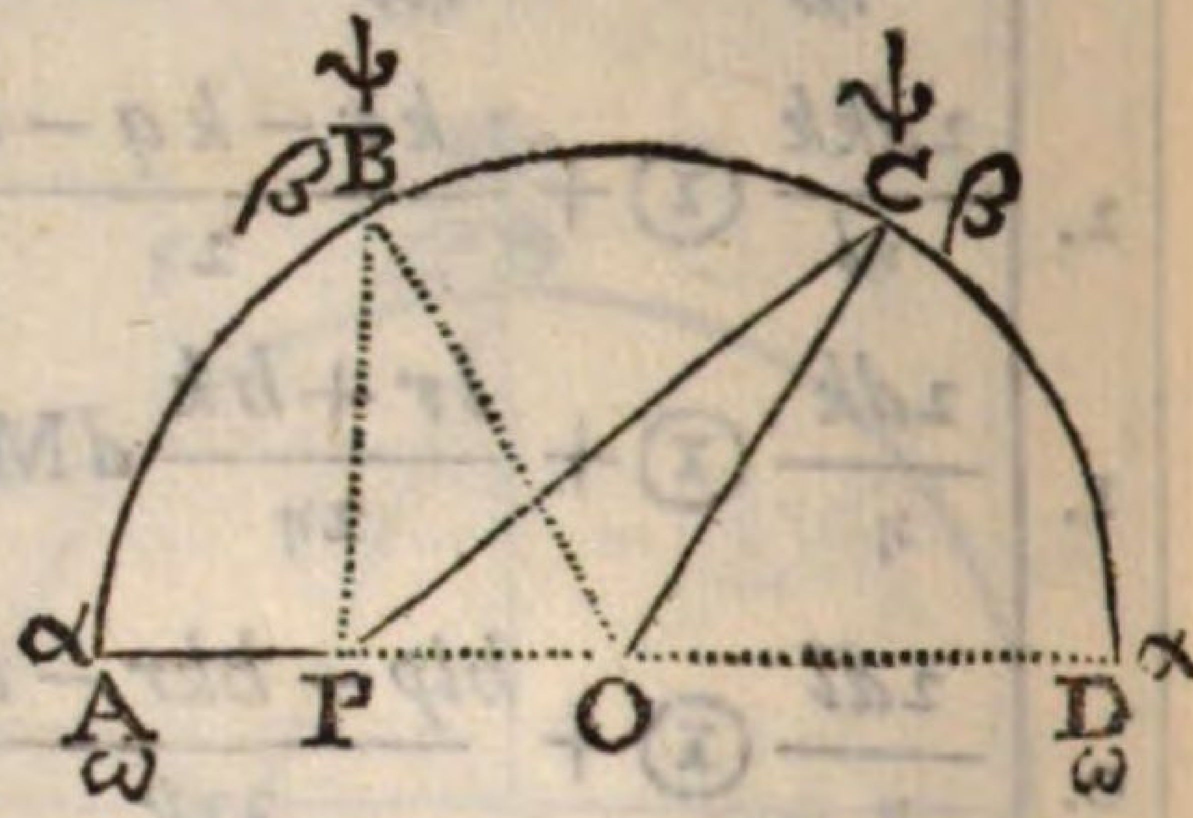
θ	FORMÆ XLII. $\dfrac{dz\,z^{\theta\eta+\frac{1}{2}\eta-1}}{e+fz^{\eta}+gz^{2\eta}+hz^{3\eta}}$ *Cas.* 2.
&c.	
3.	$\frac{2dz^{\frac{1}{2}\eta}}{\eta h}+\frac{2dk^{3}}{\eta ll}\textcircled{1}+\frac{-hkp\,\overline{kq+lp,\;q-r}}{2\eta h}dM+\frac{+hkp\,\overline{-kq-lp,\;q+r}}{2\eta h}d\text{ in }\textcircled{2}+\textcircled{3}$
2.	$\frac{2dkk}{\eta l}\textcircled{1}+\frac{kr-kq-lp}{2\eta}dM+\frac{kr+kq+lp}{2\eta}d\text{ in }\textcircled{2}+\textcircled{3}$
1.	$\frac{2dk}{\eta}\textcircled{1}+\frac{lr+hk}{2\eta}dM+\frac{lr-hk}{2\eta}d\text{ in }\textcircled{2}+\textcircled{3}$
0.	$\frac{2dl}{\eta}\textcircled{1}+\frac{hlp-hkr-lqr}{2\eta p}dM+\frac{-hlp-hkr-lqr}{2\eta p}d\text{ in }\textcircled{2}+\textcircled{3}$
−1.	$\frac{-2d}{\eta e z^{\frac{1}{2}\eta}}+\frac{2dll}{\eta k}\textcircled{1}+\frac{-hlpr\,\overline{hk+lq,\;qr-hp}}{2\eta pp}dM+\frac{-hlpr\,\overline{hk+lq,\;qr+hp}}{2\eta pp}d\text{ in }\textcircled{2}+\textcircled{3}$
&c.	

Æquationis $e+fz^{\eta}+gz^{2\eta}+hz^{3\eta}=0$ esto $\frac{k}{l}$ Radix unica possibilis ubi reliquæ sunt impossibiles, & ponantur $p=-e\frac{l}{k}$, $q=-e\frac{ll}{kk}-f\frac{l}{k}$; vel $p=f+g\frac{k}{l}+h\frac{kk}{ll}$, $q=g+h\frac{k}{l}$; $r=\sqrt{hp}$, $s=\sqrt{-hp+2\sqrt{h^{3}p}}$, $t=pll+qkl+hkk$; $M=\frac{l}{st}(r+sz^{\frac{1}{2}\eta}+hz^{\eta}:r-sz^{\frac{1}{2}\eta}+hz^{\eta})$.

Cas. 1.
$$\begin{cases} OP:OA::\sqrt[2]{z^{\eta}}:\sqrt[2]{\frac{k}{l}}\,;\;\ldots\;,\;\textcircled{1}=\frac{-l}{2t}\sqrt[2]{\frac{l}{k}}\text{ in }(CP:AP). \\ OP:OA::+\frac{1}{2}s+hz^{\frac{1}{2}\eta}:\sqrt[2]{\frac{1}{4}ss-hr}\,;\;\textcircled{2}=\frac{-l}{2t}\sqrt[2]{\frac{1}{\frac{1}{4}ss-hr}}\text{ in }(CP:AP). \\ OP:OA::-\frac{1}{2}s+hz^{\frac{1}{2}\eta}:\sqrt[2]{\frac{1}{4}ss-hr}\,;\;\textcircled{3}=\frac{-l}{2t}\sqrt[2]{\frac{1}{\frac{1}{4}ss-hr}}\text{ in }(CP:AP). \end{cases}$$

Cas. 2.
$$\begin{cases} OP:OA::\sqrt[2]{z^{\eta}}:\sqrt[2]{\frac{-k}{l}}\,;\;\ldots\;\textcircled{1}=\frac{+l}{2t}\sqrt[2]{\frac{-l}{k}}\text{ in }\psi(PBO). \\ OP:OA::+\frac{1}{2}s+hz^{\frac{1}{2}\eta}:\sqrt[2]{hr-\frac{1}{4}ss}\,;\;\textcircled{2}=\frac{+l}{2t}\sqrt[2]{\frac{1}{hr-\frac{1}{4}ss}}\text{ in }\psi(PBO). \\ OP:OA::-\frac{1}{2}s+hz^{\frac{1}{2}\eta}:\sqrt[2]{hr-\frac{1}{4}ss}\,;\;\textcircled{3}=\frac{+l}{2t}\sqrt[2]{\frac{1}{hr-\frac{1}{4}ss}}\text{ in }\psi(PBO). \end{cases}$$

θ	FORMA XLIII. $\dfrac{d\dot{z}z^{\theta\eta+\frac{1}{3}\eta-1}}{e+fz^{\eta}+gz^{2\eta}+hz^{3\eta}}$
&c.	
3.	$\frac{3dz^{\frac{1}{3}\eta}}{\eta h} + \frac{3dk^{3}}{\eta\varkappa\varkappa}\textcircled{1} - \frac{3dl^{3}}{\eta\lambda\lambda}\textcircled{2} + \frac{3dm^{3}}{\eta\mu\mu}\textcircled{3}$
2.	$\frac{3dkk}{\eta\varkappa}\textcircled{1} - \frac{3dll}{\eta\lambda}\textcircled{2} + \frac{3dmm}{\eta\mu}\textcircled{3}$
1.	$\frac{3dk}{\eta}\textcircled{1} - \frac{3dl}{\eta}\textcircled{2} + \frac{3dm}{\eta}\textcircled{3}$
0.	$\frac{3d\varkappa}{\eta}\textcircled{1} - \frac{3d\lambda}{\eta}\textcircled{2} + \frac{3d\mu}{\eta}\textcircled{3}$
$\overline{1}$.	$\frac{-3d}{\eta e z^{\frac{1}{3}\eta}} + \frac{3d\varkappa\varkappa}{\eta k}\textcircled{1} - \frac{3d\lambda\lambda}{\eta l}\textcircled{2} + \frac{3d\mu\mu}{\eta m}\textcircled{3}$
&c.	



Æquationis $e+fz^{\eta}+gz^{2\eta}+hz^{3\eta}=0$ si tres sunt Radices possibiles seu valores ipsius z^{η} vocentur $\frac{k}{\varkappa}$; $\frac{l}{\lambda}$; $\frac{m}{\mu}$; & ponantur $p=k\lambda-\varkappa l$; $q=k\mu-\varkappa m$; $r=l\mu-\lambda m$;

Cas. 1.

$OP:OA::\sqrt[3]{z^{\eta}}:\sqrt[3]{\frac{k}{\varkappa}}$; $\textcircled{1}=\frac{-1}{3pq}\sqrt[3]{\frac{\varkappa\varkappa}{kk}}$ in $-(AP:AO)$ $+\beta(CP:CO)$ $+\psi(PCO)$.

$OP:OA::\sqrt[3]{z^{\eta}}:\sqrt[3]{\frac{l}{\lambda}}$; $\textcircled{2}=\frac{-1}{3pr}\sqrt[3]{\frac{\lambda\lambda}{ll}}$ in $-(AP:AO)$ $+\beta(CP:CO)$ $+\psi(PCO)$.

$OP:OA::\sqrt[3]{z^{\eta}}:\sqrt[3]{\frac{m}{\mu}}$; $\textcircled{3}=\frac{-1}{3qr}\sqrt[3]{\frac{\mu\mu}{mm}}$ in $-(AP:AO)$ $+\beta(CP:CO)$ $+\psi(PCO)$.

Cas. 2.

$OP:OA::\sqrt[3]{z^{\eta}}:\sqrt[3]{\frac{-k}{\varkappa}}$; $\textcircled{1}=\frac{1}{3pq}\sqrt[3]{\frac{\varkappa\varkappa}{kk}}$ in $-\beta(BP:BO)$ $+(DP:DO)$ $+\psi(PBO)$.

$OP:OA::\sqrt[3]{z^{\eta}}:\sqrt[3]{\frac{-l}{\lambda}}$; $\textcircled{2}=\frac{1}{3pr}\sqrt[3]{\frac{\lambda\lambda}{ll}}$ in $-\beta(BP:BO)$ $+(DP:DO)$ $+\psi(PBO)$.

$OP:OA::\sqrt[3]{z^{\eta}}:\sqrt[3]{\frac{-m}{\mu}}$; $\textcircled{3}=\frac{1}{3qr}\sqrt[3]{\frac{\mu\mu}{mm}}$ in $-\beta(BP:BO)$ $+(DP:DO)$ $+\psi(PBO)$.

θ	FORMA XLIV. $\dfrac{dzz^{\theta n+\frac{2}{3}n-1}}{e+fz^n+gz^{2n}+hz^{3n}}$
&c.	
3.	$\dfrac{3dz^{\frac{2}{3}n}}{2\eta h}+\dfrac{3dk^3}{\eta\kappa\kappa}$ ① $-\dfrac{3dl^3}{\eta\lambda\lambda}$ ② $+\dfrac{3dm^3}{\eta\mu\mu}$ ③
2.	$\dfrac{3dkk}{\eta\kappa}$ ① $-\dfrac{3dll}{\eta\lambda}$ ② $+\dfrac{3dmm}{\eta\mu}$ ③
1.	$\dfrac{3dk}{\eta}$ ① $-\dfrac{3dl}{\eta}$ ② $+\dfrac{3dm}{\eta}$ ③
0.	$\dfrac{3d\kappa}{\eta}$ ① $-\dfrac{3d\lambda}{\eta}$ ② $+\dfrac{3d\mu}{\eta}$ ③
$-1.$	$\dfrac{-3d}{2\eta e z^{\frac{2}{3}n}}+\dfrac{3d\kappa\kappa}{\eta k}$ ① $-\dfrac{3d\lambda\lambda}{\eta l}$ ② $+\dfrac{3d\mu\mu}{\eta m}$ ③
&c.	

Æquationis $e+fz^n+gz^{2n}+hz^{3n}=0$ si tres sunt Radices possibiles seu valores ipsius z^n vocentur $\frac{k}{\kappa}$; $\frac{l}{\lambda}$; $\frac{m}{\mu}$; & ponantur $p=k\lambda-\kappa l$; $q=k\mu-\kappa m$; $r=l\mu-\lambda m$;

Cas. 1.

$OP:OA::\sqrt[3]{z^n}:\sqrt[3]{\frac{k}{\kappa}}$; ① $=\frac{-1}{3pq}\sqrt[3]{\frac{\kappa}{k}}$ in $-(AP:AO)$ $+\beta(CP:CO)-\psi(PCO)$.

$OP:OA::\sqrt[3]{z^n}:\sqrt[3]{\frac{l}{\lambda}}$; ② $=\frac{-1}{3pr}\sqrt[3]{\frac{\lambda}{l}}$ in $-(AP:AO)$ $+\beta(CP:CO)-\psi(PCO)$.

$OP:OA::\sqrt[3]{z^n}:\sqrt[3]{\frac{m}{\mu}}$; ③ $=\frac{-1}{3qr}\sqrt[3]{\frac{\mu}{m}}$ in $-(AP:AO)$ $+\beta(CP:CO)-\psi(PCO)$.

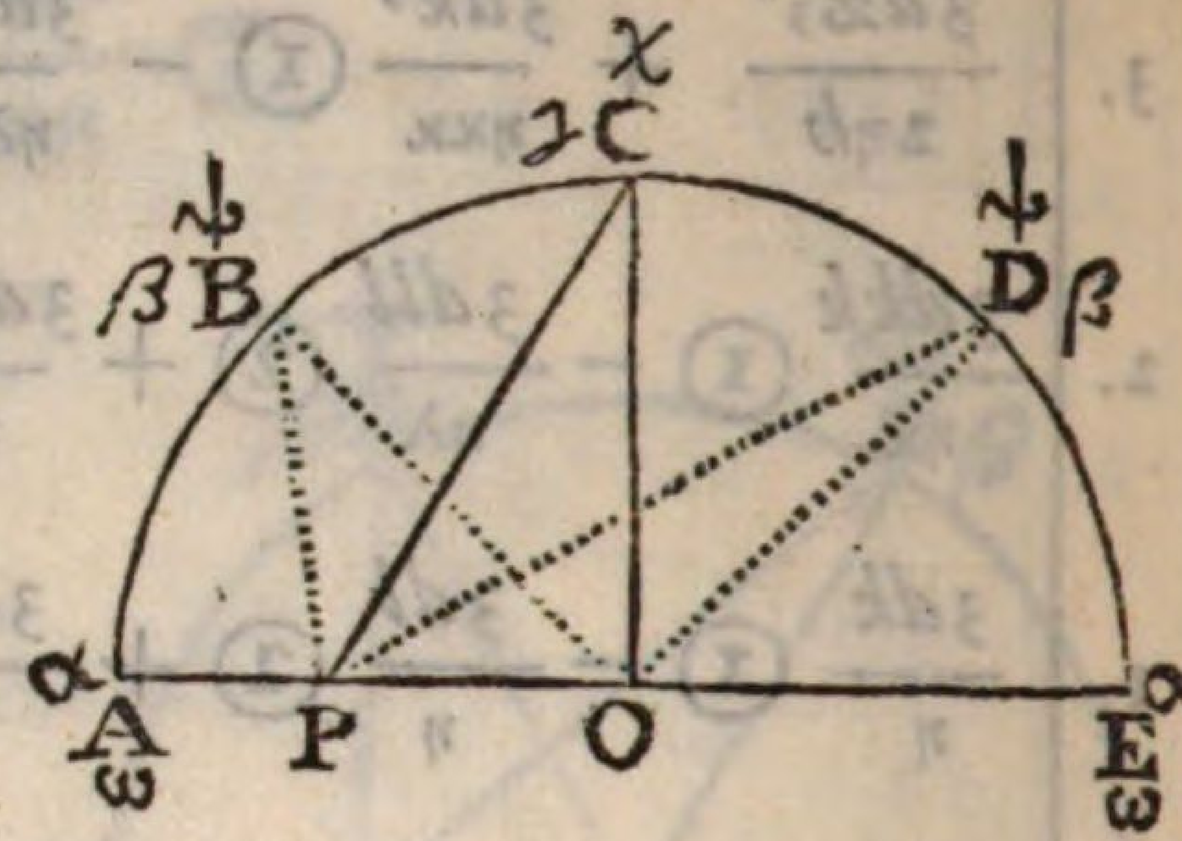
Cas. 2.

$OP:OA::\sqrt[3]{z^n}:\sqrt[3]{\frac{-k}{\kappa}}$; ① $=\frac{1}{3pq}\sqrt[3]{\frac{-\kappa}{k}}$ in $\beta(BP:BO)$ $-(DP:DO)+\psi(PBO)$.

$OP:OA::\sqrt[3]{z^n}:\sqrt[3]{\frac{-l}{\lambda}}$; ② $=\frac{1}{3pr}\sqrt[3]{\frac{-\lambda}{l}}$ in $\beta(BP:BO)$ $-(DP:DO)+\psi(PBO)$.

$OP:OA::\sqrt[3]{z^n}:\sqrt[3]{\frac{-m}{\mu}}$; ③ $=\frac{1}{3qr}\sqrt[3]{\frac{-\mu}{m}}$ in $\beta(BP:BO)$ $-(DP:DO)+\psi(PBO)$.

θ	FORMÆ XLV. $\dfrac{dzz^{\theta n+\frac{1}{4}n-1}}{e+fz^n+gz^{2n}+hz^{3n}}$ *Cas.* 1.
&c.	
2.	$\frac{4dkk}{\eta\varkappa}$ (1) $-\frac{4dll}{\eta\lambda}$ (2) $+\frac{4dmm}{\eta\mu}$ (3)
1.	$\frac{4dk}{\eta}$ (1) $-\frac{4dl}{\eta}$ (2) $+\frac{4dm}{\eta}$ (3)
0.	$\frac{4d\varkappa}{\eta}$ (1) $-\frac{4d\lambda}{\eta}$ (2) $+\frac{4d\mu}{\eta}$ (3)
&c.	



Æquationis $e+fz^n+gz^{2n}+hz^{3n}=0$ si tres sunt Radices possibiles seu valores ipsius z^n vocentur $\frac{k}{\varkappa}$, $\frac{l}{\lambda}$, $\frac{m}{\mu}$, & ponantur $p=k\lambda-\varkappa l$, $q=k\mu-\varkappa m$, $r=l\mu-\lambda m$.

Cas. 1.

$$OP:OA::\sqrt[4]{z^n}:\sqrt[4]{\frac{k}{\varkappa}};\quad (1)=\frac{-1}{4pq}\sqrt[4]{\frac{\varkappa^3}{k^3}}\ \text{in}\ -(AP:AO)+(EP:EO)+\chi(PCO).$$

$$OP:OA::\sqrt[4]{z^n}:\sqrt[4]{\frac{l}{\lambda}};\quad (2)=\frac{-1}{4pr}\sqrt[4]{\frac{\lambda^3}{l^3}}\ \text{in}\ -(AP:AO)+(EP:EO)+\chi(PCO).$$

$$OP:OA::\sqrt[4]{z^n}:\sqrt[4]{\frac{m}{\mu}};\quad (3)=\frac{-1}{4qr}\sqrt[4]{\frac{\mu^3}{m^3}}\ \text{in}\ -(AP:AO)+(EP:EO)+\chi(PCO).$$

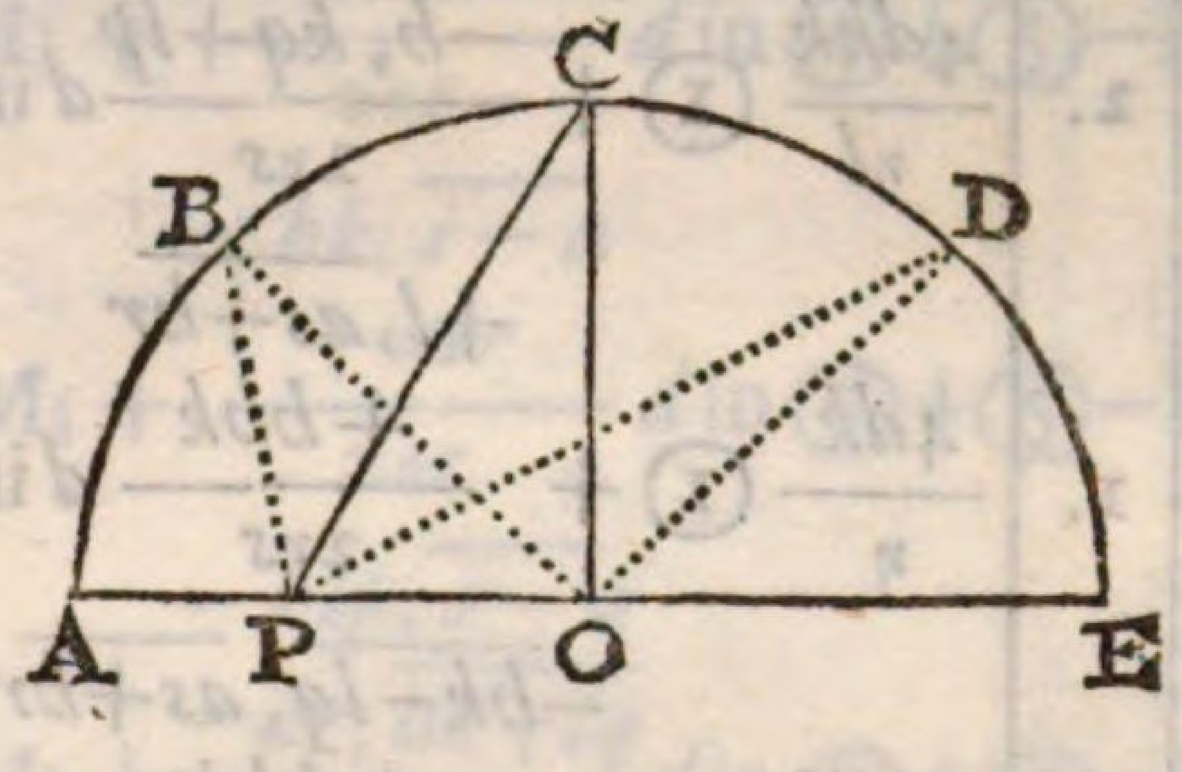
Cas. 2.

$$OP:OA::\sqrt[4]{z^n}:\sqrt[4]{\frac{-k}{\varkappa}};\quad (1)=\frac{1}{4pq}\sqrt[4]{\frac{-\varkappa^3}{k^3}}\ \text{in}\ -\beta(BP:BO)+\beta(DP:DO)+\psi(PBO)+\psi(PDO).$$

$$OP:OA::\sqrt[4]{z^n}:\sqrt[4]{\frac{-l}{\lambda}};\quad (2)=\frac{1}{4pr}\sqrt[4]{\frac{-\lambda^3}{l^3}}\ \text{in}\ -\beta(BP:BO)+\beta(DP:DO)+\psi(PBO)+\psi(PDO).$$

$$OP:OA::\sqrt[4]{z^n}:\sqrt[4]{\frac{-m}{\mu}};\quad (3)=\frac{1}{4qr}\sqrt[4]{\frac{-\mu^3}{m^3}}\ \text{in}\ -\beta(BP:BO)+\beta(DP:DO)+\psi(PBO)+\psi(PDO).$$

θ	FORMÆ XLVI. $\dfrac{dzz^{\theta n+\frac{3}{4}n-1}}{e+fz^n+gz^{2n}+hz^{3n}}$ *Cas.* I.
&c.	
2.	$\frac{4dkk}{\eta\kappa}$ ① $-\frac{4dll}{\eta\lambda}$ ② $+\frac{4dmm}{\eta\mu}$ ③
I.	$\frac{4dk}{\eta}$ ① $-\frac{4dl}{\eta}$ ② $+\frac{4dm}{\eta}$ ③
o.	$\frac{4d\kappa}{\eta}$ ① $-\frac{4d\lambda}{\eta}$ ② $+\frac{4d\mu}{\eta}$ ③
&c.	



Æquationis $e+fz^n+gz^{2n}+hz^{3n}=0$ si tres sunt Radices possibiles, seu valores ipsius z^n vocentur $\frac{k}{\kappa}$, $\frac{l}{\lambda}$, $\frac{m}{\mu}$, & ponantur $p=k\lambda-\kappa l$, $q=k\mu-\kappa m$, $r=l\mu-\lambda m$.

Cas. I.

$$OP:OA::\sqrt[4]{z^n}:\sqrt[4]{\frac{k}{\kappa}};\quad ①=\frac{-1}{4pq}\sqrt[4]{\frac{l}{k}}\text{ in }-(AP:AO)+(EP:EO)-\chi(PCO).$$

$$OP:OA::\sqrt[4]{z^n}:\sqrt[4]{\frac{l}{\lambda}};\quad ②=\frac{-1}{4pr}\sqrt[4]{\frac{\lambda}{l}}\text{ in }-(AP:AO)+(EP:EO)-\chi(PCO.)$$

$$OP:OA::\sqrt[4]{z^n}:\sqrt[4]{\frac{m}{\mu}};\quad ③=\frac{-1}{4qr}\sqrt[4]{\frac{\mu}{m}}\text{ in }-(AP:AO)+(EP:EO)-\chi(PCO).$$

Cas. 2.

$$OP:OA::\sqrt[4]{z^n}:\sqrt[4]{\frac{-k}{\kappa}};\quad ①=\frac{+1}{4pq}\sqrt[4]{\frac{-\kappa}{k}}\text{ in }\beta(BP:BO)-\beta(DP:DO)+\psi(PBO)+\psi(PDO).$$

$$OP:OA::\sqrt[4]{z^n}:\sqrt[4]{\frac{-l}{\lambda}};\quad ②=\frac{+1}{4pr}\sqrt[4]{\frac{-\lambda}{l}}\text{ in }\beta(BP:BO)-\beta(DP:DO)+\psi(PBO)+\psi(PDO).$$

$$OP:OA::\sqrt[4]{z^n}:\sqrt[4]{\frac{-m}{\mu}};\quad ③=\frac{+1}{4qr}\sqrt[4]{\frac{-\mu}{m}}\text{ in }\beta(BP:BO)-\beta(DP:DO)+\psi(PBO)+\psi(PDO).$$

θ	FORMÆ XLV. $\frac{d\dot{z}z^{\theta n+\frac{1}{4}n-1}}{e+fz^{n}+gz^{2n}+hz^{3n}}$ *Caſ.* 2.
&c.	
2.	$\frac{4dkk}{\eta l}\textcircled{1}+\frac{\begin{matrix}+k,\overline{as+hr}\\-h,\overline{kq+lp}\end{matrix}}{2\eta s}d\text{ in }\textcircled{2}+\textcircled{3}+\text{M};+\frac{\begin{matrix}+k,\overline{as-hr}\\+h,\overline{kq+lp}\end{matrix}}{2\eta s}d\text{ in }\textcircled{4}+\textcircled{5}+\text{N}.$
1.	$\frac{4dk}{\eta}\textcircled{1}+\frac{\begin{matrix}+l,\overline{as+hr}\\+hhk\end{matrix}}{2\eta s}d\text{ in }\textcircled{2}+\textcircled{3}+\text{M};+\frac{\begin{matrix}+l,\overline{as-hr}\\-hhk\end{matrix}}{2\eta s}d\text{ in }\textcircled{4}+\textcircled{5}+\text{N}.$
0.	$\frac{4dl}{\eta}\textcircled{1}+\frac{\begin{matrix}\overline{-hk-lq},\overline{as+hr}\\+hhlp\end{matrix}}{2\eta ps}d\text{ in }\textcircled{2}+\textcircled{3}+\text{M};+\frac{\begin{matrix}\overline{-hk-lq},\overline{as-hr}\\-hhlp\end{matrix}}{2\eta ps}d\text{ in }\textcircled{4}+\textcircled{5}+\text{N}.$
&c.	

Valores quantitatum $k,l,p,q,r,s,t,a,b,c,$ M,N petendi ſunt a pag. ſubſequente.

Caſ. 1.

$$OP:OA::\sqrt[4]{z^{n}}:\sqrt[4]{\frac{k}{l}};\quad \textcircled{1}=\frac{-l}{4t}\sqrt[4]{\frac{l^{3}}{k^{3}}}\text{ in }-(AP:AO)+(EP:EO)+\chi(PCO).$$

$$OP:OA::+\tfrac{1}{2}c+hz^{\frac{1}{4}n}:\sqrt{\tfrac{1}{4}cc-ah};\ \textcircled{2}=\frac{-l}{2t}\sqrt{\frac{1}{\frac{1}{4}cc-ah}}\text{ in }(EP:AP).$$

$$OP:OA::-\tfrac{1}{2}c+hz^{\frac{1}{4}n}:\sqrt{\tfrac{1}{4}cc-ah};\ \textcircled{3}=\frac{-l}{2t}\sqrt{\frac{1}{\frac{1}{4}cc-ah}}\text{ in }(EP:AP).$$

$$OP:OA::+\tfrac{1}{2}b+hz^{\frac{1}{4}n}:\sqrt{\tfrac{1}{4}bb-ah};\ \textcircled{4}=\frac{-l}{2t}\sqrt{\frac{1}{\frac{1}{4}bb-ah}}\text{ in }(EP:AP).$$

$$OP:OA::-\tfrac{1}{2}b+hz^{\frac{1}{4}n}:\sqrt{\tfrac{1}{4}bb-ah};\ \textcircled{5}=\frac{-l}{2t}\sqrt{\frac{1}{\frac{1}{4}bb-ah}}\text{ in }(EP:AP).$$

Caſ. 2.

$$OP:OA::\sqrt[4]{z^{n}}:\sqrt[4]{\frac{-k}{l}};\quad \textcircled{1}=\frac{+l}{4t}\sqrt[4]{\frac{-l^{3}}{k^{3}}}\text{ in }-\beta(BP:BO)+\beta(DP:DO)+\psi(PBO)+\psi(PDO).$$

$$OP:OA::+\tfrac{1}{2}c+hz^{\frac{1}{4}n}:\sqrt{ah-\tfrac{1}{4}cc};\ \textcircled{2}=\frac{+l}{2t}\sqrt{\frac{1}{ah-\frac{1}{4}cc}}\text{ in }\chi(PCO).$$

$$OP:OA::-\tfrac{1}{2}c+hz^{\frac{1}{4}n}:\sqrt{ah-\tfrac{1}{4}cc};\ \textcircled{3}=\frac{+l}{2t}\sqrt{\frac{1}{ah-\frac{1}{4}cc}}\text{ in }\chi(PCO).$$

$$OP:OA::+\tfrac{1}{2}b+hz^{\frac{1}{4}n}:\sqrt{ah-\tfrac{1}{4}bb};\ \textcircled{4}=\frac{+l}{2t}\sqrt{\frac{1}{ah-\frac{1}{4}bb}}\text{ in }\chi(PCO).$$

$$OP:OA::-\tfrac{1}{2}b+hz^{\frac{2}{4}n}:\sqrt{ah-\tfrac{1}{4}bb};\ \textcircled{5}=\frac{+l}{2t}\sqrt{\frac{1}{ah-\frac{1}{4}bb}}\text{ in }\chi(PCO).$$

θ	FORMÆ XLVI. $\frac{d\dot{z}z^{\theta\eta+\frac{1}{4}\eta-1}}{e+fz^{n}+gz^{2n}+hz^{3n}}$ *Caſ.* 2.
&c.	
2.	$\frac{4dkk}{\eta l}\textcircled{1}+\frac{\overline{lp+kq},\overline{s+a}\;-akr}{2\eta s}d\text{ in }\textcircled{2}+\textcircled{3}-\mathrm{M};+\frac{\overline{lp+kq},\overline{s-a}\;+akr}{2\eta s}d\text{ in }\textcircled{4}+\textcircled{5}-\mathrm{N}.$
1.	$\frac{4dk}{\eta}\textcircled{1}+\frac{-hk,\overline{s+a}\;-alr}{2\eta s}d\text{ in }\textcircled{2}+\textcircled{3}-\mathrm{M};+\frac{-hk,\overline{s-a}\;+alr}{2\eta s}d\text{ in }\textcircled{4}+\textcircled{5}-\mathrm{N}.$
0.	$\frac{4dl}{\eta}\textcircled{1}+\frac{-hlp,\overline{s+a}\;+ar,\overline{hk+lq}}{2\eta ps}d\text{ in }\textcircled{2}+\textcircled{3}-\mathrm{M};+\frac{-hlp,\overline{s-a}\;-ar,\overline{hk+lq}}{2\eta ps}d\text{ in }\textcircled{4}+\textcircled{5}-\mathrm{N}.$
&c.	

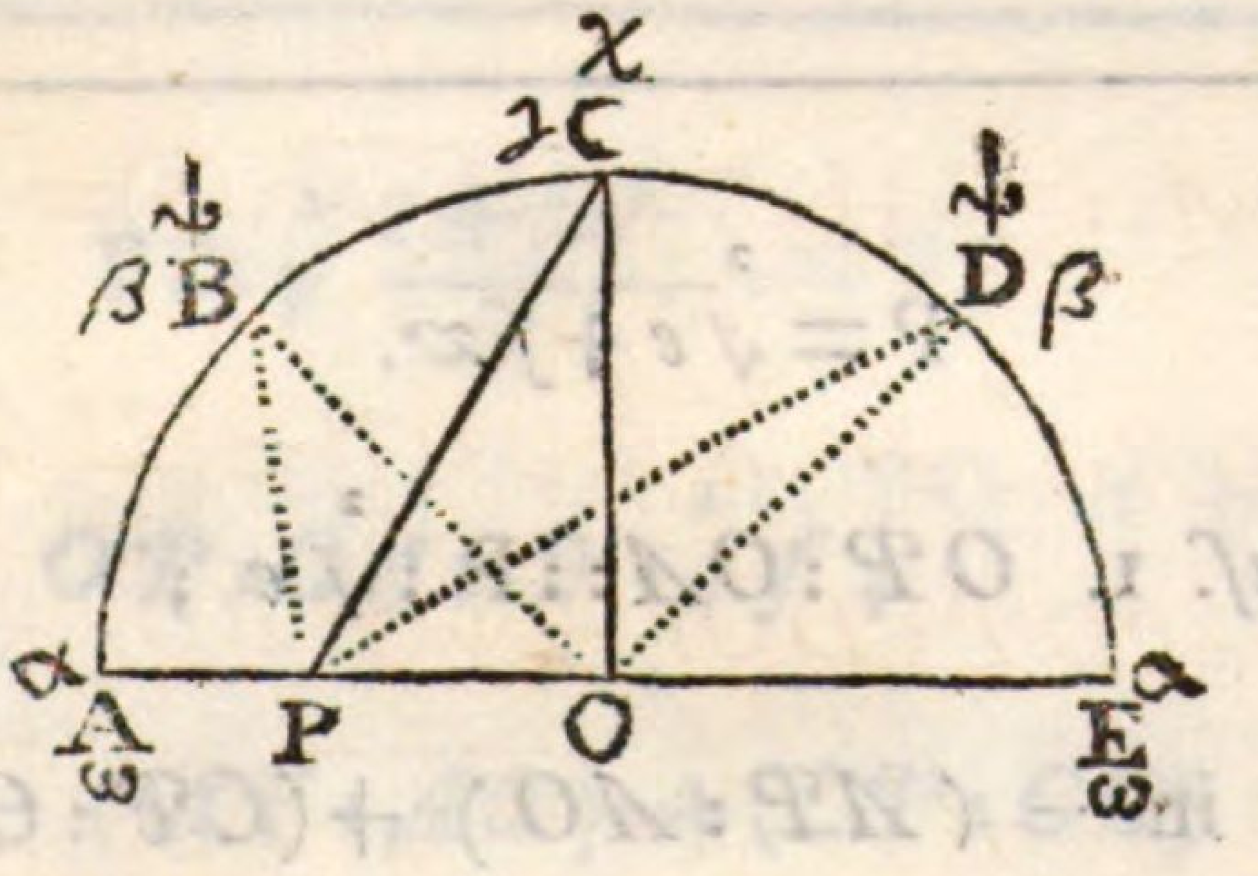
Æquationis $e+fz^{n}+gz^{2n}+hz^{3n}=0$ eſto $\frac{k}{l}$ Radix poſſibilis, ſeu valor ipſius z^{n}, ubi reliquæ ſunt impoſſibiles, & ponantur $p=-e\frac{l}{k}$, $q=-e\frac{ll}{kk}-f\frac{l}{k}$; vel $p=f+g\frac{k}{l}+h\frac{kk}{ll}$, $q=g+h\frac{k}{l}$;

$$r=\sqrt{hp},\quad s=\sqrt{-hq+2\sqrt{h^{3}p}},\quad t=pll+qkl+hkk;\quad a=\sqrt[4]{h^{3}p},$$

$$b=\sqrt{+\sqrt{-h^{3}q+2\sqrt{h^{7}p}}+2\sqrt[4]{h^{7}p}},\quad c=\sqrt{-\sqrt{-h^{3}q+2\sqrt{h^{7}p}}+2\sqrt[4]{h^{7}p}};$$

$$\mathrm{M}=\tfrac{l}{bt}(a+bz^{\frac{1}{4}n}+hz^{\frac{1}{2}n}:a-bz^{\frac{1}{4}n}+hz^{\frac{1}{2}n}.)$$

$$\mathrm{N}=\tfrac{l}{ct}(a+cz^{\frac{1}{4}n}+hz^{\frac{1}{2}n}:a-cz^{\frac{1}{4}n}+hz^{\frac{1}{2}n}).$$

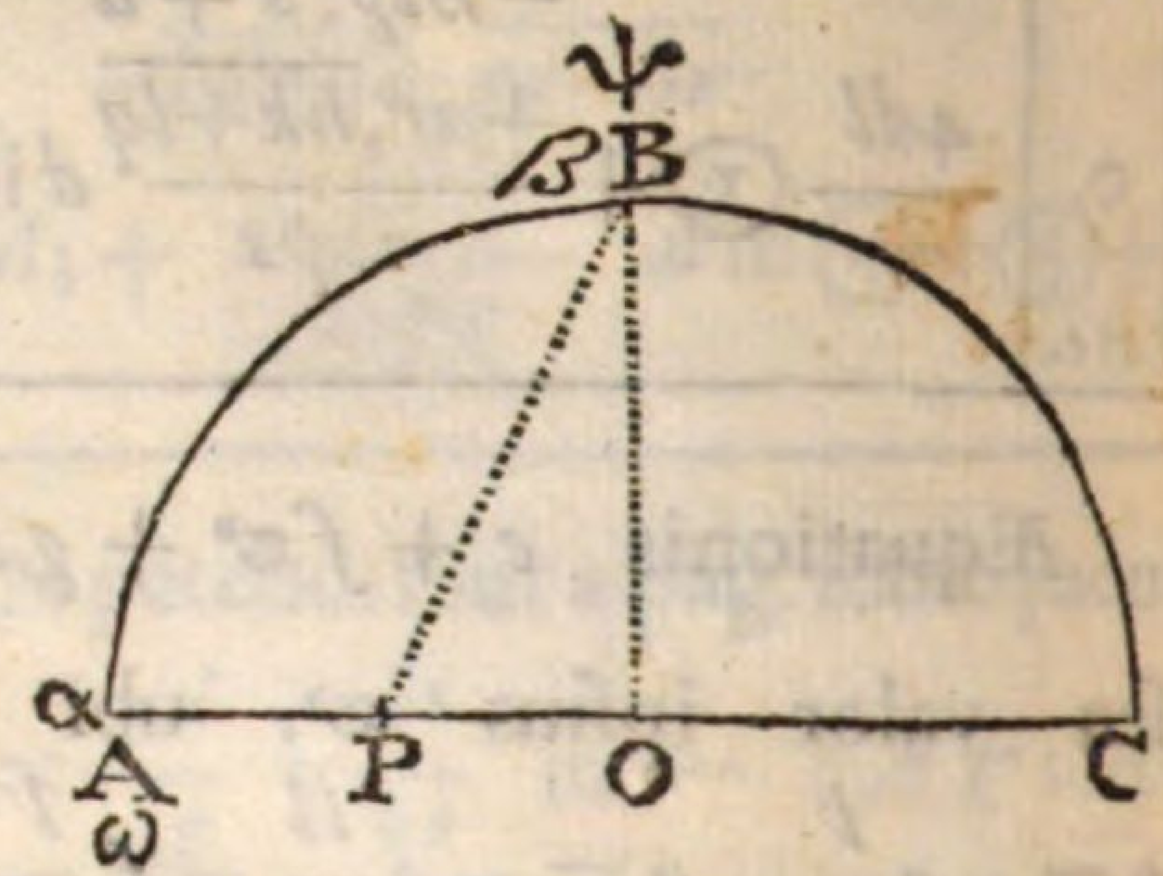


Caſ. 1. $OP:OA::\sqrt[4]{z^{n}}:\sqrt[4]{\frac{k}{l}}$; $\textcircled{1}=\frac{-l}{4t}\sqrt[4]{\frac{l}{k}}$ in $-(AP:AO)$ $+(EP:EO)-\chi(PCO)$.

Caſ. 2. $OP:OA::\sqrt[4]{z^{n}}:\sqrt[4]{\frac{-k}{l}}$; $\textcircled{1}=\frac{+l}{4t}\sqrt[4]{\frac{-l}{k}}$ in $+\beta(BP:BO)$ $-\beta(DP:DO)+\psi(PBO)+\psi(PDO)$.

Valores quantitatum $\textcircled{2}$, $\textcircled{3}$, $\textcircled{4}$, $\textcircled{5}$ petendi ſunt a pag. præcedente.

θ	FORMA XLVII. $\dfrac{d\dot{z}z^{\theta n-1}}{\sqrt[2]{e+fz^n}}$
&c.	
4.	$\dfrac{-96e^3+48eefz^n-36effz^{2n}+30f^3z^{3n}}{105\eta f^4}dP.$
3.	$\dfrac{+16ee-8efz^n+6ffz^{2n}}{15\eta f^3}dP.$
2.	$\dfrac{-4e+2fz^n}{3\eta ff}dP.$
1.	$\dfrac{+2}{\eta f}dP.$
0.	$\dfrac{+2}{\eta}d(\mathrm{I}).$
$\overline{1}.$	$\dfrac{-1}{\eta ez^n}dP-\dfrac{f}{\eta e}d(\mathrm{I}).$
$\overline{2}.$	$\dfrac{-2e+3fz^n}{4\eta eez^{2n}}dP+\dfrac{3ff}{4\eta ee}d(\mathrm{I}).$
$\overline{3}.$	$\dfrac{-8ee+10efz^n-15ffz^{2n}}{24\eta e^3z^{3n}}dP-\dfrac{5f^3}{8\eta e^3}d(\mathrm{I}).$
&c.	



$$P=\sqrt[2]{e+fz^n}.$$

Caſ. 1. $OP:OA::P:\sqrt[2]{e}$;

$$(\mathrm{I})=-\tfrac{1}{2}\sqrt[2]{\frac{1}{e}}\ \text{in}\ -(AP:AO)+(CP:CO).$$

Caſ. 2. $OP:OA::P:\sqrt[2]{-e}$;

$$(\mathrm{I})=+\tfrac{1}{2}\sqrt[2]{\frac{-1}{e}}\ \text{in}\ \psi\,(PBO).$$

θ	FORMAE XLVIII. $\frac{d\dot{z}z^{\theta n+\frac{1}{2}n-1}}{\sqrt[2]{e+fz^n}}$
&c.	
3.	$\frac{+15eez^n-10efz^{2n}+8ffz^{3n}}{24\eta f^3}dP+\frac{5e^3}{8\eta f^3}d\textcircled{I}$.
2.	$\frac{-3ez^n+2fz^{2n}}{4\eta ff}dP-\frac{3ee}{4\eta ff}d\textcircled{I}$.
1.	$\frac{+z^n}{\eta f}dP+\frac{e}{\eta f}d\textcircled{I}$.
0.	$\frac{-2}{\eta}d\textcircled{I}$.
$\overline{1}$.	$\frac{-2}{\eta e}dP$.
$\overline{2}$.	$\frac{-2e+4fz^n}{3\eta eez^n}dP$.
$\overline{3}$.	$\frac{-6ee+8efz^n-16ffz^{2n}}{15\eta e^3z^{2n}}dP$.
$\overline{4}$.	$\frac{-30e^3+36eefz^n-48effz^{2n}+96f^3z^{3n}}{105\eta e^4z^{3n}}dP$.
&c.	

$$P=\sqrt[2]{\frac{e+fz^n}{z^n}}.$$

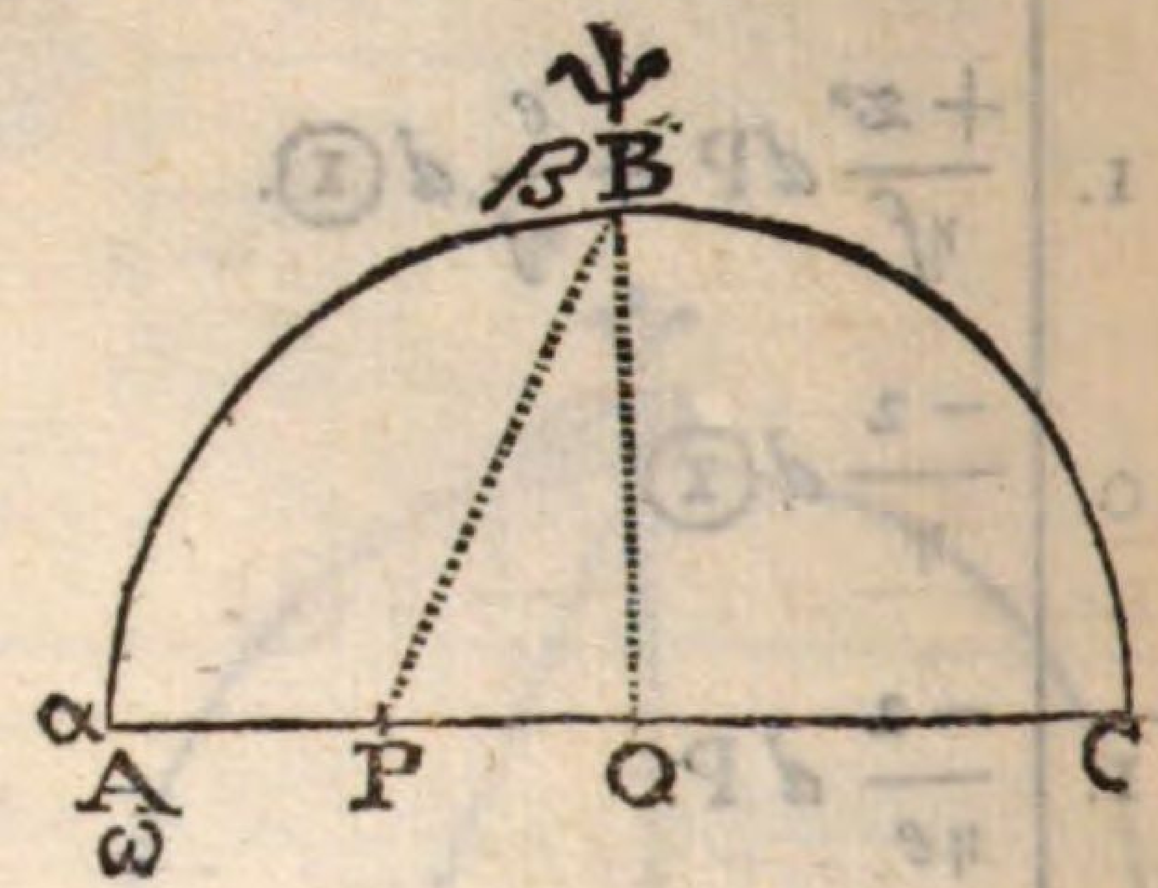
Cas. 1. $OP:OA::P:\sqrt[2]{f}$;

$$\textcircled{I}=-\tfrac{1}{2}\sqrt[2]{\frac{1}{f}}\text{ in }-(AP:AO)+(CP:CO).$$

Cas. 2. $OP:OA::P:\sqrt[2]{-f}$;

$$\textcircled{I}=+\tfrac{1}{2}\sqrt[2]{\frac{-1}{f}}\text{ in }\downarrow(PBO).$$

θ	FORMA XLIX. $dzz^{\theta n-1}\sqrt[2]{e+fz^n}$
&c.	
3.	$\dfrac{+16e^3-8eefz^n+6effz^{2n}+30f^3z^{3n}}{105\eta f^3}dP.$
2.	$\dfrac{-4ee+2efz^n+6ffz^{2n}}{15\eta ff}dP.$
1.	$\dfrac{+2e+2fz^n}{3\eta f}dP.$
0.	$\dfrac{+2}{\eta}dP+\dfrac{2e}{\eta}d\,(I).$
$\overline{1}$.	$\dfrac{-1}{\eta z^n}dP+\dfrac{f}{\eta}d\,(I).$
$\overline{2}$.	$\dfrac{-2e-fz^n}{4\eta ez^{2n}}dP-\dfrac{ff}{4\eta e}d\,(I).$
$\overline{3}$.	$\dfrac{-8ee-2efz^n+3ffz^{2n}}{24\eta eez^{3n}}dP+\dfrac{f^3}{8\eta ee}d\,(I).$
&c.	



$$P=\sqrt[2]{e+fz^n}.$$

Cas. 1. $OP:OA::P:\sqrt[2]{e}$;

$$(I)=+\tfrac{1}{2}\sqrt[2]{\frac{1}{e}}\ \text{in} -(AP:AO)+(CP:CO).$$

Cas. 2. $OP:QA::P:\sqrt[2]{-e}$;

$$(I)=+\tfrac{1}{2}\sqrt[2]{\frac{-1}{e}}\ \text{in}\ \psi(PBO).$$

θ	FORMA L. $d\dot{z}z^{\theta\eta+\frac{1}{2}\eta-1}\sqrt[2]{e+fz^{\eta}}$
&c.	
2.	$\frac{-3eez^{\eta}+2efz^{2\eta}+8ffz^{3\eta}}{24\eta ff}d\mathrm{P}-\frac{e^{3}}{8\eta ff}d\textcircled{I}.$
1.	$\frac{+ez^{\eta}+2fz^{2\eta}}{4\eta f}d\mathrm{P}+\frac{ee}{4\eta f}d\textcircled{I}.$
0.	$\frac{+z^{\eta}}{\eta}d\mathrm{P}-\frac{e}{\eta}d\textcircled{I}.$
$\overline{1}$.	$\frac{-2}{\eta}d\mathrm{P}-\frac{2f}{\eta}d\textcircled{I}.$
$\overline{2}$.	$\frac{-2e-2fz^{\eta}}{3\eta ez^{\eta}}d\mathrm{P}.$
$\overline{3}$.	$\frac{-6ee-2efz^{\eta}+4ffz^{2\eta}}{15\eta eez^{2\eta}}d\mathrm{P}.$
$\overline{4}$.	$\frac{-30e^{3}-6eefz^{\eta}+8effz^{2\eta}-16f^{3}z^{3\eta}}{105\eta e^{3}z^{3\eta}}d\mathrm{P}.$
&c.	

$$\mathrm{P}=\sqrt[2]{\frac{e+fz^{\eta}}{z^{\eta}}}.$$

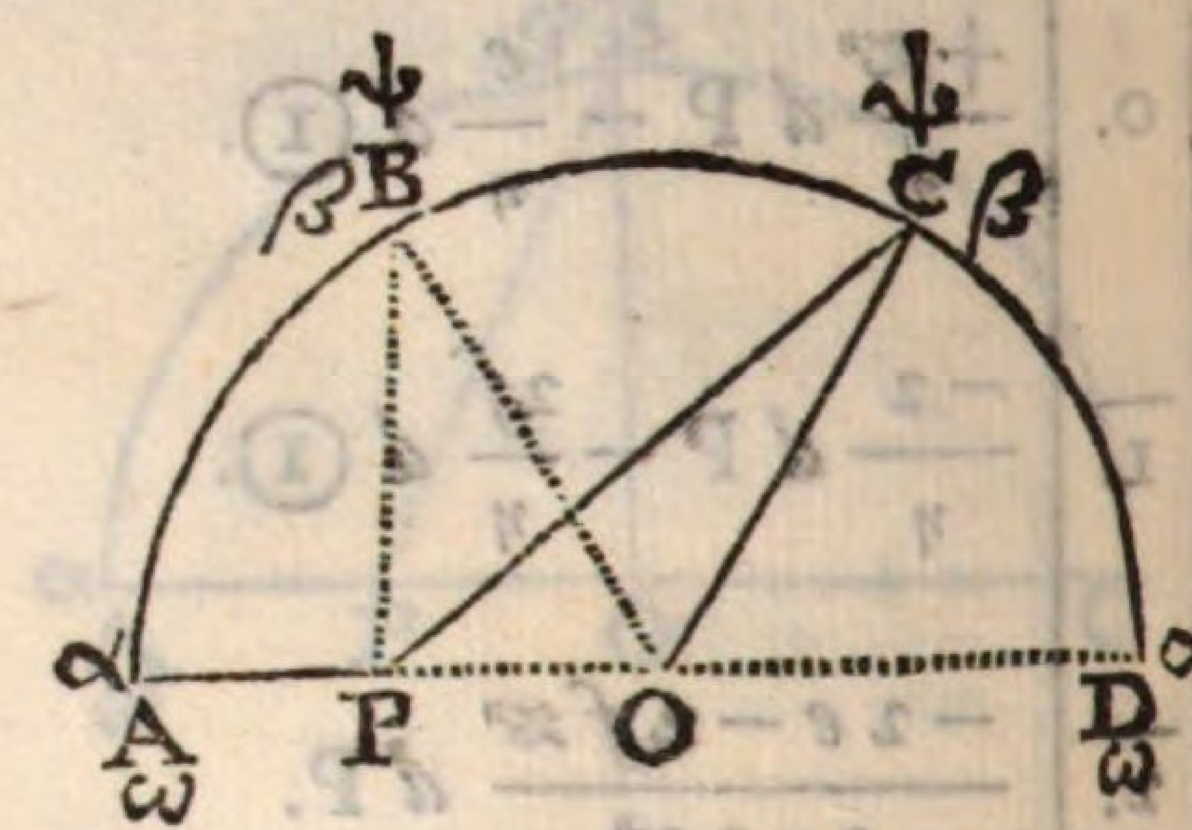
Cas. 1. $OP:OA::\mathrm{P}:\sqrt[2]{f}$;

$$\textcircled{I}=-\tfrac{1}{2}\sqrt[2]{\frac{1}{f}}\ \text{in}\ -(AP:AO)+(CP:CO).$$

Cas. 2. $OP::OA::\mathrm{P}:\sqrt[2]{-f}$;

$$\textcircled{I}=+\tfrac{1}{2}\sqrt[2]{\frac{-1}{f}}\ \text{in}\ \psi\,(PBO).$$

θ	FORMA LI. $\dfrac{d\dot{z}z^{\theta n-1}}{\sqrt[3]{e+fz^n}}$
&c.	
4.	$\dfrac{-243e^3+162eefz^n-135effz^{2n}+120f^3z^{3n}}{440\eta f^4}\, d\,PP.$
3.	$\dfrac{+27ee-18efz^n+15ffz^{2n}}{40\eta f^3}\, d\,PP.$
2.	$\dfrac{-9e+6fz^n}{10\eta ff}\, d\,PP.$
1.	$\dfrac{+3}{2\eta f}\, d\,PP.$
0.	$\dfrac{+3}{\eta}\, d\,\textcircled{\scriptsize I}.$
$\overline{1}$.	$\dfrac{-1}{\eta ez^n}\, d\,PP - \dfrac{f}{\eta e}\, d\,\textcircled{\scriptsize I}.$
$\overline{2}$.	$\dfrac{-3e+4fz^n}{6\eta eez^{2n}}\, d\,PP + \dfrac{2ff}{3\eta ee}\, d\,\textcircled{\scriptsize I}.$
$\overline{3}$.	$\dfrac{-18ee+21efz^n-28ffz^{2n}}{54\eta e^3z^{3n}}\, d\,PP - \dfrac{14f^3}{27\eta e^3}\, d\,\textcircled{\scriptsize I}.$
&c.	



$$P=\sqrt[3]{e+fz^n}.$$

Caſ. 1. $OP : OA :: P : \sqrt[3]{e}$;

$$\textcircled{\scriptsize I} = -\tfrac{1}{3}\sqrt[3]{\frac{1}{e}} \text{ in } -(AP:AO) + \beta(CP:CO) - \psi(PCO).$$

Caſ. 2. $OP : OA :: P : \sqrt[3]{-e}$;

$$\textcircled{\scriptsize I} = +\tfrac{1}{3}\sqrt[3]{\frac{-1}{e}} \text{ in } +\beta(BP:BO) - (DP:DO) + \psi(PBO).$$

θ	FORMA LII. $\frac{d\dot{z} z^{\theta\eta+\frac{1}{3}\eta-1}}{\sqrt[3]{e+fz^{\eta}}}$
&c.	
3.	$\frac{+28eez^{\eta}-21efz^{2\eta}+18ffz^{3\eta}}{54\eta f^{3}}\,d\,PP+\frac{14e^{3}}{27\eta f^{3}}\,d\,\textcircled{1}.$
2.	$\frac{-4ez^{\eta}+3fz^{2\eta}}{6\eta ff}\,d\,PP-\frac{2ee}{3\eta ff}\,d\,\textcircled{1}.$
1.	$\frac{+z^{\eta}}{\eta f}\,d\,PP+\frac{e}{\eta f}\,d\,\textcircled{1}.$
0.	$\frac{-3}{\eta}\,d\,\textcircled{1}.$
$\overline{1}$.	$\frac{-3}{2\eta e}\,d\,PP.$
$\overline{2}$.	$\frac{-6e+9fz^{\eta}}{10\eta eez^{\eta}}\,d\,PP.$
$\overline{3}$.	$\frac{-15ee+18efz^{\eta}-27ffz^{2\eta}}{40\eta e^{3}z^{2\eta}}\,d\,PP.$
$\overline{4}$.	$\frac{-120e^{3}+135eefz^{\eta}-162effz^{2\eta}+243f^{3}z^{3\eta}}{440\eta e^{4}z^{3\eta}}\,d\,PP.$
&c.	

$$P=\sqrt[3]{\frac{e+fz^{\eta}}{z^{\eta}}}.$$

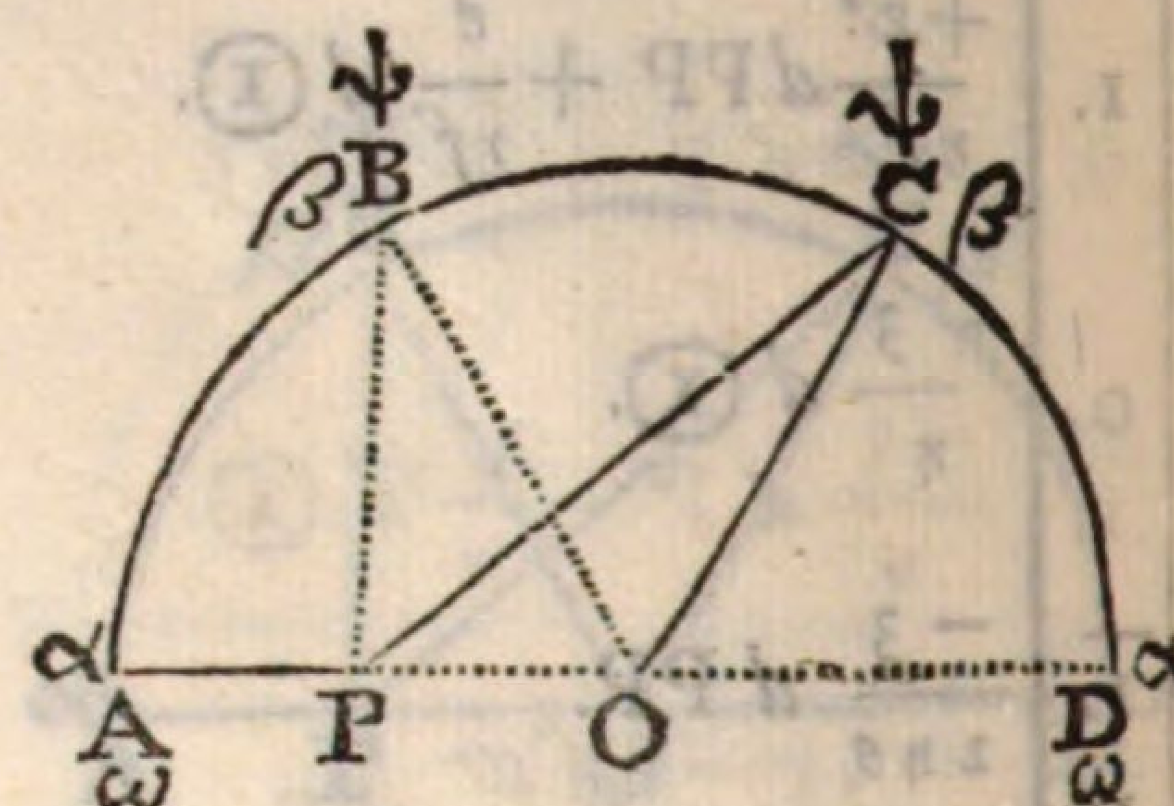
Cas. 1. $OP:OA::P:\sqrt[3]{f}$;

$$\textcircled{1}=-\tfrac{1}{3}\sqrt[3]{\frac{1}{f}}\ \text{in}\ -(AP:AO)+\beta(CP:CO)-\psi(PCO).$$

Cas. 2. $OP::OA::P:\sqrt[3]{-f}$;

$$\textcircled{1}=+\tfrac{1}{3}\sqrt[3]{\frac{-1}{f}}\ \text{in}\ +\beta(BP:BO)-(DP:DO)+\psi(PBO).$$

θ	FORMA LIII. $\quad d z z^{\theta n - 1} \sqrt[3]{e+fz^n}$
&c.	
3.	$\dfrac{+27e^3 - 9eefz^n + 6effz^{2n} + 42f^3z^{3n}}{140\eta f^3}\, d\,\mathrm{P}.$
2.	$\dfrac{-9ee + 3efz^n + 12ffz^{2n}}{28\eta ff}\, d\,\mathrm{P}.$
1.	$\dfrac{+3e + 3fz^n}{4\eta f}\, d\,\mathrm{P}.$
0.	$\dfrac{+3}{\eta}\, d\,\mathrm{P} + \dfrac{3e}{\eta}\, d\,\textcircled{1}.$
$\overline{1}$.	$\dfrac{-1}{\eta z^n}\, d\,\mathrm{P} + \dfrac{f}{\eta}\, d\,\textcircled{1}.$
$\overline{2}$.	$\dfrac{-3e - fz^n}{6\eta e z^{2n}}\, d\,\mathrm{P} - \dfrac{ff}{3\eta e}\, d\,\textcircled{1}.$
$\overline{3}$.	$\dfrac{-18ee - 3efz^n + 5ffz^{2n}}{54\eta eez^{3n}}\, d\,\mathrm{P} + \dfrac{5f^3}{27\eta ee}\, d\,\textcircled{1}.$
&c.	



$$\mathrm{P} = \sqrt[3]{e+fz^n}.$$

Cas. 1. $OP : OA :: \mathrm{P} : \sqrt[3]{e}$;

$$\textcircled{1} = -\tfrac{1}{3}\sqrt[3]{\frac{1}{ee}} \text{ in } -(AP:AO) + \beta(CP:CO) + \psi(PCO).$$

Cas. 2. $OP : OA :: \mathrm{P} : \sqrt[3]{-e}$;

$$\textcircled{1} = \mp\tfrac{1}{3}\sqrt[3]{\frac{1}{ee}} \text{ in } -\beta(BP:BO) + (DP:DO) + \psi(PBO).$$

θ	FORMA LIV. $d\dot{z}z^{\theta\eta+\frac{2}{3}\eta-1}\sqrt[3]{e+fz^{\eta}}$
&c.	
2.	$\frac{-5eez^{\eta}+3efz^{2\eta}+18ffz^{3\eta}}{54\eta ff}d\mathrm{P}-\frac{5e^{3}}{27\eta ff}d\textcircled{\small I}.$
1.	$\frac{+ez^{\eta}+3fz^{2\eta}}{6\eta f}d\mathrm{P}+\frac{ee}{3\eta f}d\textcircled{\small I}.$
0.	$\frac{+z^{\eta}}{\eta}d\mathrm{P}-\frac{e}{\eta}d\textcircled{\small I}.$
$\overline{1}$.	$\frac{-3}{\eta}d\mathrm{P}-\frac{3f}{\eta}d\textcircled{\small I}.$
$\overline{2}$.	$\frac{-3e-3fz^{\eta}}{4\eta ez^{\eta}}d\mathrm{P}.$
$\overline{3}$.	$\frac{-12ee-3efz^{\eta}+9ffz^{2\eta}}{28\eta eez^{2\eta}}d\mathrm{P}.$
$\overline{4}$.	$\frac{-42e^{3}-6eefz^{\eta}+9effz^{2\eta}-27f^{3}z^{3\eta}}{140\eta e^{3}z^{3\eta}}d\mathrm{P}.$
&c.	

$$\mathrm{P}=\sqrt[3]{\frac{e+fz^{\eta}}{z^{\eta}}}.$$

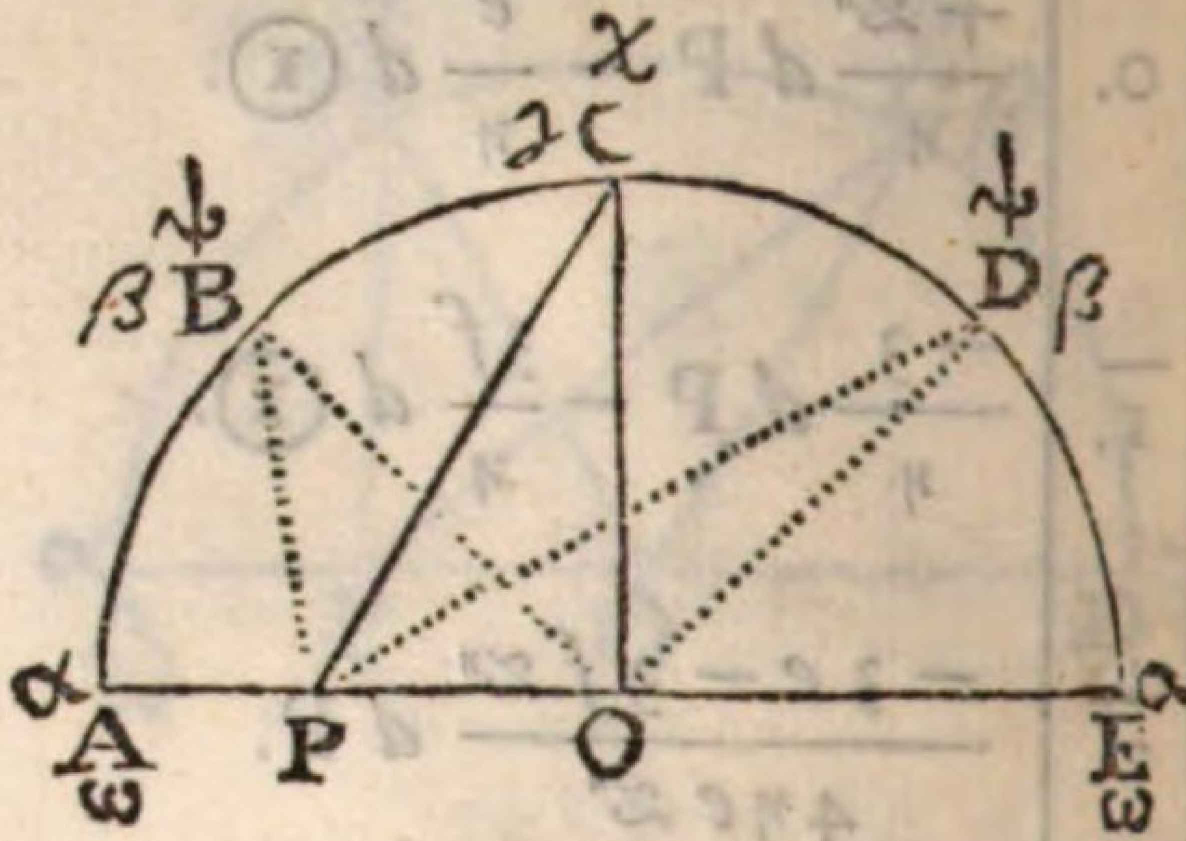
Cas. 1. $OP:OA::\mathrm{P}:\sqrt[3]{f}$;

$$\textcircled{\small I}=-\tfrac{1}{3}\sqrt[3]{\frac{1}{ff}}\text{ in }-(AP:AO)+\beta(CP:CO)+\psi(PCO).$$

Cas. 2. $OP:OA::\mathrm{P}:\sqrt[3]{-f}$;

$$\textcircled{\small I}=+\tfrac{1}{3}\sqrt[3]{\frac{1}{ff}}\text{ in }-\beta(BP:BO)+(DP:DO)+\psi(PBO).$$

θ	FORMA LV. $\frac{d\dot{z}z^{\theta n-1}}{\sqrt[4]{e+fz^n}}$
&c.	
4.	$\frac{-512e^3+384eefz^n-336effz^{2n}+308f^3z^{3n}}{1155\eta f^4}\,dP^3.$
3.	$\frac{+128ee-96efz^n+84ffz^{2n}}{231\eta f^3}\,dP^3.$
2.	$\frac{-16e+12fz^n}{21\eta ff}\,dP^3.$
1.	$\frac{+4}{3\eta f}\,dP^3.$
0.	$\frac{+4}{\eta}\,d\,\textcircled{1}.$
$\overline{1}$.	$\frac{-1}{\eta ez^n}\,dP^3-\frac{f}{\eta e}\,d\,\textcircled{1}.$
$\overline{2}$.	$\frac{-4e+5fz^n}{8\eta eez^{2n}}\,dP^3+\frac{5ff}{8\eta ee}\,d\,\textcircled{1}.$
$\overline{3}$.	$\frac{-32ee+36efz^n-45ffz^{2n}}{96\eta e^3z^{3n}}\,dP^3-\frac{15f^3}{32\eta e^3}\,d\,\textcircled{1}.$
&c.	



$$P=\sqrt[4]{e+fz^n}.$$

Caſ. 1. $OP:OA::P:\sqrt[4]{e}$;

$$\textcircled{1}=-\tfrac{1}{4}\sqrt[4]{\frac{1}{e}}\text{ in }-(AP:AO)+(EP:EO)-\chi(PCO).$$

Caſ. 2. $OP:OA::P:\sqrt[4]{-e}$;

$$\textcircled{1}=+\tfrac{1}{4}\sqrt[4]{\frac{-1}{e}}\text{ in }+\beta(BP:BO)-\beta(DP:DO)+\psi(PBO)+\psi(PDO).$$

θ	FORMA LVI. $\dfrac{dzz^{\theta\eta+\frac{1}{4}\eta-1}}{\sqrt[4]{e+fz^{\eta}}}$
&c.	
3.	$\dfrac{+45eez^{\eta}-36efz^{2\eta}+32ffz^{3\eta}}{96\eta f^{3}}dP^{3}+\dfrac{15e^{3}}{32\eta f^{3}}d\,\text{①}.$
2.	$\dfrac{-5ez^{\eta}+4fz^{2\eta}}{8\eta ff}dP^{3}-\dfrac{5ee}{8\eta ff}d\,\text{①}.$
1.	$\dfrac{+z^{\eta}}{\eta f}dP^{3}+\dfrac{e}{\eta f}d\,\text{①}.$
0.	$\dfrac{-4}{\eta}d\,\text{①}.$
$\overline{1}$.	$\dfrac{-4}{3\eta e}dP^{3}.$
$\overline{2}$.	$\dfrac{-12e+16fz^{\eta}}{21\eta eez^{\eta}}dP^{3}.$
$\overline{3}$.	$\dfrac{-84ee+96efz^{\eta}-128ffz^{2\eta}}{231\eta e^{3}z^{2\eta}}dP^{3}.$
$\overline{4}$.	$\dfrac{-308e^{3}+336eefz^{\eta}-384effz^{2\eta}+512f^{3}z^{3\eta}}{1155\eta e^{4}z^{3\eta}}dP^{3}.$
&c.	

$$P=\sqrt[4]{\frac{e+fz^{\eta}}{z^{\eta}}}.$$

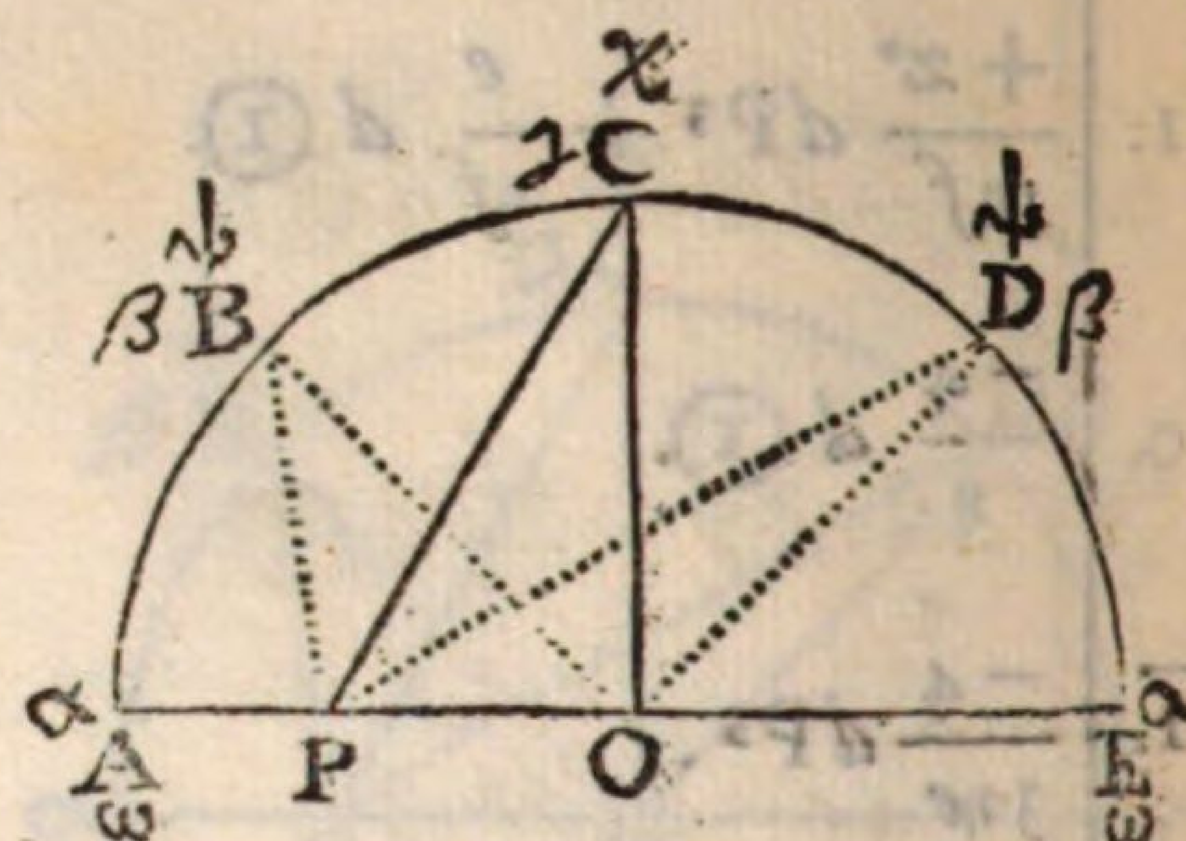
Cas. 1. $OP:OA::P:\sqrt[4]{f}$;

$$\text{①}=\tfrac{-1}{4}\sqrt[4]{\frac{1}{f}}\ \text{in}\ -(AP:AO)+(EP:EO)-\chi(PCO).$$

Cas. 2. $OP:OA::P:\sqrt[4]{-f}$;

$$\text{①}=\tfrac{+1}{4}\sqrt[4]{\frac{-1}{f}}\ \text{in}\ +\beta(BP:BO)-\beta(DP:DO)+\psi(PBO)+\psi(PDO).$$

θ	FORMA LVII. $dzz^{\theta n-1}\sqrt[4]{e+fz^n}$
&c.	
3.	$\frac{+128e^3-32eefz^n+20effz^{2n}+180f^3z^{3n}}{585\eta f^3}dP.$
2.	$\frac{-16ee+4efz^n+20ffz^{2n}}{45\eta ff}dP.$
1.	$\frac{+4e+4fz^n}{5\eta f}dP.$
0.	$\frac{+4}{\eta}dP+\frac{4e}{\eta}d\,\textcircled{I}.$
$\bar{1}$.	$\frac{-1}{\eta z^n}dP+\frac{f}{\eta}d\,\textcircled{I}.$
$\bar{2}$.	$\frac{-4e-fz^n}{8\eta ez^{2n}}dP-\frac{3ff}{8\eta e}d\,\textcircled{I}.$
$\bar{3}$.	$\frac{-32ee-4efz^n+7ffz^{2n}}{96\eta eez^{3n}}dP+\frac{7f^3}{32\eta ee}d\,\textcircled{I}.$
&c.	



$$P=\sqrt[4]{e+fz^n}.$$

Cas. 1. $OP:OA::P:\sqrt[4]{e}$;

$$\textcircled{I}=-\tfrac{1}{4}\sqrt[4]{\frac{1}{e^3}}\text{ in }-(AP:AO)+(EP:EO)+\chi(PCO).$$

Cas. 2. $OP:OA::P:\sqrt[4]{-e}$;

$$\textcircled{I}=+\tfrac{1}{4}\sqrt[4]{\frac{-1}{e^3}}\text{ in }-\beta(BP:BO)+\beta(DP:DO)+\psi(PBO)+\psi(PDO).$$

θ	FORMA LVIII. $dzz^{\theta n+\frac{3}{4}n-1}\sqrt[4]{e+fz^n}$
&c.	
2.	$\frac{-7eez^n+4efz^{2n}+32ffz^{3n}}{96\eta ff}dP - \frac{7e^3}{32\eta ff}d\,\textcircled{1}$.
1.	$\frac{+ez^n+4fz^{2n}}{8\eta f}dP + \frac{3ee}{8\eta f}d\,\textcircled{1}$.
0.	$\frac{+z^n}{\eta}dP - \frac{e}{\eta}d\,\textcircled{1}$.
$\overline{1}$.	$\frac{-4}{\eta}dP - \frac{4f}{\eta}d\,\textcircled{1}$.
$\overline{2}$.	$\frac{-4e-4fz^n}{5\eta ez^n}dP$.
$\overline{3}$.	$\frac{-20ee-4efz^n+16ffz^{2n}}{45\eta eez^{2n}}dP$.
$\overline{4}$.	$\frac{-180e^3-20effz^n+32eefz^{2n}-128f^3z^{3n}}{585\eta e^3z^{3n}}dP$.
&c.	

$$P=\sqrt[4]{\frac{e+fz^n}{z^n}}.$$

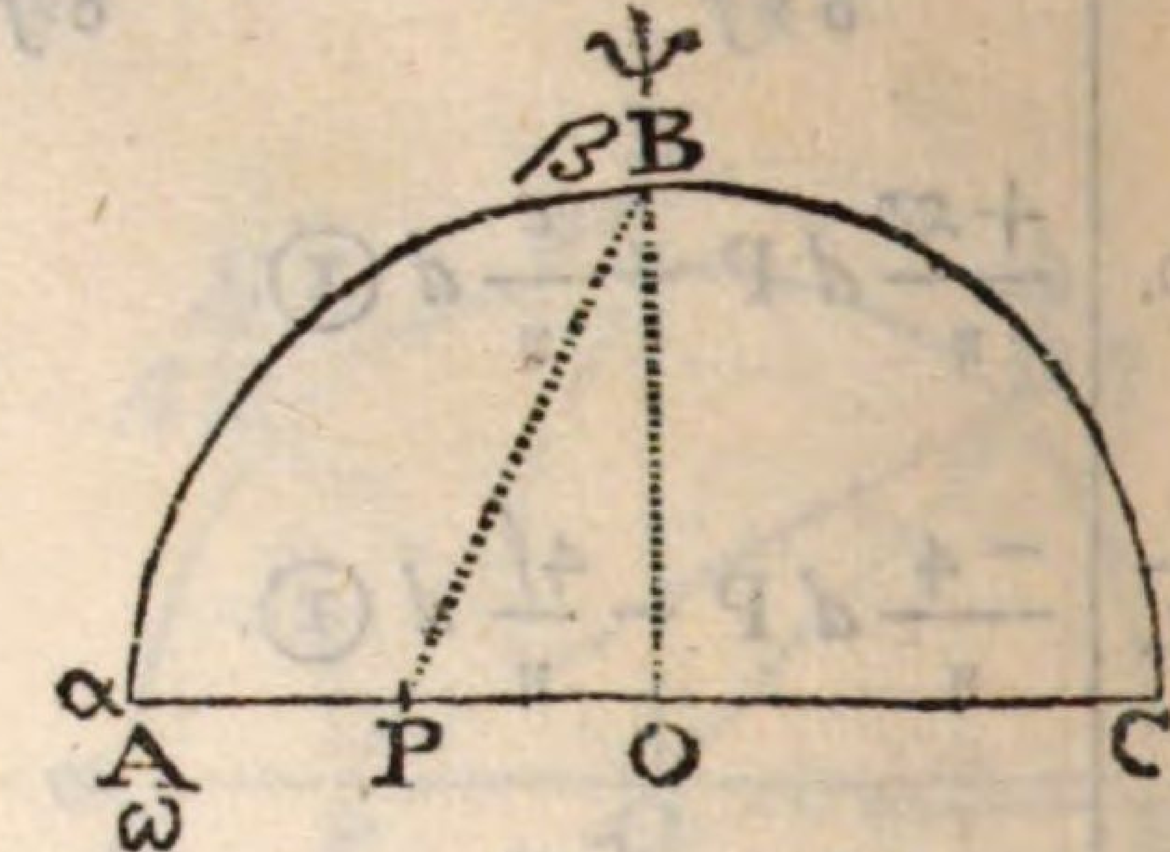
Cas. 1. $OP:OA::P:\sqrt[4]{f}$;

$$\textcircled{1}=-\tfrac{1}{4}\sqrt[4]{\frac{1}{f^3}}\text{ in }-(AP:AO)+(EP:EO)+\chi(PCO).$$

Cas. 2. $OP:OA::P:\sqrt[4]{-f}$;

$$\textcircled{1}=+\tfrac{1}{4}\sqrt[4]{\frac{-1}{f^3}}\text{ in }-\beta(BP:BO)+\beta(DP:DO)+\psi(PBO)+\psi(PDO).$$

θ	FORMA LIX. $\dfrac{d\dot{z}z^{\theta n-1}}{\overline{g+hz^n}\sqrt[2]{e+fz^n}}$
$\overline{}$ &c.	
3.	$\dfrac{\begin{matrix}-6fg\\-4eh+2fhz^n\end{matrix}}{3\eta ffhh}dP+\dfrac{2gg}{\eta h^3}d\,\textcircled{1}.$
2.	$\dfrac{+2}{\eta fh}dP-\dfrac{2g}{\eta hh}d\,\textcircled{1}.$
1.	$\dfrac{+2}{\eta h}d\,\textcircled{1}.$
0.	$\dfrac{-2}{\eta g}d\,\textcircled{1}-\dfrac{2}{\eta g}d\,\textcircled{2}.$
$\bar{1}$.	$\dfrac{-1}{\eta egz^n}dP+\dfrac{2h}{\eta gg}d\,\textcircled{1}+\dfrac{\begin{matrix}+fg\\2eh\end{matrix}}{\eta egg}d\,\textcircled{2}.$
&c.	



$$P=\sqrt[2]{e+fz^n},\quad p=fg-eh.$$

$$Cas.\ 1.\begin{cases}OP:OA::P:\sqrt[2]{\dfrac{p}{h}};\quad \textcircled{1}=+\tfrac{1}{2}\sqrt[2]{\dfrac{h}{p}}\ \text{in}\ \psi(PBO).\\ OP:OA::P:\sqrt[2]{e};\quad \textcircled{2}=+\tfrac{1}{2}\sqrt[2]{\dfrac{1}{e}}\ \text{in}-(AP:AO)+\\ \quad (CP:CO).\end{cases}$$

$$Cas.\ 2.\begin{cases}OP::OA::P:\sqrt[2]{\dfrac{-p}{h}};\quad \textcircled{1}=-\tfrac{1}{2}\sqrt[2]{\dfrac{-h}{p}}\ \text{in}-(AP:AO)+\\ \quad (CP:CO).\\ OP:OA::P:\sqrt[2]{-e};\quad \textcircled{2}=-\tfrac{1}{2}\sqrt[2]{\dfrac{-1}{e}}\ \text{in}\ \psi(PBO).\end{cases}$$

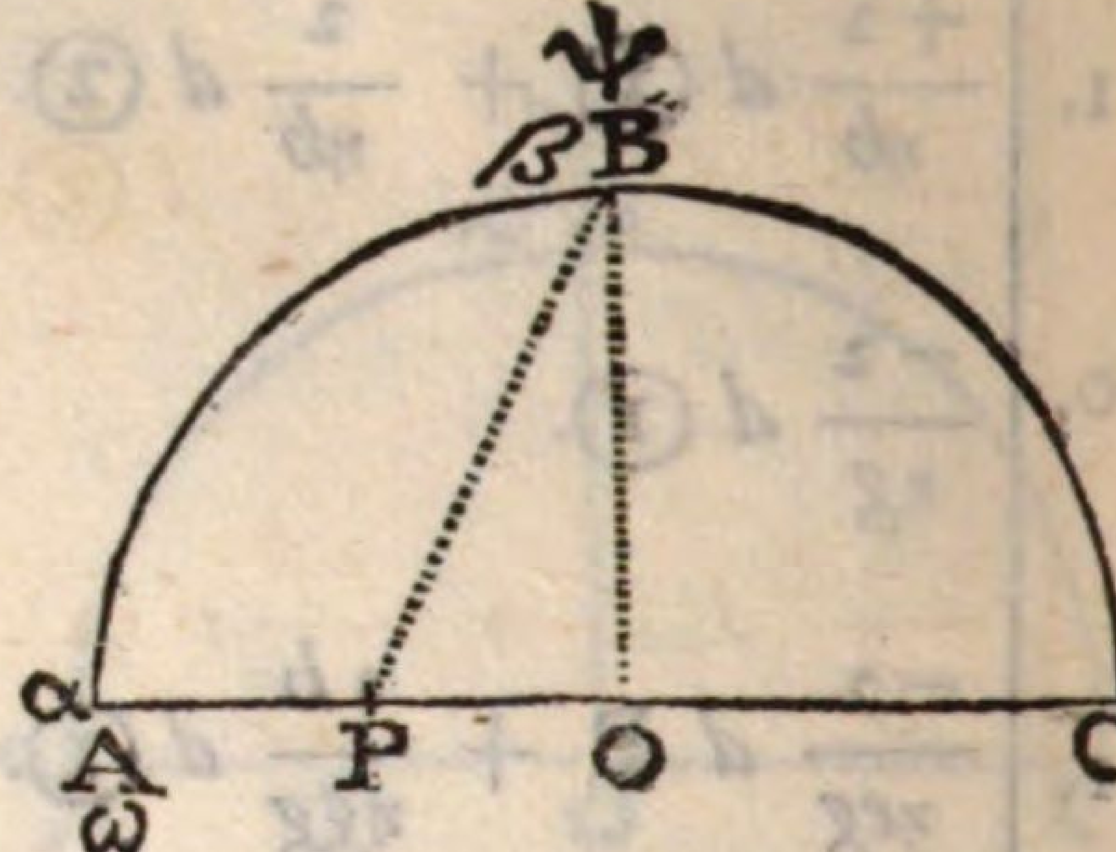
θ	FORMA LX. $\dfrac{d\dot{z}\, z^{\theta\eta+\frac{1}{2}\eta-1}}{\overline{g+hz^{\eta}}\,\sqrt[2]{e+fz^{\eta}}}$
&c.	
2.	$\dfrac{+z^{\eta}}{\eta fh}\,dP - \dfrac{2g}{\eta hh}\,d\textcircled{1} + \dfrac{\begin{matrix}-eh\\-2fg\end{matrix}}{\eta fhh}\,d\textcircled{2}.$
1.	$\dfrac{+2}{\eta h}\,d\textcircled{1} + \dfrac{2}{\eta h}\,d\textcircled{2}.$
0.	$\dfrac{-2}{\eta g}\,d\textcircled{1}.$
$\overline{1}$.	$\dfrac{-2}{\eta eg}\,dP + \dfrac{2h}{\eta gg}\,d\textcircled{1}.$
$\overline{2}$.	$\dfrac{\begin{matrix}+6ehz^{\eta}\\-2eg+4fgz^{\eta}\end{matrix}}{3\eta eeggz^{\eta}}\,dP - \dfrac{2hh}{\eta gg}\,d\textcircled{1}.$
&c.	

$$P=\sqrt[2]{\frac{e+fz^{\eta}}{z^{\eta}}}.\quad p=eh-fg.$$

Cas. 1. $\left\{\begin{array}{l} OP:OA::P:\sqrt[2]{\dfrac{p}{g}};\quad \textcircled{1}=+\tfrac{1}{2}\sqrt[2]{\dfrac{g}{p}}\ \text{in}\ \downarrow(PBO).\\ OP:OA::P:\sqrt[2]{f};\quad \textcircled{2}=+\tfrac{1}{2}\sqrt[2]{\dfrac{1}{f}}\ \text{in}-(AP:AO)+\\ \quad (CP:CO).\end{array}\right.$

Cas. 2. $\left\{\begin{array}{l} OP:OA::P:\sqrt[2]{\dfrac{-p}{g}};\quad \textcircled{1}=-\tfrac{1}{2}\sqrt[2]{\dfrac{-g}{p}}\ \text{in}-(AP:AO)+\\ \quad (CP:CO).\\ OP:OA::P:\sqrt[2]{-f};\quad \textcircled{2}=-\tfrac{1}{2}\sqrt[2]{\dfrac{-1}{f}}\ \text{in}\ \downarrow(PBO).\end{array}\right.$

θ	FORMA LXI. $\dfrac{d\dot{z}z^{\theta\eta-1}\sqrt[2]{e+fz^{\eta}}}{g+hz^{\eta}}$
&c.	
2.	$\dfrac{\begin{matrix}-6fg\\+2eh+2fhz^{\eta}\end{matrix}}{3\eta fhh}\,d\mathrm{P}+\dfrac{2gp}{\eta h^{3}}\,d①.$
1.	$\dfrac{+2}{\eta h}\,d\mathrm{P}-\dfrac{2p}{\eta hh}\,d①.$
0.	$\dfrac{+2p}{\eta gh}\,d①-\dfrac{2e}{\eta g}\,d②.$
$\overline{1}$.	$\dfrac{-1}{\eta gz^{\eta}}\,d\mathrm{P}-\dfrac{2p}{\eta gg}\,d①+\dfrac{\begin{matrix}-fg\\+2eh\end{matrix}}{\eta gg}\,d②.$
&c.	



$$\mathrm{P}=\sqrt[2]{e+fz^{\eta}},\quad p=fg-eh.$$

Caf. 1.
$OP:OA::\mathrm{P}:\sqrt[2]{\dfrac{p}{h}}$; $① = +\tfrac{1}{2}\sqrt[2]{\dfrac{h}{p}}$ in $\psi\,(PBO)$.
$OP:OA::\mathrm{P}:\sqrt[2]{e}$; $② = +\tfrac{1}{2}\sqrt[2]{\dfrac{1}{e}}$ in $-\,(AP:AO)+(CP:CO)$.

Caf. 2.
$OP::OA::\mathrm{P}:\sqrt[2]{\dfrac{-p}{h}}$; $① = -\tfrac{1}{2}\sqrt[2]{\dfrac{-h}{p}}$ in $-(AP:AO)+(CP:CO)$.
$OP:OA::\mathrm{P}:\sqrt[2]{-e}$; $② = -\tfrac{1}{2}\sqrt[2]{\dfrac{-1}{e}}$ in $\psi(PBO)$.

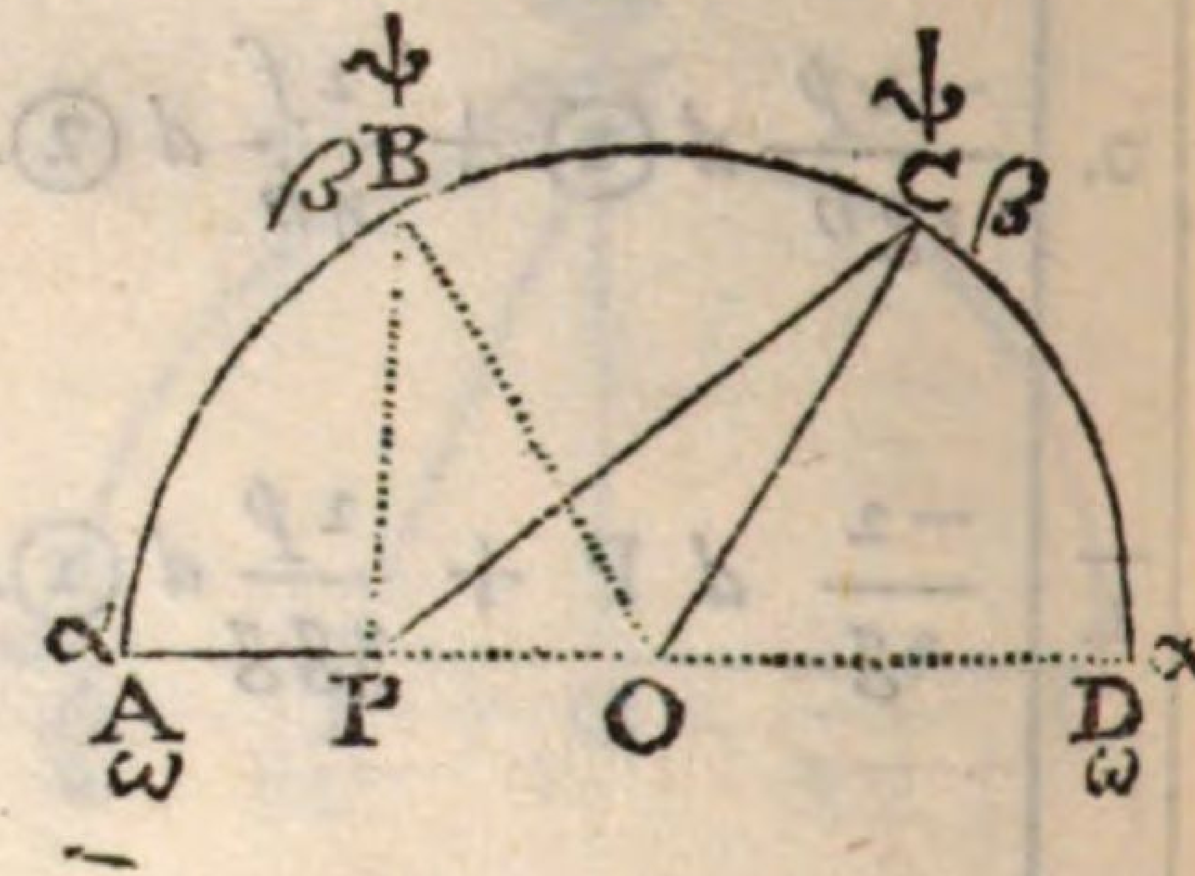
θ	FORMA LXII. $\dfrac{d\dot{z}z^{\theta\eta+\frac{1}{2}\eta-1}\sqrt[2]{e+fz^{\eta}}}{g+hz^{\eta}}$
&c.	
1.	$\dfrac{+z^{\eta}}{\eta h}d\mathrm{P}+\dfrac{2p}{\eta hh}d\textcircled{1}-\dfrac{\begin{matrix}-eh\\+2fg\end{matrix}}{\eta hh}d\textcircled{2}.$
0.	$\dfrac{-2p}{\eta gh}d\textcircled{1}+\dfrac{2f}{\eta h}d\textcircled{2}.$
$\overline{1}$.	$\dfrac{-2}{\eta g}d\mathrm{P}+\dfrac{2p}{\eta gg}d\textcircled{1}.$
$\overline{2}$.	$\dfrac{\begin{matrix}+6ehz^{\eta}\\-2eg-2fgz^{\eta}\end{matrix}}{3\eta eggz^{\eta}}d\mathrm{P}-\dfrac{2hp}{\eta g^{3}}d\textcircled{1}.$
&c.	

$$\mathrm{P}=\sqrt[2]{\frac{e+fz^{\eta}}{z^{\eta}}}.\quad p=eh-fg.$$

Cas. 1.
$$\left\{\begin{array}{l} OP:OA::\mathrm{P}:\sqrt[2]{\frac{p}{g}};\quad \textcircled{1}=+\tfrac{1}{2}\sqrt[2]{\frac{g}{p}}\ \text{in}\ \downarrow(PBO).\\ OP:OA::\mathrm{P}:\sqrt[2]{f};\quad \textcircled{2}=+\tfrac{1}{2}\sqrt[2]{\frac{1}{f}}\ \text{in}-(AP:AO)+\\ (CP:CO).\end{array}\right.$$

Cas. 2.
$$\left\{\begin{array}{l} OP:OA::\mathrm{P}:\sqrt[2]{\frac{-p}{g}};\quad \textcircled{1}=-\tfrac{1}{2}\sqrt[2]{\frac{-g}{p}}\ \text{in}-(AP:AO)+\\ (CP:CO).\\ OP:OA::\mathrm{P}:\sqrt[2]{-f};\quad \textcircled{2}=-\tfrac{1}{2}\sqrt[2]{\frac{-1}{f}}\ \text{in}\ \downarrow(PBO).\end{array}\right.$$

θ	FORMA LXIII. $\dfrac{d\dot{z}z^{\theta n-1}}{g+hz^n\,\sqrt[3]{e+fz^n}}$
&c.	
3.	$\dfrac{-15fg-9eh+6fhz^n}{10\eta ffhh}\,d\,PP+\dfrac{3gg}{\eta h^3}\,d\,\textcircled{1}$.
2.	$\dfrac{+3}{2\eta fh}\,d\,PP-\dfrac{3g}{\eta hh}\,d\,\textcircled{1}$.
1.	$\dfrac{+3}{\eta h}\,d\,\textcircled{1}$.
0.	$\dfrac{-3}{\eta g}\,d\,\textcircled{1}-\dfrac{3}{\eta g}\,d\,\textcircled{2}$.
$\overline{1}$.	$\dfrac{-1}{\eta egz^n}\,d\,PP+\dfrac{3h}{\eta gg}\,d\,\textcircled{1}+\dfrac{+fg+3eh}{\eta egg}\,d\,\textcircled{2}$.
&c.	



$$P=\sqrt[3]{e+fz^n}.\quad p=fg-eh.$$

Cas. 1.

$$OP:OA::P:\sqrt[3]{\tfrac{p}{h}};\quad \textcircled{1}=+\tfrac{1}{3}\sqrt[3]{\tfrac{h}{p}}\ \text{in}\ +\beta(BP:BO)-(DP:DO)+\psi(PBO).$$

$$OP:OA::P:\sqrt[3]{e};\quad \textcircled{2}=+\tfrac{1}{3}\sqrt[3]{\tfrac{1}{e}}\ \text{in}\ -(AP:AO)+\beta(CP:CO)-\psi(PCO).$$

Cas. 2.

$$OP:OA::P:\sqrt[3]{\tfrac{-p}{h}};\quad \textcircled{1}=-\tfrac{1}{3}\sqrt[3]{\tfrac{-h}{p}}\ \text{in}\ -(AP:AO)+\beta(CP:CO)-\psi(PCO).$$

$$OP:OA::P:\sqrt[3]{-e};\quad \textcircled{2}=-\tfrac{1}{3}\sqrt[3]{\tfrac{-1}{e}}\ \text{in}\ +\beta(BP:BO)-(DP:DO)+\psi(PBO).$$

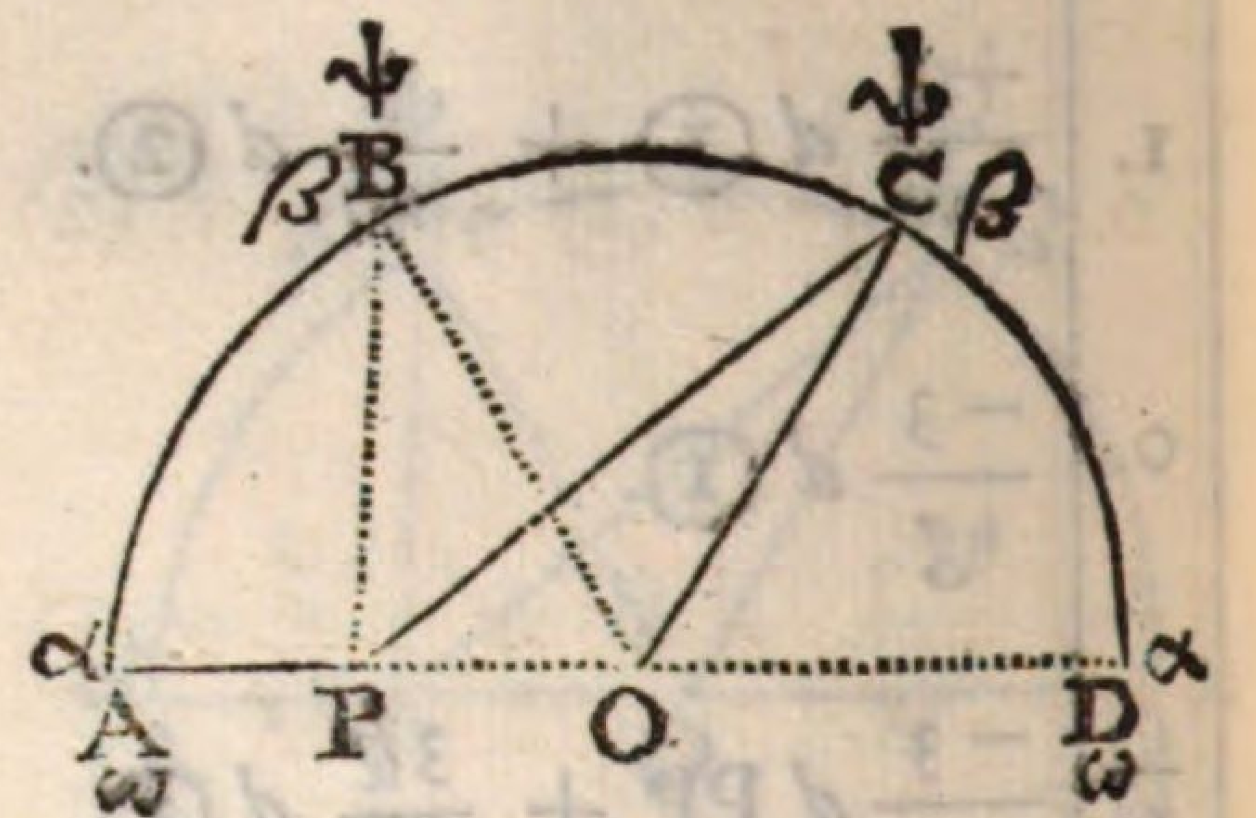
θ	FORMA LXIV. $\dfrac{dz\,z^{\theta\eta+\frac{1}{3}\eta-1}}{\overline{g+hz^{\eta}}\,\sqrt[3]{e+fz^{\eta}}}$
&c.	
2.	$\dfrac{+z^{\eta}}{\eta fh}\,d\,PP-\dfrac{3g}{\eta hh}\,d\,\textcircled{1}+\dfrac{\begin{matrix}-eh\\-3fg\end{matrix}}{\eta fhh}\,d\,\textcircled{2}.$
1.	$\dfrac{+3}{\eta h}\,d\,\textcircled{1}+\dfrac{3}{\eta h}\,d\,\textcircled{2}.$
0.	$\dfrac{-3}{\eta g}\,d\,\textcircled{1}.$
$\overline{1}$.	$\dfrac{-3}{2\eta eg}\,d\,PP+\dfrac{3h}{\eta gg}\,d\,\textcircled{1}.$
$\overline{2}$.	$\dfrac{\begin{matrix}+15ehz^{\eta}\\-6eg+9fgz^{\eta}\end{matrix}}{10\eta eeggz^{\eta}}\,d\,PP-\dfrac{3hh}{\eta g^{3}}\,d\,\textcircled{1}.$
&c.	

$$P=\sqrt[3]{\frac{e+fz^{\eta}}{z^{\eta}}}.\quad p=eh-fg.$$

Cas. 1.
$$\left\{\begin{aligned}&OP:OA::P:\sqrt[3]{\frac{p}{g}};\quad \textcircled{1}=\frac{+1}{3}\sqrt[3]{\frac{g}{p}}\text{ in }+\beta(BP:BO)\\&\qquad-(DP:DO)+\psi(PBO).\\&OP:OA::P:\sqrt[3]{f};\quad \textcircled{2}=\frac{+1}{3}\sqrt[3]{\frac{1}{f}}\text{ in }-(AP:AO)\\&\qquad+\beta(CP:CO)-\psi(PCO).\end{aligned}\right.$$

Cas. 2.
$$\left\{\begin{aligned}&OP:OA::P:\sqrt[3]{\frac{-p}{g}};\quad \textcircled{1}=\frac{-1}{3}\sqrt[3]{\frac{-g}{p}}\text{ in }-(AP:AO)\\&\qquad+\beta(CP:CO)-\psi(PCO).\\&OP:OA::P:\sqrt[3]{-f};\quad \textcircled{2}=\frac{-1}{3}\sqrt[3]{\frac{-1}{f}}\text{ in }+\beta(BP:BO)\\&\qquad-(DP:DO)+\psi(PBO).\end{aligned}\right.$$

θ	FORMA LXV. $\dfrac{d\dot{z}z^{\theta\eta-1}\sqrt[3]{e+fz^{\eta}}}{g+hz^{\eta}}$
&c.	
2.	$\dfrac{-12fg+3eh+3fhz^{\eta}}{4\eta fhh}dP+\dfrac{3gp}{\eta h^{3}}d\textcircled{1}.$
1.	$\dfrac{+3}{\eta h}dP-\dfrac{3p}{\eta hh}d\textcircled{1}.$
0.	$\dfrac{+3p}{\eta gh}d\textcircled{1}-\dfrac{3e}{\eta g}d\textcircled{2}.$
$\bar{1}$.	$\dfrac{-1}{\eta gz^{\eta}}dP-\dfrac{3p}{\eta gg}d\textcircled{1}+\dfrac{-fg+3eh}{\eta gg}d\textcircled{2}.$
&c.	



$$P=\sqrt[3]{e+fz^{\eta}},\quad p=fg-eh.$$

Cas. 1.

$$OP:OA::P:\sqrt[3]{\frac{p}{h}};\quad \textcircled{1}=+\tfrac{1}{3}\sqrt[3]{\frac{hh}{pp}}\ \text{in}\ -\beta(BP:BO)+(DP:DO)+\psi(PBO).$$

$$OP:OA::P:\sqrt[3]{e};\quad \textcircled{2}=+\tfrac{1}{3}\sqrt[3]{\frac{1}{ee}}\ \text{in}\ -(AP:AO)+\beta(CP:CO)+\psi(PCO).$$

Cas. 2.

$$OP:OA::P:\sqrt[3]{\frac{-p}{h}};\quad \textcircled{1}=-\tfrac{1}{3}\sqrt[3]{\frac{hh}{pp}}\ \text{in}\ -(AP:AO)+\beta(CP:CO)+\psi(PCO).$$

$$OP:OA::P:\sqrt[3]{-e};\quad \textcircled{2}=-\tfrac{1}{3}\sqrt[3]{\frac{1}{ee}}\ \text{in}\ -\beta(BP:BO)+(DP:DO)+\psi(PBO).$$

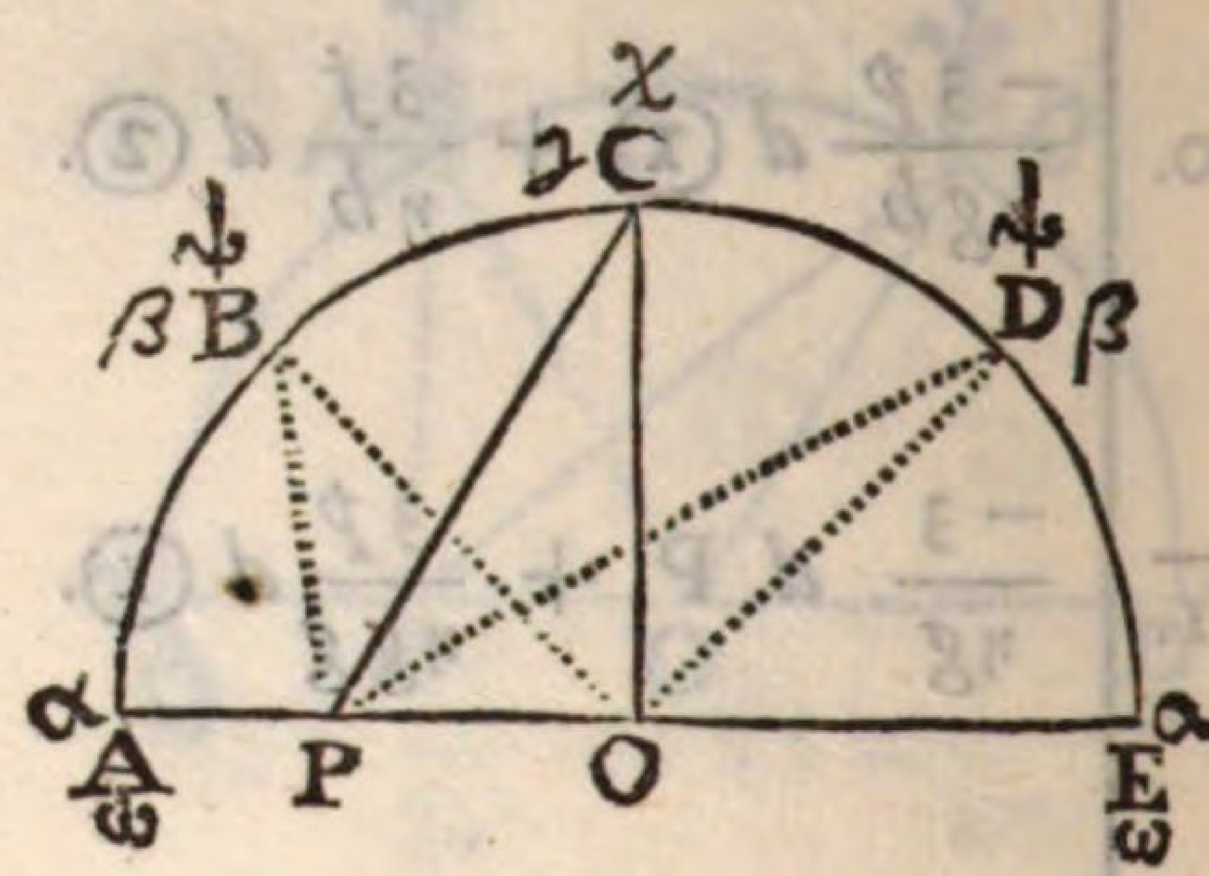
θ	FORMA LXVI. $\dfrac{dzz^{\theta n+\frac{2}{3}n-1}\sqrt[3]{e+fz^n}}{g+hz^n}$
&c.	
1.	$\dfrac{+z^n}{\eta h}dP+\dfrac{3p}{\eta hh}d\textcircled{1}-\dfrac{\overset{-eh}{+3fg}}{\eta hh}d\textcircled{2}.$
0.	$\dfrac{-3p}{\eta gh}d\textcircled{1}+\dfrac{3f}{\eta h}d\textcircled{2}.$
$\bar{1}$.	$\dfrac{-3}{\eta g}dP+\dfrac{3p}{\eta gg}d\textcircled{1}.$
$\bar{2}$.	$\dfrac{\overset{+12ehz^n}{-3eg-3fgz^n}}{4\eta eggz^n}dP-\dfrac{3hp}{\eta g^3}d\textcircled{1}.$
&c.	

$$P=\sqrt[3]{\frac{e+fz^n}{z^n}}.\quad p=eh-fg.$$

Cas. 1.
$$\left\{\begin{array}{l} OP:OA::P:\sqrt[3]{\dfrac{p}{g}};\quad \textcircled{1}=\overset{+}{}\tfrac{1}{3}\sqrt[3]{\dfrac{gg}{pp}}\ \text{in}\ -\beta(BP:BO) \\ \quad +(DP:DO)+\psi(PBO). \\ OP:OA::P:\sqrt[3]{f};\quad \textcircled{2}=\overset{+}{}\tfrac{1}{3}\sqrt[3]{\dfrac{1}{ff}}\ \text{in}\ -(AP:AO) \\ \quad +\beta(CP:CO)+\psi(PCO). \end{array}\right.$$

Cas. 2.
$$\left\{\begin{array}{l} OP:OA::P:\sqrt[3]{\dfrac{-p}{g}};\quad \textcircled{1}=\overset{-}{}\tfrac{1}{3}\sqrt[3]{\dfrac{gg}{pp}}\ \text{in}\ -(AP:AO) \\ \quad +\beta(CP:CO)+\psi(PCO). \\ OP:OA::P:\sqrt[3]{-f};\quad \textcircled{2}=\overset{-}{}\tfrac{1}{3}\sqrt[3]{\dfrac{1}{ff}}\ \text{in}\ -\beta(BP:BO) \\ \quad +(DP:DO)+\psi(PBO). \end{array}\right.$$

θ	FORMA LXVII. $\dfrac{d\dot{z}z^{\theta n-1}}{g+hz^n\sqrt[4]{e+fz^n}}$
&c.	
3.	$\dfrac{-28fg-16eh+12fhz^n}{21\eta ffhh}dP^3+\dfrac{4gg}{\eta h^3}d\textcircled{1}.$
2.	$\dfrac{+14}{3\eta fh}dP^3-\dfrac{4g}{\eta hh}d\textcircled{1}.$
1.	$\dfrac{+4}{\eta h}d\textcircled{1}.$
0.	$\dfrac{-4}{\eta g}d\textcircled{1}-\dfrac{4}{\eta g}d\textcircled{2}.$
$\overline{1}$.	$\dfrac{-1}{\eta egz^n}dP^3+\dfrac{4h}{\eta gg}d\textcircled{1}+\dfrac{\substack{+fg\\4eh}}{\eta egg}d\textcircled{2}.$
&c.	



$$P=\sqrt[4]{e+fz^n}.\quad p=fg-eh.$$

Cas. 1.

$$OP:OA::P:\sqrt[4]{\frac{p}{h}};\quad \textcircled{1}=\frac{+1}{4}\sqrt[4]{\frac{h}{p}}\text{ in }+\beta(BP:BO)-\beta(DP:DO)+\psi(PBO)+\psi(PDO).$$

$$OP:OA::P:\sqrt[4]{e};\quad \textcircled{2}=\frac{+1}{4}\sqrt[4]{\frac{1}{e}}\text{ in }-(AP:AO)+(EP:EO)-\chi(PCO).$$

Cas. 2.

$$OP:OA::P:\sqrt[4]{\frac{-p}{h}};\quad \textcircled{1}=\frac{-1}{4}\sqrt[4]{\frac{-h}{p}}\text{ in }-(AP:AO)+(EP:EO)-\chi(PCO).$$

$$OP:OA::P:\sqrt[4]{-e};\quad \textcircled{2}=\frac{-1}{4}\sqrt[4]{\frac{-1}{e}}\text{ in }+\beta(BP:BO)-\beta(DP:DO)+\psi(PBO)+\psi(PDO).$$

θ	FORMA LXVIII. $\dfrac{dz\,z^{\theta n+\frac{1}{4}n-1}}{g+hz^n\sqrt[4]{e+fz^n}}$
&c.	
2.	$\dfrac{+z^n}{\eta fh}\,dP^3-\dfrac{4g}{\eta hh}\,d\,\textcircled{1}+\dfrac{-eh-4fg}{\eta fhh}\,d\,\textcircled{2}.$
1.	$\dfrac{+4}{\eta h}\,d\,\textcircled{1}+\dfrac{4}{\eta h}\,d\,\textcircled{2}.$
0.	$\dfrac{-4}{\eta g}\,d\,\textcircled{1}.$
$\overline{1}$.	$\dfrac{-4}{3\eta eg}\,dP^3+\dfrac{4h}{\eta gg}\,d\,\textcircled{1}.$
$\overline{2}$.	$\dfrac{+28ehz^n-12eg+16fgz^n}{21\eta eegg\,z^n}\,dP^3-\dfrac{4hh}{\eta g^3}\,d\,\textcircled{1}.$
&c.	

$$P=\sqrt[3]{\frac{e+fz^n}{z^n}}.\quad p=eh-fg.$$

Cas. 1.

$OP:OA::P:\sqrt[4]{\frac{p}{g}}$; $\textcircled{1}=+\frac{1}{4}\sqrt[4]{\frac{g}{p}}$ in $+\beta(PB:OB)-\beta(DP:DO)+\psi(PBO)+\psi(PDO)$.

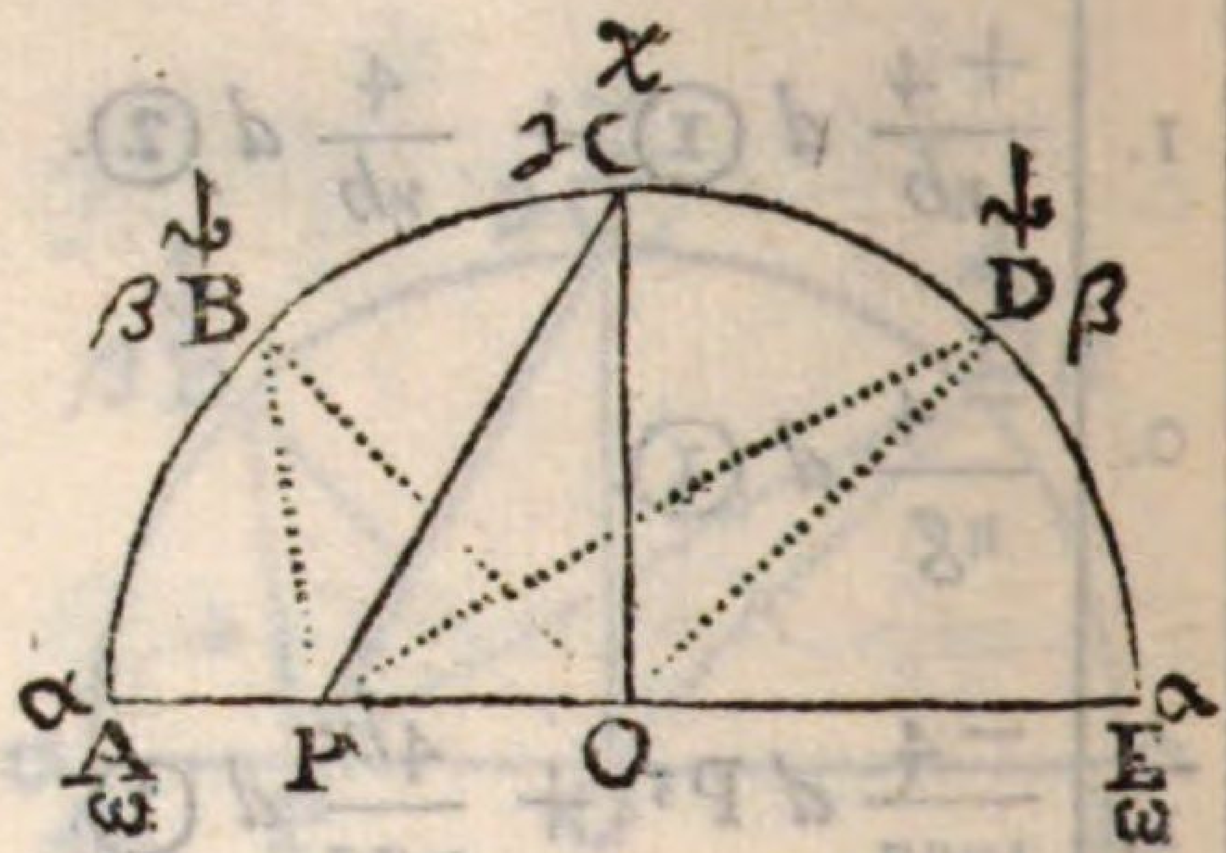
$OP:OA::P:\sqrt[4]{f}$; $\textcircled{2}=+\frac{1}{4}\sqrt[4]{\frac{1}{f}}$ in $-(AP:AO)+(EP:EO)-\chi(PCO)$.

Cas. 2.

$OP:OA::P:\sqrt[4]{\frac{-p}{g}}$; $\textcircled{1}=-\frac{1}{4}\sqrt[4]{\frac{-g}{p}}$ in $-(AP:AO)+(EP:EO)-\chi(PCO)$.

$OP:OA::P:\sqrt[4]{-f}$; $\textcircled{2}=-\frac{1}{4}\sqrt[4]{\frac{-1}{f}}$ in $+\beta(BP:BO)-\beta(DP:DO)+\psi(PBO)+\psi(PDO)$.

θ	FORMA LXIX. $\frac{dz z^{\theta n-1}\sqrt[4]{e+fz^n}}{g+hz^n}$
&c.	
2.	$\frac{-20fg+4eh+4fhz^n}{5\eta fhh}dP + \frac{4gp}{\eta h^3}d\textcircled{1}.$
1.	$\frac{+4}{\eta h}dP - \frac{4p}{\eta hh}d\textcircled{1}.$
0.	$\frac{+4p}{\eta gh}d\textcircled{1} - \frac{4e}{\eta g}d\textcircled{2}.$
$\overline{1}$.	$\frac{-1}{\eta g z^n}dP - \frac{4p}{\eta gg}d\textcircled{1} + \frac{-fg+4eh}{\eta gg}d\textcircled{2}.$
&c.	



$$P=\sqrt[4]{e+fz^n}. \quad p=fg-eh.$$

Cas. 1.
$$OP:OA::P:\sqrt[4]{\frac{p}{h}};\quad \textcircled{1}=\frac{+1}{4}\sqrt[4]{\frac{h^3}{p^3}} \text{ in } -\beta(BP:BO) +\beta(DP:DO) +\psi(PBO) +\psi(PDO).$$
$$OP:OA::P:\sqrt[4]{e};\quad \textcircled{2}=\frac{+1}{4}\sqrt[4]{\frac{1}{e^3}} \text{ in } -(AP:AO) +(EP:EO) +\chi(PCO).$$

Cas. 2.
$$OP:OA::P:\sqrt[4]{\frac{-p}{h}};\quad \textcircled{1}=\frac{-1}{4}\sqrt[4]{\frac{-h^3}{p^3}} \text{ in } -(AP:AO) +(EP:EO) +\chi(PCO).$$
$$OP:OA::P:\sqrt[4]{-e};\quad \textcircled{2}=\frac{-1}{4}\sqrt[4]{\frac{-1}{e^3}} \text{ in } -\beta(BP:BO) +\beta(DP:DO) +\psi(PBO) +\psi(PDO).$$

θ	FORMA LXX. $\dfrac{dz\,z^{\theta\eta+\frac{1}{4}\eta-1}\sqrt[4]{e+fz^{\eta}}}{g+hz^{\eta}}$
&c.	
1.	$\dfrac{+z^{\eta}}{\eta h}\,d\,\mathrm{P}+\dfrac{4p}{\eta hh}\,d\,\textcircled{1}-\dfrac{\begin{matrix}-eh\\+4fg\end{matrix}}{\eta hh}\,d\,\textcircled{2}.$
0.	$\dfrac{-4p}{\eta gh}\,d\,\textcircled{1}+\dfrac{4f}{\eta h}\,d\,\textcircled{2}.$
$\bar{1}.$	$\dfrac{-4}{\eta g}\,d\,\mathrm{P}+\dfrac{4p}{\eta gg}\,d\,\textcircled{1}.$
$\bar{2}.$	$\dfrac{\begin{matrix}+20ehz^{\eta}\\-4eg-4fgz^{\eta}\end{matrix}}{5\eta eggz^{\eta}}\,d\,\mathrm{P}-\dfrac{4hp}{\eta g^{3}}\,d\,\textcircled{1}.$
&c.	

$$\mathrm{P}=\sqrt[4]{\frac{e+fz^{\eta}}{z^{\eta}}}.\quad p=eh-fg.$$

Cas. 1.

$$OP:OA::\mathrm{P}:\sqrt[4]{\frac{p}{g}};\quad \textcircled{1}=+\tfrac{1}{4}\sqrt[4]{\frac{g^{3}}{p^{3}}}\ \text{in}\ -\beta(BP:BO)+\beta(DP:DO)+\psi(PBO)+\psi(PDO).$$

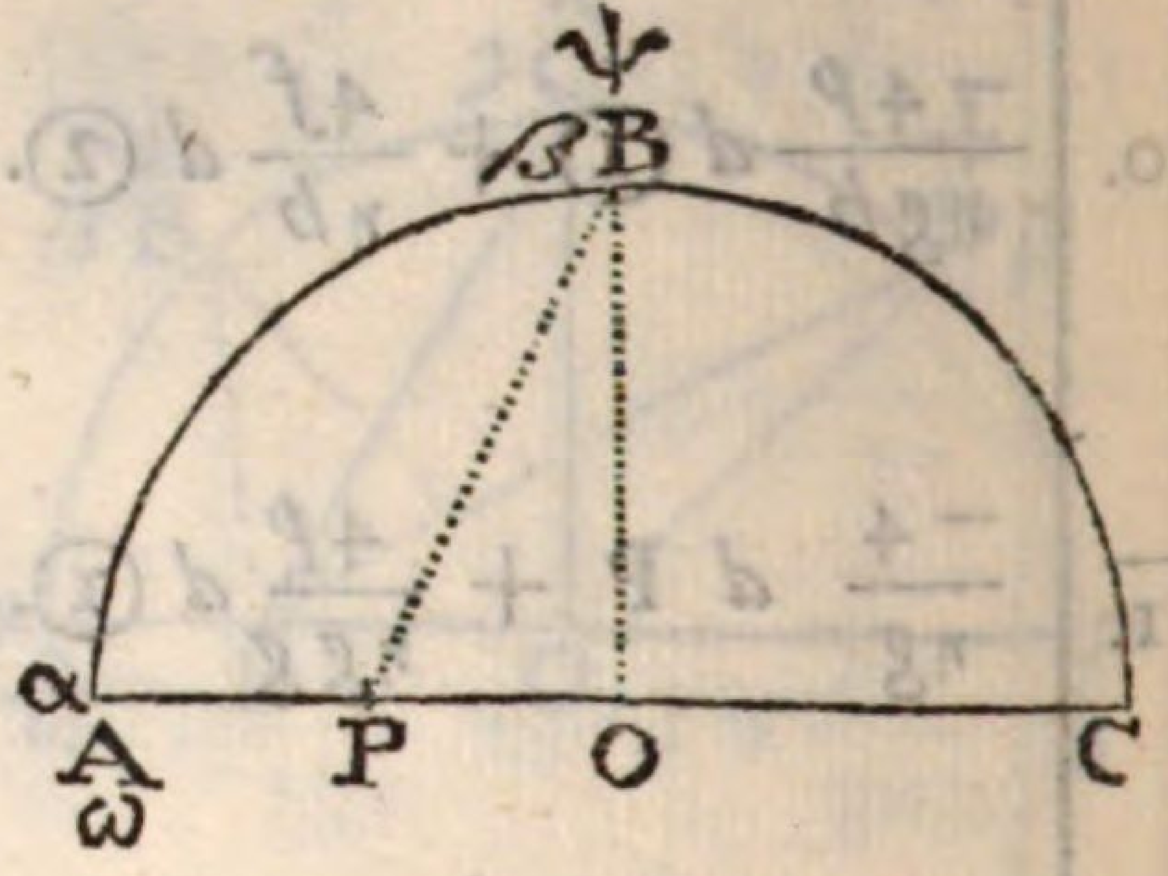
$$OP:OA::\mathrm{P}:\sqrt[4]{f};\quad \textcircled{2}=+\tfrac{1}{4}\sqrt[4]{\frac{1}{f^{3}}}\ \text{in}\ -(AP:AO)+(EP:EO)+\chi(PCO).$$

Cas. 2.

$$OP:OA::\mathrm{P}:\sqrt[4]{\frac{-p}{g}};\quad \textcircled{1}=-\tfrac{1}{4}\sqrt[4]{\frac{-g^{3}}{p^{3}}}\ \text{in}\ -(AP:AO)+(EP:EO)+\chi(PCO).$$

$$OP:OA::\mathrm{P}:\sqrt[4]{-f};\quad \textcircled{2}=-\tfrac{1}{4}\sqrt[4]{\frac{-1}{f^{3}}}\ \text{in}\ -\beta(BP:BO)+\beta(DP:DO)+\psi(PBO)+\psi(PDO).$$

θ	FORMA LXXI. $\dfrac{d\dot{z}z^{\theta\eta-1}}{e+fz^{\eta}+gz^{2\eta}\sqrt[2]{k+lz^{\eta}}}$ Cas. 1.
—	
&c.	
3.	$\dfrac{2}{\eta g l}\,dP - \dfrac{\overline{+fl+a}\vert^{2}}{2\eta ggla}\,d\,\textcircled{1} + \dfrac{\overline{fl-a}\vert^{2}}{2\eta ggla}\,d\,\textcircled{2}.$
2.	$\dfrac{a+fl}{\eta g a}\,d\,\textcircled{1} + \dfrac{a-fl}{\eta g a}\,d\,\textcircled{2}.$
1.	$\dfrac{-2l}{\eta a}\,d\,\textcircled{1} + \dfrac{2l}{\eta a}\,d\,\textcircled{2}.$
0.	$\dfrac{-2}{\eta e}\,d\,\textcircled{0} + \dfrac{fl-a}{\eta e a}\,d\,\textcircled{1} - \dfrac{+fl+a}{\eta e a}\,d\,\textcircled{2}.$
1, &c.	



$P=\sqrt[2]{k+lz^{\eta}}.\quad a=\sqrt[2]{ffll-4egll}.\quad p=\tfrac{1}{2}fl-gk+\tfrac{1}{2}a.\quad q=\tfrac{1}{2}fl-gk-\tfrac{1}{2}a.$

Cas. 1.

$OP:OA::P:\sqrt[2]{k}$; $\textcircled{0}={}^{+}\tfrac{1}{2}\sqrt[2]{\dfrac{1}{k}}$ in $-(AP:AO)+(CP:CO)$.

$OP:OA::P:\sqrt[2]{\dfrac{p}{g}}$; $\textcircled{1}={}^{+}\tfrac{1}{2}\sqrt[2]{\dfrac{g}{p}}$ in $\psi(PBO)$.

$OP:OA::P:\sqrt[2]{\dfrac{q}{g}}$; $\textcircled{2}={}^{+}\tfrac{1}{2}\sqrt[2]{\dfrac{g}{q}}$ in $\psi(PBO)$.

Cas. 2.

$OP:OA::P:\sqrt[2]{-k}$; $\textcircled{0}={}^{-}\tfrac{1}{2}\sqrt[2]{\dfrac{-1}{k}}$ in $\psi(PBO)$.

$OP::OA::P:\sqrt[2]{\dfrac{-p}{g}}$; $\textcircled{1}={}^{-}\tfrac{1}{2}\sqrt[2]{\dfrac{-g}{p}}$ in $-(AP:AO)+(CP:CO)$.

$OP::OA::P:\sqrt[2]{\dfrac{-q}{g}}$; $\textcircled{2}={}^{-}\tfrac{1}{2}\sqrt[2]{\dfrac{-g}{q}}$ in $-(AP:AO)+(CP:CO)$.

θ	FORMÆ LXXII. $\frac{dz\,z^{\theta\eta+\frac{1}{2}\eta-1}}{e+fz^{\eta}+gz^{2\eta}\sqrt[2]{k+lz^{\eta}}}$ Caf. 1.
&c.	
2.	$\frac{+2}{\eta g}\,d\odot - \frac{+fk-a}{\eta g a}\,d\textcircled{1} + \frac{fk+a}{\eta g a}\,d\textcircled{2}.$
1.	$\frac{+2k}{\eta a}\,d\textcircled{1} - \frac{2k}{\eta a}\,d\textcircled{2}.$
0.	$-\frac{+fk+a}{\eta e a}\,d\textcircled{1} + \frac{fk-a}{\eta e a}\,d\textcircled{2}.$
$\overline{1}$.	$\frac{2}{\eta e k}\,d\mathrm{P} + \frac{\overline{fk+a}\vert^{2}}{2\eta eeka}\,d\textcircled{1} - \frac{\overline{+fk-a}\vert^{2}}{2\eta eeka}\,d\textcircled{2}.$
&c.	

$\mathrm{P}=\sqrt[2]{\frac{k+lz^{\eta}}{z^{\eta}}}.\quad a=\sqrt[2]{ffkk-4egkk}.\quad p=\tfrac{1}{2}fk-el+\tfrac{1}{2}a.\quad q=\tfrac{1}{2}fk-el-\tfrac{1}{2}a.$

Caf. 1.

$OP:OA::\mathrm{P}:\sqrt[2]{l}\,;\quad \odot=+\tfrac{1}{2}\sqrt[2]{\frac{1}{l}}$ in $-(AP:AO)+(CP:CO).$

$OP:OA::\mathrm{P}:\sqrt[2]{\frac{p}{e}}\,;\quad \textcircled{1}=+\tfrac{1}{2}\sqrt[2]{\frac{e}{p}}$ in $\downarrow(PBO).$

$OP:OA::\mathrm{P}:\sqrt[2]{\frac{q}{e}}\,;\quad \textcircled{2}=+\tfrac{1}{2}\sqrt[2]{\frac{e}{q}}$ in $\downarrow(PBO).$

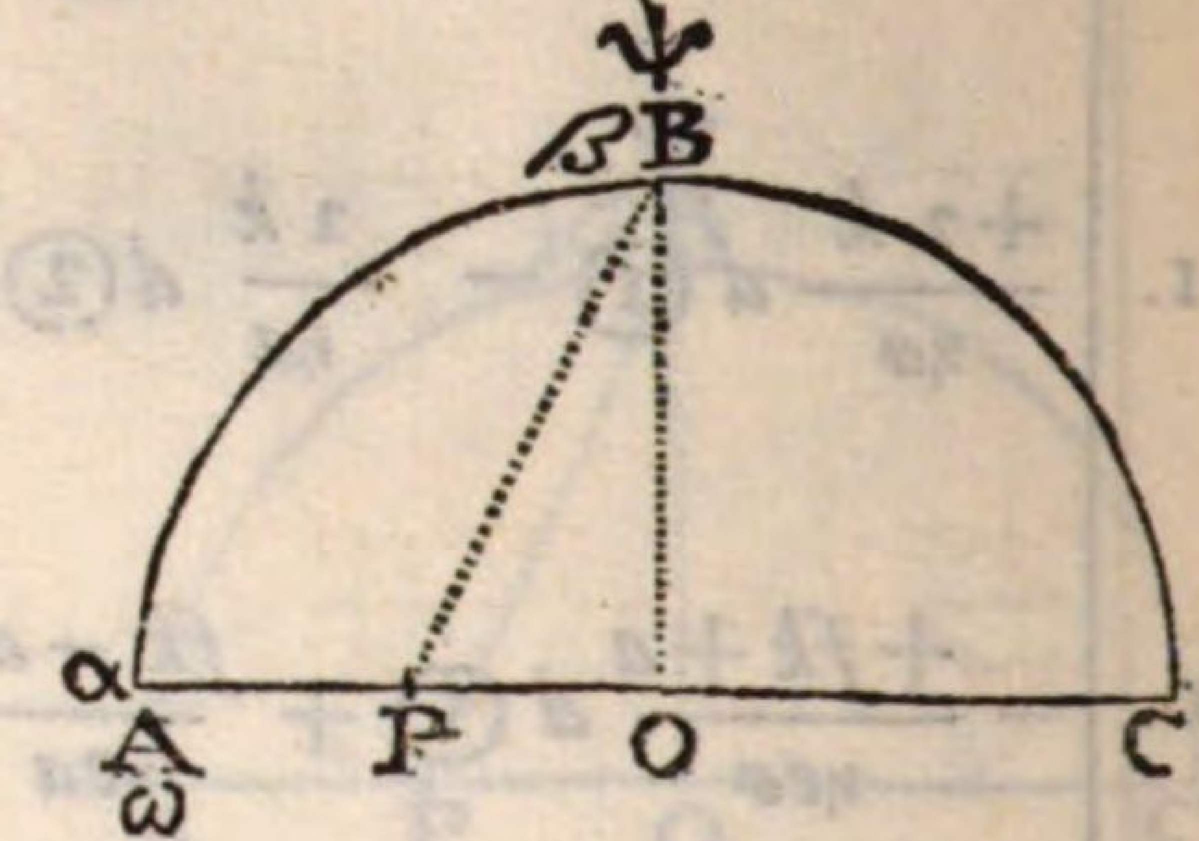
Caf. 2.

$OP:OA::\mathrm{P}:\sqrt[2]{-l}\,;\quad \odot=-\tfrac{1}{2}\sqrt[2]{\frac{-1}{l}}$ in $\downarrow(PBO).$

$OP:OA::\mathrm{P}:\sqrt[2]{\frac{-p}{e}}\,;\quad \textcircled{1}=-\tfrac{1}{2}\sqrt[2]{\frac{-e}{p}}$ in $-(AP:AO)+(CP:CO).$

$OP:OA::\mathrm{P}:\sqrt[2]{\frac{-q}{e}}\,;\quad \textcircled{2}=-\tfrac{1}{2}\sqrt[2]{\frac{-e}{q}}$ in $-(AP:AO)+(CP:CO).$

θ	FORMÆ LXXI. $\dfrac{d\dot{z}z^{\theta\eta-1}}{e+fz^{\eta}+gz^{2\eta}\,\sqrt[2]{k+lz^{\eta}}}$ Cas. 2.
&c.	
3.	$\frac{2}{\eta g l}\,d\mathrm{P}+\frac{\overset{f,\overline{r+gk}}{-egl}}{2\eta gr}\,d\mathrm{M}+\frac{\overset{f,\overline{r-gk}}{+egl}}{2\eta gr}\,d$ in $\textcircled{1}+\textcircled{2}$.
2.	$-\frac{+r+gk}{2\eta r}\,d\mathrm{M}-\frac{+r-gk}{2\eta r}\,d$ in $\textcircled{1}+\textcircled{2}$.
1.	$\frac{+gl}{2\eta r}\,d\mathrm{M}-\frac{gl}{2\eta r}\,d$ in $\textcircled{1}+\textcircled{2}$.
0.	$\frac{-2}{\eta e}\,d\circledcirc+\frac{\overset{g,\overline{r-fl}}{+ggk}}{2\eta er}\,d\mathrm{M}+\frac{\overset{g,\overline{r+fl}}{-ggk}}{2\eta er}\,d$ in $\textcircled{1}+\textcircled{2}$.
&c.	



$\mathrm{P}=\sqrt[2]{k+lz^{\eta}}$. $q=ell-fkl+gkk$. $r=\sqrt{gq}$. $s=\sqrt{-fgl+2ggk+2\sqrt{g^3q}}$.

$$\mathrm{M}=\tfrac{2}{s}(r+s\mathrm{P}+g\mathrm{PP}:r-s\mathrm{P}+g\mathrm{PP}).$$

Cas. 1.
$$\begin{cases} OP:OA::\mathrm{P}:\sqrt[2]{k}; & \circledcirc={}^{+}\tfrac{1}{2}\sqrt[2]{\dfrac{1}{k}}\ \text{in}\ (CP:AP). \\ OP:OA::{}^{+}\tfrac{1}{2}s+g\mathrm{P}:\sqrt{\tfrac{1}{4}ss-gr}; & \textcircled{1}={}^{+}\tfrac{1}{2}\sqrt{\dfrac{1}{\frac{1}{4}ss-gr}}\ \text{in}\ (CP:AP). \\ OP:OA::{}^{-}\tfrac{1}{2}s+g\mathrm{P}:\sqrt{\tfrac{1}{4}ss-gr}; & \textcircled{2}={}^{+}\tfrac{1}{2}\sqrt{\dfrac{1}{\frac{1}{4}ss-gr}}\ \text{in}\ (CP:AP). \end{cases}$$

Cas. 2.
$$\begin{cases} OP:OA::\mathrm{P}:\sqrt[2]{-k}; & \circledcirc={}^{-}\tfrac{1}{2}\sqrt[2]{\dfrac{-1}{k}}\ \text{in}\ \psi(PBO). \\ OP:OA::{}^{+}\tfrac{1}{2}s+g\mathrm{P}:\sqrt{gr-\tfrac{1}{4}ss}; & \textcircled{1}={}^{-}\tfrac{1}{2}\sqrt{\dfrac{1}{gr-\frac{1}{4}ss}}\ \text{in}\ \psi(PBO). \\ OP:OA::{}^{-}\tfrac{1}{2}s+g\mathrm{P}:\sqrt{gr-\tfrac{1}{4}ss}; & \textcircled{2}={}^{-}\tfrac{1}{2}\sqrt{\dfrac{1}{gr-\frac{1}{4}ss}}\ \text{in}\ \psi(PBO). \end{cases}$$

θ	FORMÆ LXXII. $\frac{dz z^{\theta\eta+\frac{1}{2}\eta-1}}{e+fz^{\eta}+gz^{2\eta}\sqrt[2]{k+lz^{\eta}}}$ Cas. 2.
&c.	
2.	$\frac{2}{\eta g} d\,\textcircled{0} + \frac{-e,\overline{r-fk}\;-eel}{2\eta g r} d\mathrm{M} + \frac{-e,\overline{r+fk}\;+eel}{2\eta g r} d \text{ in } \textcircled{1}+\textcircled{2}.$
1.	$\frac{-ek}{2\eta r} d\mathrm{M} + \frac{ek}{2\eta r} d \text{ in } \textcircled{1}+\textcircled{2}.$
0.	$\frac{r+el}{2\eta r} d\mathrm{M} + \frac{r-el}{2\eta r} d \text{ in } \textcircled{1}+\textcircled{2}.$
$\bar{1}$.	$\frac{-2}{\eta e k} d\mathrm{P} + \frac{-f,\overline{r+el}\;+egk}{2\eta e r} d\mathrm{M} + \frac{-f,\overline{r-el}\;-egk}{2\eta e r} d \text{ in } \textcircled{1}+\textcircled{2}.$
&c.	

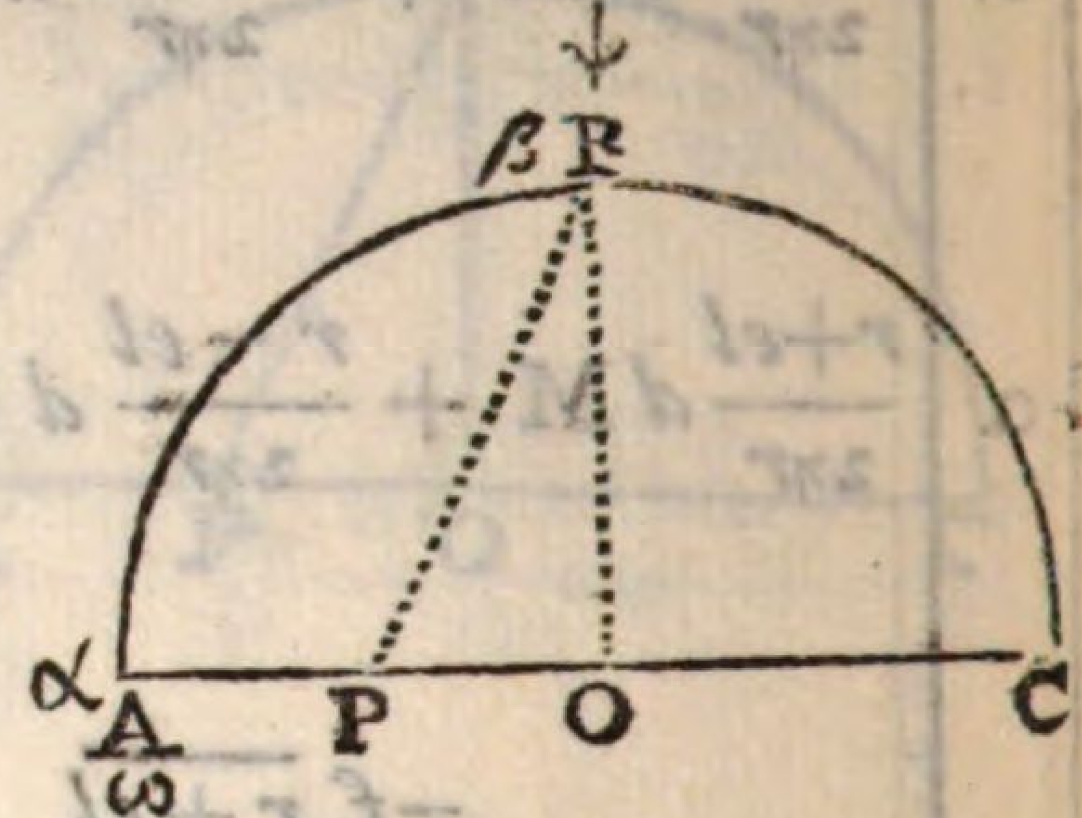
$\mathrm{P}=\sqrt[2]{\frac{k+lz^{\eta}}{z^{\eta}}}$. $q=ell-fkl+gkk$. $r=\sqrt{eq}$. $s=\sqrt{-efk+2eel+2\sqrt{e^3q}}$.

$$\mathrm{M}=\tfrac{1}{s}(r+s\mathrm{P}+e\mathrm{PP} : r-s\mathrm{P}+e\mathrm{PP}).$$

Cas. 1.
$$OP:OA::\mathrm{P}:\sqrt[2]{l}\,; \quad \ldots\ldots \quad \textcircled{0}=\pm\tfrac{1}{2}\sqrt[2]{\frac{1}{l}} \text{ in } (CP:AP).$$
$$OP:OA::\pm\tfrac{1}{2}s+e\mathrm{P}:\sqrt{\tfrac{1}{4}ss-er}\,;\quad \textcircled{1}=\pm\tfrac{1}{2}\sqrt{\frac{1}{\frac{1}{4}ss-er}} \text{ in } (CP:AP).$$
$$OP:OA::-\tfrac{1}{2}s+e\mathrm{P}:\sqrt{\tfrac{1}{4}ss-er}\,;\quad \textcircled{2}=\pm\tfrac{1}{2}\sqrt{\frac{1}{\frac{1}{4}ss-er}} \text{ in } (CP:AP).$$

Cas. 2.
$$OP:OA::\mathrm{P}:\sqrt[2]{-l}\,; \quad \ldots\ldots \quad \textcircled{0}=-\tfrac{1}{2}\sqrt[2]{\frac{-1}{l}} \text{ in } \downarrow(PBO).$$
$$OP:OA::\pm\tfrac{1}{2}s+e\mathrm{P}:\sqrt{er-\tfrac{1}{4}ss}\,;\quad \textcircled{1}=-\tfrac{1}{2}\sqrt{\frac{1}{er-\frac{1}{4}ss}} \text{ in } \downarrow(PBO).$$
$$OP:OA::-\tfrac{1}{2}s+e\mathrm{P}:\sqrt{er-\tfrac{1}{4}ss}\,;\quad \textcircled{2}=-\tfrac{1}{2}\sqrt{\frac{1}{er-\frac{1}{4}ss}} \text{ in } \downarrow(PBO).$$

ϑ	FORMÆ LXXIII. $\dfrac{d\dot{z}z^{\theta\eta-1}\sqrt[2]{k+lz^{\eta}}}{e+fz^{\eta}+gz^{2\eta}}$ *Cas.* I.
—	
&c.	
3.	$\dfrac{-6fl+2gk+2glz^{\eta}}{3\eta ggl}d\mathrm{P}+\dfrac{p,\overline{fl+a}\vert^{2}}{2\eta g^{3}la}d\textcircled{1}+\dfrac{-q,\overline{fl-a}\vert^{2}}{2\eta g^{3}la}d\textcircled{2}.$
2.	$\dfrac{+2}{\eta g}d\mathrm{P}+\dfrac{-p,\overline{fl+a}}{\eta gga}d\textcircled{1}+\dfrac{q,\overline{fl-a}}{\eta gga}d\textcircled{2}.$
I.	$\dfrac{+2pl}{\eta ga}d\textcircled{1}-\dfrac{2ql}{\eta ga}d\textcircled{2}.$
0.	$\dfrac{-2k}{\eta e}d\textcircled{0}+\dfrac{+k,\overline{fl-a}\;-2ell}{\eta ea}d\textcircled{1}+\dfrac{-k,\overline{fl+a}\;+2ell}{\eta ea}d\textcircled{2}.$
&c.	



$\mathrm{P}=\sqrt[2]{k+lz^{\eta}}.\quad a=\sqrt[2]{ffll-4egll}.\quad p=\tfrac{1}{2}fl-gk+\tfrac{1}{2}a.\quad q=\tfrac{1}{2}fl-gk-\tfrac{1}{2}a.$

Cas. I.

$OP:OA::\mathrm{P}:\sqrt[2]{k};\quad \textcircled{0}=+\tfrac{1}{2}\sqrt[2]{\tfrac{1}{k}}$ in $-(AP:AO)+(CP:CO)$.

$OP:OA::\mathrm{P}:\sqrt[2]{\tfrac{p}{g}};\quad \textcircled{1}=+\tfrac{1}{2}\sqrt[2]{\tfrac{g}{p}}$ in $\psi(PBO)$.

$OP:OA::\mathrm{P}:\sqrt[2]{\tfrac{q}{g}};\quad \textcircled{2}=+\tfrac{1}{2}\sqrt[2]{\tfrac{g}{q}}$ in $\psi(PBO)$.

Cas. 2.

$OP:OA::\mathrm{P}:\sqrt[2]{-k};\quad \textcircled{0}=-\tfrac{1}{2}\sqrt[2]{\tfrac{-1}{k}}$ in $\psi(PBO)$.

$OP::OA::\mathrm{P}:\sqrt[2]{\tfrac{-p}{g}};\quad \textcircled{1}=-\tfrac{1}{2}\sqrt[2]{\tfrac{-g}{p}}$ in $-(AP:AO)+(CP:CO)$.

$OP::OA::\mathrm{P}:\sqrt[2]{\tfrac{-q}{g}};\quad \textcircled{2}=-\tfrac{1}{2}\sqrt[2]{\tfrac{-g}{q}}$ in $-(AP:AO)+(CP:CO)$.

θ	FORMÆ LXXIV. $\dfrac{dz\,z^{\theta\eta+\frac{1}{2}\eta-1}\sqrt[2]{k+lz^{\eta}}}{e+fz^{\eta}+gz^{2\eta}}$ *Cas.* 1.
&c.	
1.	$\dfrac{+2l}{\eta g}d\circledcirc+\dfrac{-l,\overline{fk-a}\;+2gkk}{\eta ga}d\textcircled{1}+\dfrac{+l,\overline{fl+a}\;-2gkk}{\eta ga}d\textcircled{2}$.
0.	$\dfrac{-2pk}{\eta ea}d\textcircled{1}+\dfrac{2qk}{\eta ea}d\textcircled{2}$.
$\overline{1}$.	$\dfrac{-2}{\eta e}d\mathrm{P}+\dfrac{p.\overline{fk+a}}{\eta eea}d\textcircled{1}+\dfrac{-q,\overline{fl+a}}{\eta eea}d\textcircled{2}$.
$\overline{2}$.	$\dfrac{+6fkz^{\eta}-2ek-2elz^{\eta}}{3\eta eekz^{\eta}}d\mathrm{P}+\dfrac{-p,\overline{fk+a}\vert^{2}}{2\eta e^{3}ka}d\textcircled{1}+\dfrac{+q,\overline{fk-a}\vert^{2}}{2\eta e^{3}ka}d\textcircled{2}$.
&c.	

$\mathrm{P}=\sqrt[2]{\dfrac{k+lz^{\eta}}{z^{\eta}}}$. $a=\sqrt[2]{ffkk-4egkk}$. $p=\frac{1}{2}fk-el+\frac{1}{2}a$. $q=\frac{1}{2}fk-el-\frac{1}{2}a$.

Cas. 1.

$OP:OA::\mathrm{P}:\sqrt[2]{l}$; $\circledcirc=+\frac{1}{2}\sqrt[2]{\dfrac{1}{l}}$ in $-(AP:AO)+(CP:CO)$.

$OP:OA::\mathrm{P}:\sqrt[2]{\dfrac{p}{e}}$; $\textcircled{1}=+\frac{1}{2}\sqrt[2]{\dfrac{e}{p}}$ in $\downarrow(PBO)$.

$OP:OA::\mathrm{P}:\sqrt[2]{\dfrac{q}{e}}$; $\textcircled{2}=+\frac{1}{2}\sqrt[2]{\dfrac{e}{q}}$ in $\downarrow(PBO)$.

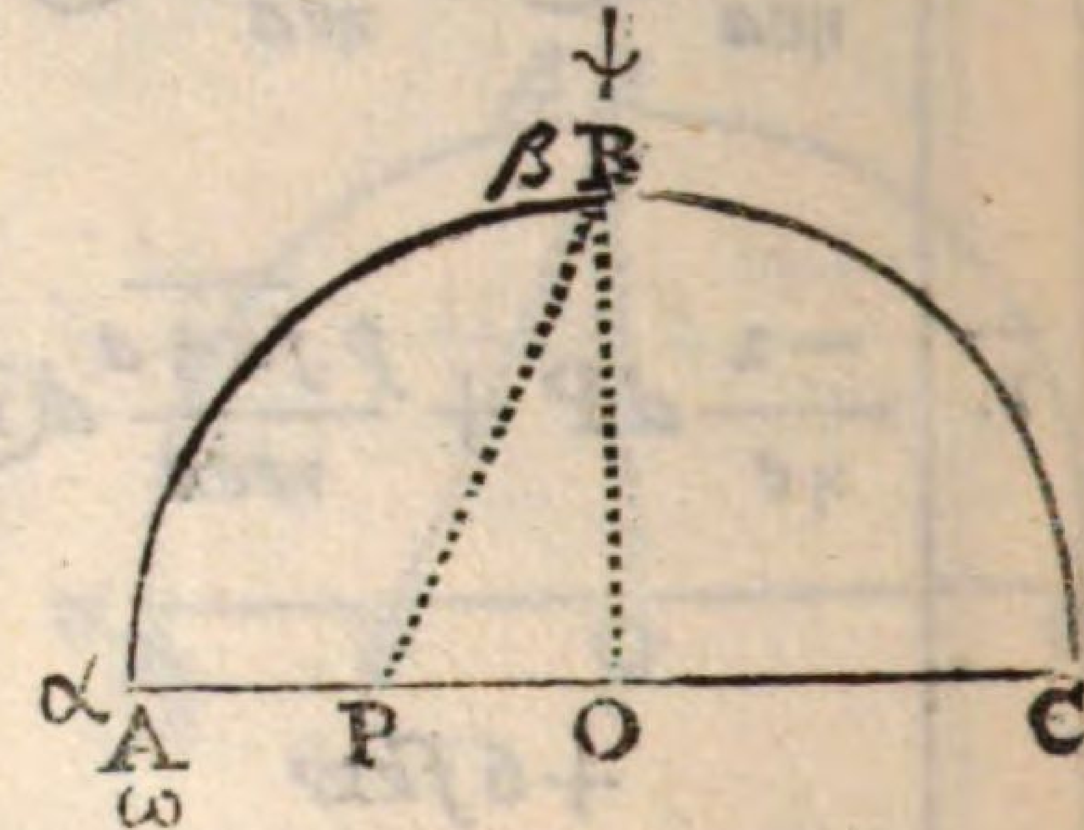
Cas. 2.

$OP:OA::\mathrm{P}:\sqrt[2]{-l}$; $\circledcirc=-\frac{1}{2}\sqrt[2]{\dfrac{-1}{l}}$ in $\downarrow(PBO)$.

$OP:OA::\mathrm{P}:\sqrt[2]{\dfrac{-p}{e}}$; $\textcircled{1}=-\frac{1}{2}\sqrt[2]{\dfrac{-e}{p}}$ in $-(AP:AO)+(CP:CO)$.

$OP:OA::\mathrm{P}:\sqrt[2]{\dfrac{-q}{e}}$; $\textcircled{2}=-\frac{1}{2}\sqrt[2]{\dfrac{-e}{q}}$ in $-(AP:AO)+(CP:CO)$.

θ	FORMÆ LXXIII. $\frac{d\dot{z}z^{\theta\eta-1}\sqrt[2]{k+lz^{\eta}}}{e+fz^{\eta}+gz^{2\eta}}$ *Cas.* 2.
&c.	
3.	$\frac{\begin{matrix}-6fl\\+2gk+2glz^{\eta}\end{matrix}}{3\eta ggl}d\mathrm{P}+\frac{\begin{matrix}+l,\overline{eg-ff}\\f,\overline{r+gk}\end{matrix}}{2\eta gg}d\mathrm{M}+\frac{\begin{matrix}+l,\overline{eg-ff}\\-f,\overline{r-gk}\end{matrix}}{2\eta gg}d\textcircled{1}+\textcircled{2}.$
2.	$\frac{2}{\eta g}d\mathrm{P}+\frac{fl-r-gk}{2\eta g}d\mathrm{M}+\frac{fl+r-gk}{2\eta g}d\,\mathrm{in}\,\textcircled{1}+\textcircled{2}.$
1.	$\frac{-l}{2\eta}d\mathrm{M}-\frac{l}{2\eta}d\,\mathrm{in}\,\textcircled{1}+\textcircled{2}.$
0.	$\frac{-2k}{\eta e}d\circledcirc+\frac{r+gk}{2\eta e}d\mathrm{M}-\frac{+r-gk}{2\eta e}d\,\mathrm{in}\,\textcircled{1}+\textcircled{2}.$
&c.	



$\mathrm{P}=\sqrt[2]{k+lz^{\eta}}$. $q=ell-fkl+gkk$. $r=\sqrt{gq}$. $s=\sqrt{-fgl+2ggk+2\sqrt{g^3q}}$.

$$\mathrm{M}=\tfrac{1}{s}(r+s\mathrm{P}+g\mathrm{PP}:r-s\mathrm{P}+g\mathrm{PP}).$$

Cas. 1.

$OP:OA::\mathrm{P}:\sqrt[2]{k}$; . . . $\circledcirc={}^{+}\tfrac{1}{2}\sqrt[2]{\frac{1}{k}}$ in $(CP:AP)$.

$OP:OA::{}^{+}\tfrac{1}{2}s+g\mathrm{P}:\sqrt{\tfrac{1}{4}ss-gr}$; $\textcircled{1}={}^{+}\tfrac{1}{2}\sqrt{\frac{1}{\frac{1}{4}ss-gr}}$ in $(CP:AP)$.

$OP:OA::{}^{-}\tfrac{1}{2}s+g\mathrm{P}:\sqrt{\tfrac{1}{4}ss-gr}$; $\textcircled{2}={}^{+}\tfrac{1}{2}\sqrt{\frac{1}{\frac{1}{4}ss-gr}}$ in $(CP:AP)$.

Cas. 2.

$OP:OA::\mathrm{P}:\sqrt[2]{-k}$; . . . $\circledcirc={}^{-}\tfrac{1}{2}\sqrt[2]{\frac{-1}{k}}$ in $\downarrow(PBO)$.

$OP:OA::{}^{+}\tfrac{1}{2}s+g\mathrm{P}:\sqrt{gr-\tfrac{1}{4}ss}$; $\textcircled{1}={}^{-}\tfrac{1}{2}\sqrt{\frac{1}{gr-\frac{1}{4}ss}}$ in $\downarrow(PBO)$.

$OP:OA::{}^{-}\tfrac{1}{2}s+g\mathrm{P}:\sqrt{gr-\tfrac{1}{4}ss}$; $\textcircled{2}={}^{-}\tfrac{1}{2}\sqrt{\frac{1}{gr-\frac{1}{4}ss}}$ in $\downarrow(PBO)$.

θ	FORMÆ LXXIV. $\dfrac{dzz^{\theta\eta+\frac{1}{2}\eta-1}\sqrt[2]{k+lz^{\eta}}}{e+fz^{\eta}+gz^{2\eta}}$ *Cas.* 2.
&c.	
1.	$\dfrac{+2l}{\eta g}d$ ◎ $-\dfrac{+r+el}{2\eta g}d\mathrm{M}+\dfrac{r-el}{2\eta g}d$ in ① + ②.
0.	$\dfrac{+k}{2\eta}d\mathrm{M}+\dfrac{k}{2\eta}d$ in ① + ②.
$\overline{1}$.	$\dfrac{-2}{\eta e}d\mathrm{P}+\dfrac{r+el-fk}{2\eta e}d\mathrm{M}-\dfrac{+r-el+fk}{2\eta e}d$ in ① + ②.
$\overline{2}$.	$\dfrac{\begin{matrix}+6fkz^{\eta}\\-2ek-2elz^{\eta}\end{matrix}}{3\eta eekz^{\eta}}d\mathrm{P}+\dfrac{\begin{matrix}+k,\overline{ff-eg}\\-f,\overline{r+el}\end{matrix}}{2\eta ee}d\mathrm{M}+\dfrac{\begin{matrix}+k,\overline{ff-eg}\\f,\overline{r-el}\end{matrix}}{2\eta ee}d$ in ① + ②.
&c.	

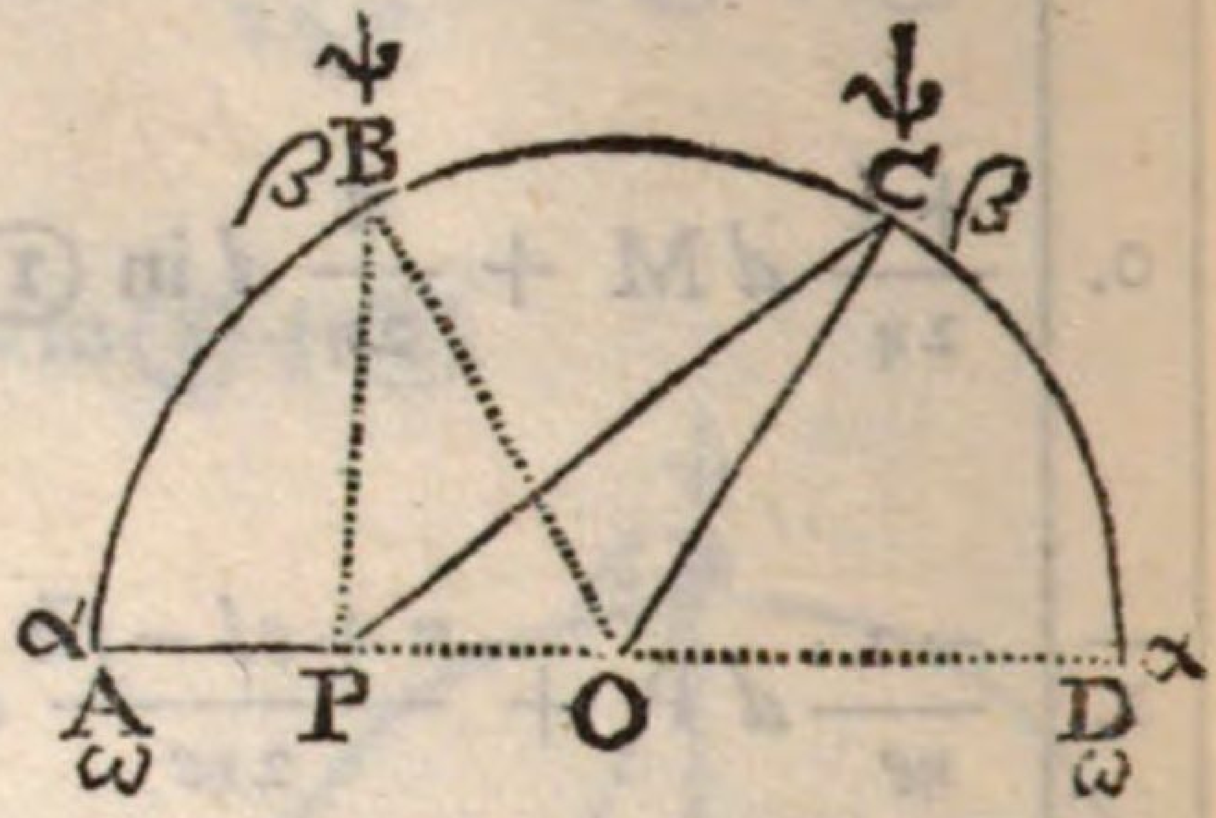
$\mathrm{P}=\sqrt[2]{\dfrac{k+lz^{\eta}}{z^{\eta}}}$. $q=ell-fkl+gkk$. $r=\sqrt{eq}$. $s=\sqrt{-efk+2eel+2\sqrt{e^{3}q}}$.

$$\mathrm{M}=\tfrac{1}{s}(r+s\mathrm{P}+e\mathrm{PP}:r-s\mathrm{P}+e\mathrm{PP}).$$

Cas. 1.
- $OP:OA::\mathrm{P}:\sqrt[2]{l}$; ◎ $=\,^{+}\tfrac{1}{2}\sqrt[2]{\dfrac{1}{l}}$ in $(CP:AP)$.
- $OP:OA::\,^{+}\tfrac{1}{2}s+e\mathrm{P}:\sqrt{\tfrac{1}{4}ss-er}$; ① $=\,^{+}\tfrac{1}{2}\sqrt{\dfrac{1}{\tfrac{1}{4}ss-er}}$ in $(CP:AP)$.
- $OP:OA::\,^{-}\tfrac{1}{2}s+e\mathrm{P}:\sqrt{\tfrac{1}{4}ss-er}$; ② $=\,^{+}\tfrac{1}{2}\sqrt{\dfrac{1}{\tfrac{1}{4}ss-er}}$ in $(CP:AP)$.

Cas. 2.
- $OP:OA::\mathrm{P}:\sqrt[2]{-l}$; ◎ $=\,^{-}\tfrac{1}{2}\sqrt[2]{\dfrac{-1}{l}}$ in ↓(PBO).
- $OP:OA::\,^{+}\tfrac{1}{2}s+e\mathrm{P}:\sqrt{er-\tfrac{1}{4}ss}$; ① $=\,^{-}\tfrac{1}{2}\sqrt{\dfrac{1}{er-\tfrac{1}{4}ss}}$ in ↓(PBO).
- $OP:OA::\,^{-}\tfrac{1}{2}s+e\mathrm{P}:\sqrt{er-\tfrac{1}{4}ss}$; ② $=\,^{-}\tfrac{1}{2}\sqrt{\dfrac{1}{er-\tfrac{1}{4}ss}}$ in ↓(PBO).

θ	FORMA LXXV. $\dfrac{d\dot{z}z^{\theta n-1}}{e+fz^{n}+gz^{2n}\sqrt[3]{k+lz^{n}}}$
&c.	
3.	$\frac{3}{2\eta gl}dPP - 3\times\frac{+\overline{fl+a}\vert^{2}}{4\eta ggla}d\textcircled{1} + 3\times\frac{\overline{fl-a}\vert^{2}}{4\eta ggla}d\textcircled{2}$.
2.	$\frac{3a+3fl}{2\eta ga}d\textcircled{1} + \frac{3a-3fl}{2\eta ga}d\textcircled{2}$.
1.	$\frac{-3l}{\eta a}d\textcircled{1} + \frac{3l}{\eta a}d\textcircled{2}$.
0.	$\frac{-3}{\eta e}d\textcircled{0} + \frac{3fl-3a}{2\eta ea}d\textcircled{1} - \frac{+3fl+3a}{2\eta ea}a\textcircled{2}$.
&c.	



$P=\sqrt[3]{k+lz^{n}}$. $a=\sqrt[2]{ffll-4egll}$. $p=\frac{1}{2}fl-gk+\frac{1}{2}a$. $q=\frac{1}{2}fl-gk-\frac{1}{2}a$.

Cas. 1.

$OP:OA::P:\sqrt[3]{k}$; $\textcircled{0}=+\frac{1}{3}\sqrt[3]{\frac{1}{k}}$ in $-(AP:AO)$ $+\beta(CP:CO)-\psi(PCO)$.

$OP:OA::P:\sqrt[3]{\frac{p}{g}}$; $\textcircled{1}=+\frac{1}{3}\sqrt[3]{\frac{g}{p}}$ in $\beta\,(BP:BO)$ $-(DP:DO)+\psi(PBO)$.

$OP:OA::P:\sqrt[3]{\frac{q}{g}}$; $\textcircled{2}=+\frac{1}{3}\sqrt[3]{\frac{g}{q}}$ in $\beta\,(BP:BO)$ $-(DP:DO)+\psi(PBO)$.

Cas. 2.

$OP:OA::P:\sqrt[3]{-k}$; $\textcircled{0}=-\frac{1}{3}\sqrt[3]{\frac{-1}{k}}$ in $\beta\,(BP:BO)$ $-(DP:DO)+\psi(PBO)$.

$OP::OA::P:\sqrt[3]{\frac{-p}{g}}$; $\textcircled{1}=-\frac{1}{3}\sqrt[3]{\frac{-g}{p}}$ in $-(AP:AO)$ $+\beta(CP:CO)-\psi(PCO)$.

$OP::OA::P:\sqrt[3]{\frac{-q}{g}}$; $\textcircled{2}=-\frac{1}{3}\sqrt[3]{\frac{-g}{q}}$ in $-(AP:AO)$ $+\beta(CP:CO)-\psi(PCO)$.

θ	FORMA LXXVI. $\dfrac{d\dot{z}z^{\theta n+\frac{1}{3}n-1}}{e+fz^n+gz^{2n}\sqrt[3]{k+lz^n}}$
&c.	
2.	$+\dfrac{2}{ng}d\textcircled{0}-\dfrac{+3fk-3a}{2nga}d\textcircled{1}+\dfrac{3fk+3a}{2nga}d\textcircled{2}.$
1.	$\dfrac{+3k}{na}d\textcircled{1}-\dfrac{3k}{na}d\textcircled{2}.$
0.	$-\dfrac{+3fk+3a}{2nea}d\textcircled{1}+\dfrac{3fk-3a}{2nea}d\textcircled{2}.$
$\overline{1}$.	$\dfrac{-3}{2nek}dPP+3\times\dfrac{\overline{fk+a}\vert^2}{4neeka}d\textcircled{1}-3\times\dfrac{\overline{+fk-a}\vert^2}{4neeka}d\textcircled{2}.$
&c.	

$P=\sqrt[3]{\dfrac{k+lz^n}{z^n}}.\ a=\sqrt[2]{ffkk-4egkk}.\ p=\tfrac{1}{2}fk-el+\tfrac{1}{2}a.\ q=\tfrac{1}{2}fk-el-\tfrac{1}{2}a.$

Caf. 1.

$OP:OA::P:\sqrt[3]{l}$; $\textcircled{0}=+\tfrac{1}{3}\sqrt[3]{\dfrac{1}{l}}$ in $-(AP:AO)$ $+\beta(CP:CO)-\downarrow(PCO).$

$OP:OA::P:\sqrt[3]{\dfrac{p}{e}}$; $\textcircled{1}=+\tfrac{1}{3}\sqrt[3]{\dfrac{e}{p}}$ in $\beta(BP:BO)$ $-(DP:DO)+\downarrow(PBO).$

$OP:OA::P:\sqrt[3]{\dfrac{q}{e}}$; $\textcircled{2}=+\tfrac{1}{3}\sqrt[3]{\dfrac{e}{q}}$ in $\beta(BP:BO)$ $-(DP:DO)+\downarrow(PBO).$

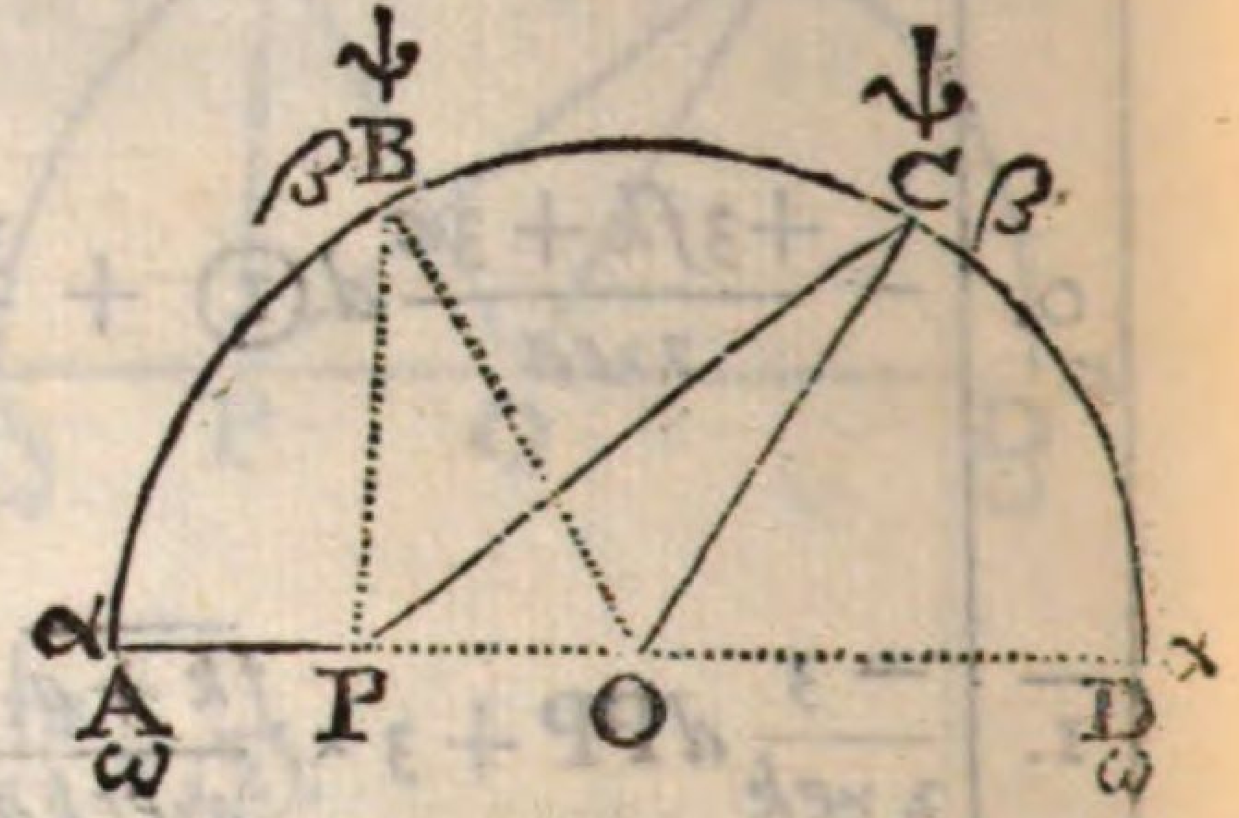
Caf. 2.

$OP:OA::P:\sqrt[3]{-l}$; $\textcircled{0}=-\tfrac{1}{3}\sqrt[3]{\dfrac{-1}{l}}$ in $\beta(BP:BO)$ $-(DP:DO)+\downarrow(PBO).$

$OP:OA::P:\sqrt[3]{\dfrac{-p}{e}}$; $\textcircled{1}=-\tfrac{1}{3}\sqrt[3]{\dfrac{-e}{p}}$ in $-(AP:AO)$ $+\beta(CP:CO)-\downarrow(PCO).$

$OP:OA::P:\sqrt[3]{\dfrac{-q}{e}}$; $\textcircled{2}=-\tfrac{1}{3}\sqrt[3]{\dfrac{-e}{q}}$ in $-(AP:AO)$ $+\beta(CP:CO)-\downarrow(PCO).$

θ	FORMA LXXVII. $\dfrac{d\dot{z}z^{\theta n-1}\sqrt[3]{k+lz^n}}{e+fz^n+gz^{2n}}$
&c.	
3.	$\dfrac{-12fl+3gk+3glz^n}{4nggl}dP+\dfrac{3p,\overline{fl+a}\vert^2}{4ng^3la}d\textcircled{1}+\dfrac{-3q,\overline{fl-a}\vert^2}{4ng^3la}d\textcircled{2}.$
2.	$\dfrac{+3}{ng}dP+\dfrac{-3p,\overline{fl+a}}{2ngga}d\textcircled{1}+\dfrac{3q,\overline{fl-a}}{2ngga}d\textcircled{2}.$
1.	$\dfrac{+3pl}{nga}d\textcircled{1}-\dfrac{3ql}{nga}d\textcircled{2}.$
0.	$\dfrac{-3k}{ne}d\circledcirc+\dfrac{+3k,\overline{fl-a}-6ell}{2nea}d\textcircled{1}+\dfrac{-3k,\overline{fl+a}+6ell}{2nea}d\textcircled{2}.$
&c.	



$P=\sqrt[3]{k+lz^n}$. $a=\sqrt[2]{ffll-4egll}$. $p=\frac{1}{2}fl-gk+\frac{1}{2}a$. $q=\frac{1}{2}fl-gk-\frac{1}{2}a$.

Cas. 1.

$OP:OA::P:\sqrt[3]{k}$; $\circledcirc=+\frac{1}{3}\sqrt[3]{\frac{1}{kk}}$ in $-(AP:AO)$ $+\beta(CP:CO)$ $+\psi(PCO)$.

$OP:OA::P:\sqrt[3]{\frac{p}{g}}$; $\textcircled{1}=+\frac{1}{3}\sqrt[3]{\frac{gg}{pp}}$ in $-\beta(BP:BO)$ $+(DP:DO)$ $+\psi(PBO)$.

$OP:OA::P:\sqrt[3]{\frac{q}{g}}$; $\textcircled{2}=+\frac{1}{3}\sqrt[3]{\frac{gg}{qq}}$ in $-\beta(BP:BO)$ $+(DP:DO)$ $+\psi(PBO)$.

Cas. 2.

$OP:OA::P:\sqrt[3]{-k}$; $\circledcirc=-\frac{1}{3}\sqrt[3]{\frac{1}{kk}}$ in $-\beta(BP:BO)$ $+(DP:DO)$ $+\psi(PBO)$.

$OP::OA::P:\sqrt[3]{\frac{-p}{g}}$; $\textcircled{1}=-\frac{1}{3}\sqrt[3]{\frac{gg}{pp}}$ in $-(AP:AO)$ $+\beta(CP:CO)$ $+\psi(PCO)$.

$OP::OA::P:\sqrt[3]{\frac{-q}{g}}$; $\textcircled{2}=-\frac{1}{3}\sqrt[3]{\frac{gg}{qq}}$ in $-(AP:AO)$ $+\beta(CP:CO)$ $+\psi(PCO)$.

θ	FORMA LXXVIII. $\dfrac{dzz^{\theta n+\frac{2}{3}n-1}\sqrt[3]{k+lz^n}}{e+fz^n+gz^{2n}}$
&c.	
1.	$\dfrac{+3l}{\eta g}d\circledcirc+\dfrac{\overset{-3l,\overline{fk-a}}{+6gkk}}{2\eta ga}d\textcircled{1}+\dfrac{\overset{+3l,\overline{fk+a}}{-6gkk}}{2\eta ga}d\textcircled{2}.$
0.	$\dfrac{-3pk}{\eta ea}d\textcircled{1}+\dfrac{3qk}{\eta ea}d\textcircled{2}.$
$\overline{1}$.	$\dfrac{-3}{\eta e}dP+\dfrac{3p,\overline{fk+a}}{2\eta eea}d\textcircled{1}+\dfrac{-3q,\overline{fk-a}}{2\eta eea}d\textcircled{2}.$
$\overline{2}$.	$\dfrac{\overset{+12fkz^n}{-3ek-3elz^n}}{4\eta eekz^n}dP+\dfrac{-3p,\overline{fk+a}\vert^2}{4\eta e^3ka}d\textcircled{1}+\dfrac{+3q,\overline{fk-a}\vert^2}{4\eta e^3ka}d\textcircled{2}.$
&c.	

$P=\sqrt[3]{\dfrac{k+lz^n}{z^n}}.\quad a=\sqrt[2]{ffkk-4egkk}.\quad p=\tfrac{1}{2}fk-el+\tfrac{1}{2}a.\quad q=\tfrac{1}{2}fk-el-\tfrac{1}{2}a.$

Cas. 1.

$$\left\{\begin{array}{l} OP:OA::P:\sqrt[3]{l};\quad \circledcirc=+\tfrac{1}{3}\sqrt[3]{\dfrac{1}{ll}}\text{ in }-(AP:AO) \\ \quad +\beta(CP:CO)+\psi(PCO.) \end{array}\right.$$

$$\left\{\begin{array}{l} OP:OA::P:\sqrt[3]{\dfrac{p}{e}};\quad \textcircled{1}=+\tfrac{1}{3}\sqrt[3]{\dfrac{ee}{pp}}\text{ in }-\beta(BP:BO) \\ \quad +(DP:DO)+\psi(PBO). \end{array}\right.$$

$$\left\{\begin{array}{l} OP:OA::P:\sqrt[3]{\dfrac{q}{e}};\quad \textcircled{2}=+\tfrac{1}{3}\sqrt[3]{\dfrac{ee}{qq}}\text{ in }-\beta(BP:BO) \\ \quad +(DP:DO)+\psi(PBO). \end{array}\right.$$

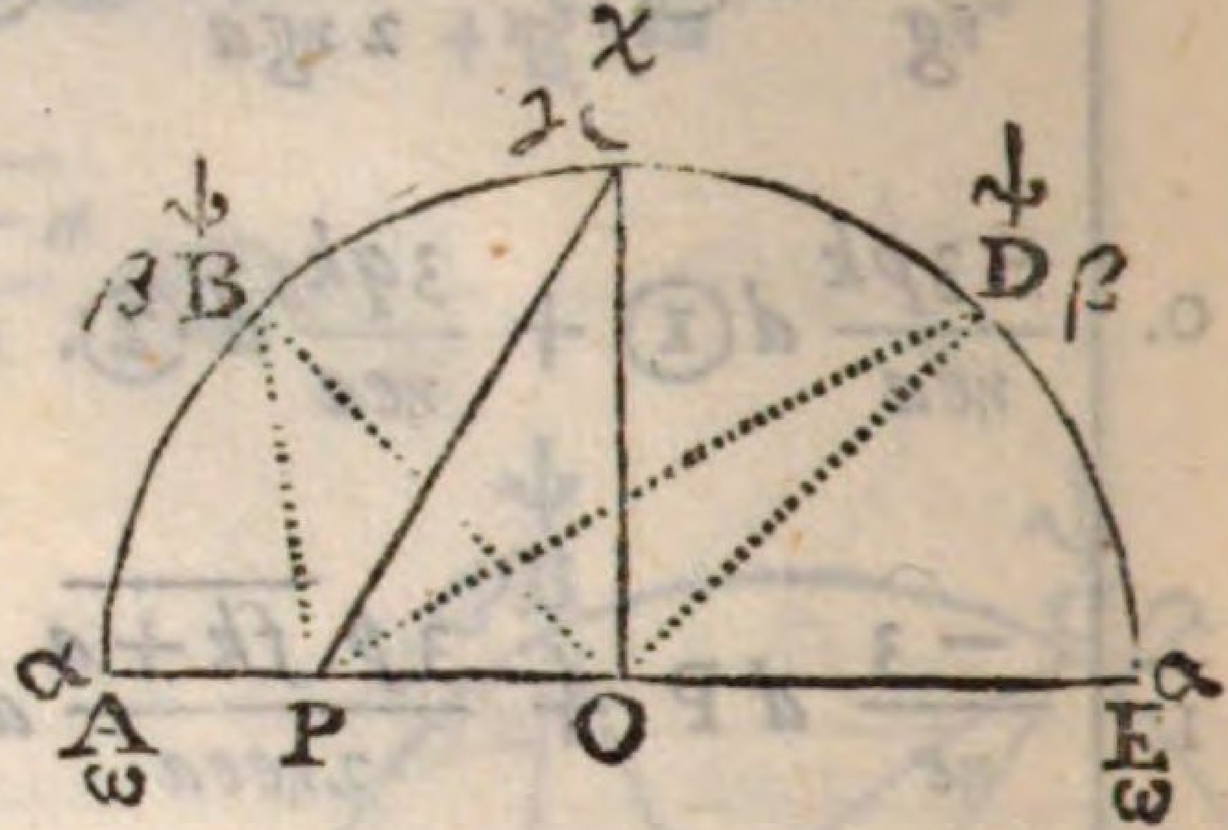
Cas. 2.

$$\left\{\begin{array}{l} OP:OA::P:\sqrt[3]{-l};\quad \circledcirc=-\tfrac{1}{3}\sqrt[3]{\dfrac{1}{ll}}\text{ in }-\beta(BP:BO) \\ \quad +(DP:DO)+\psi(PBO). \end{array}\right.$$

$$\left\{\begin{array}{l} OP:OA::P:\sqrt[3]{\dfrac{-p}{e}};\quad \textcircled{1}=-\tfrac{1}{3}\sqrt[3]{\dfrac{ee}{pp}}\text{ in }-(AP:AO) \\ \quad +\beta(CP:CO)+\psi(PCO). \end{array}\right.$$

$$\left\{\begin{array}{l} OP:OA::P:\sqrt[3]{\dfrac{-q}{e}};\quad \textcircled{2}=-\tfrac{1}{3}\sqrt[3]{\dfrac{ee}{qq}}\text{ in }-(AP:AO) \\ \quad +\beta(CP:CO)+\psi(PCO). \end{array}\right.$$

θ	FORMÆ LXXIX. $\dfrac{d\dot{z}z^{\theta n-1}}{e+fz^n+gz^{2n}\,\sqrt[4]{k+lz^n}}$ *Cas.* 1.
&c.	
3.	$\dfrac{4}{3\eta gl}d\,\dot{P}^3 - \dfrac{+\overline{fl+a}\vert^2}{\eta ggla}d\,\textcircled{1} + \dfrac{\overline{fl-a}\vert^2}{\eta ggla}d\,\textcircled{2}.$
2.	$\dfrac{2a+2fl}{\eta ga}d\,\textcircled{1} + \dfrac{2a-2fl}{\eta ga}d\,\textcircled{2}.$
1.	$\dfrac{-4l}{\eta a}d\,\textcircled{1} + \dfrac{4l}{\eta a}d\,\textcircled{2}.$
0.	$\dfrac{-4}{\eta e}d\,\circledcirc + \dfrac{2fl-2a}{\eta ea}d\,\textcircled{1} - \dfrac{+2fl+2a}{\eta ea}d\,\textcircled{2}.$
&c.	



$P=\sqrt[4]{k+lz^n}.\ a=\sqrt[2]{ffll-4egll}.\ p=\tfrac{1}{2}fl-gk+\tfrac{1}{2}a.\ q=\tfrac{1}{2}fl-gk-\tfrac{1}{2}a.$

Cas. 1.

$OP:OA::P:\sqrt[4]{k}$; $\circledcirc = +\tfrac{1}{4}\sqrt[4]{\tfrac{1}{k}}$ in $-(AP:AO)$ $+(EP:EO)-\chi(PCO)$.

$OP:OA::P:\sqrt[4]{\tfrac{p}{g}}$; $\textcircled{1} = +\tfrac{1}{4}\sqrt[4]{\tfrac{g}{p}}$ in $\beta(BP:BO)$ $-\beta(DP:DO)+\downarrow(PBO)+\downarrow(PDO)$.

$OP:OA::P:\sqrt[4]{\tfrac{q}{g}}$; $\textcircled{2} = +\tfrac{1}{4}\sqrt[4]{\tfrac{g}{q}}$ in $\beta(BP:BO)$ $-\beta(DP:DO)+\downarrow(PBO)+\downarrow(PDO)$.

Cas. 2.

$OP:OA::P:\sqrt[4]{-k}$; $\circledcirc = -\tfrac{1}{4}\sqrt[4]{\tfrac{-1}{k}}$ in $\beta(BP:BO)$ $-\beta(DP:DO)+\downarrow(PBO)+\downarrow(PDO)$.

$OP:OA::P:\sqrt[4]{\tfrac{-p}{g}}$; $\textcircled{1} = -\tfrac{1}{4}\sqrt[4]{\tfrac{-g}{p}}$ in $-(AP:AO)$ $+(EP:EO)-\chi(PCO)$.

$OP:OA::P:\sqrt[4]{\tfrac{-q}{g}}$; $\textcircled{2} = -\tfrac{1}{4}\sqrt[4]{\tfrac{-g}{q}}$ in $-(AP:AO)$ $+(EP:EO)-\chi(PCO)$.

θ	FORMÆ LXXX. $\dfrac{dzz^{\theta n+\frac{1}{4}n-1}}{e+fz^n+gz^{2n}\sqrt[4]{k+lz^n}}$ *Caſ.* 1.
&c.	
2.	$\dfrac{+4}{ng}d\circledcirc - \dfrac{+2fk-2a}{nga}d\textcircled{1} + \dfrac{2fk+2a}{nga}d\textcircled{2}.$
1.	$\dfrac{+4k}{na}d\textcircled{1} - \dfrac{4k}{na}d\textcircled{2}.$
0.	$-\dfrac{+2fk+2a}{nea}d\textcircled{1} + \dfrac{2fk-2a}{nea}d\textcircled{2}.$
$\overline{1}$.	$\dfrac{-4}{3nek}d\,P^3 + \dfrac{\overline{fk+a}\vert^2}{neeka}d\textcircled{1} - \dfrac{+\overline{fk-a}\vert^2}{neeka}d\textcircled{2}.$
&c.	

$P=\sqrt[4]{\dfrac{k+lz^n}{z^n}}.\ a=\sqrt[2]{ffkk-4egkk}.\ p=\tfrac{1}{2}fk-el+\tfrac{1}{2}a.\ q=\tfrac{1}{2}fk-el-\tfrac{1}{2}a.$

Caſ. 1.

$OP:OA::P:\sqrt[4]{l};\quad \circledcirc = +\tfrac{1}{4}\sqrt[4]{\dfrac{1}{l}}$ in $-(AP:AO)$
$+(EP:EO)-\chi(PCO).$

$OP:OA::P:\sqrt[4]{\dfrac{p}{e}};\quad \textcircled{1} = +\tfrac{1}{4}\sqrt[4]{\dfrac{e}{p}}$ in $\beta(BP:BO)$
$-\beta(DP:DO)+\psi(PBO)+\psi(PDO).$

$OP:OA::P:\sqrt[4]{\dfrac{q}{e}};\quad \textcircled{2} = +\tfrac{1}{4}\sqrt[4]{\dfrac{e}{q}}$ in $\beta(BP:BO)$
$-\beta(DP:DO)+\psi(PBO)+\psi(PDO).$

Caſ. 2.

$OP:OA::P:\sqrt[4]{-l};\quad \circledcirc = -\tfrac{1}{4}\sqrt[4]{\dfrac{-1}{l}}$ in $\beta(BP:BO)$
$-\beta(DP:DO)+\psi(PBO)+\psi(PDO).$

$OP:OA::P:\sqrt[4]{\dfrac{-p}{e}};\quad \textcircled{1} = -\tfrac{1}{4}\sqrt[4]{\dfrac{-e}{p}}$ in $-(AP:AO)$
$+(EP:EO)-\chi(PCO).$

$OP:OA::P:\sqrt[4]{\dfrac{-q}{e}};\quad \textcircled{2} = -\tfrac{1}{4}\sqrt[4]{\dfrac{-e}{q}}$ in $-(AP:AO)$
$+(EP:EO)-\chi(PCO).$

θ	FORMÆ LXXIX. $\dfrac{d\dot{z}z^{\theta n-1}}{e+fz^{n}+gz^{2n}\sqrt[4]{k+lz^{n}}}$ Caſ. 2.
&c.	
2.	$\overset{-q,\overline{s+a}}{\dfrac{-akr}{2\eta qs}}d$ in $\textcircled{1}+\textcircled{2}+\mathrm{M}$; $+\overset{-q,\overline{s-a}}{\dfrac{+akr}{2\eta qs}}d$ in $\textcircled{3}+\textcircled{4}+\mathrm{N}$.
1.	$\dfrac{+alr}{2\eta qs}d$ in $\textcircled{1}+\textcircled{2}+\mathrm{M}$; $+\dfrac{-alr}{2\eta qs}d$ in $\textcircled{3}+\textcircled{4}+\mathrm{N}$.
0.	$\dfrac{-4}{\eta e}d\odot+\dfrac{\substack{+gq,\overline{s+a}\\ -ar,\overline{fl-gk}}}{2\eta eqs}d$ in $\textcircled{1}+\textcircled{2}+\mathrm{M}$; $+\dfrac{\substack{+gq,\overline{s-a}\\ ar,\overline{fl-gk}}}{2\eta eqs}d$ in $\textcircled{3}+\textcircled{4}+\mathrm{N}$.
&c.	

$$\mathrm{P}=\sqrt[4]{k+lz^{n}}.\quad q=ell-fkl+gkk.$$

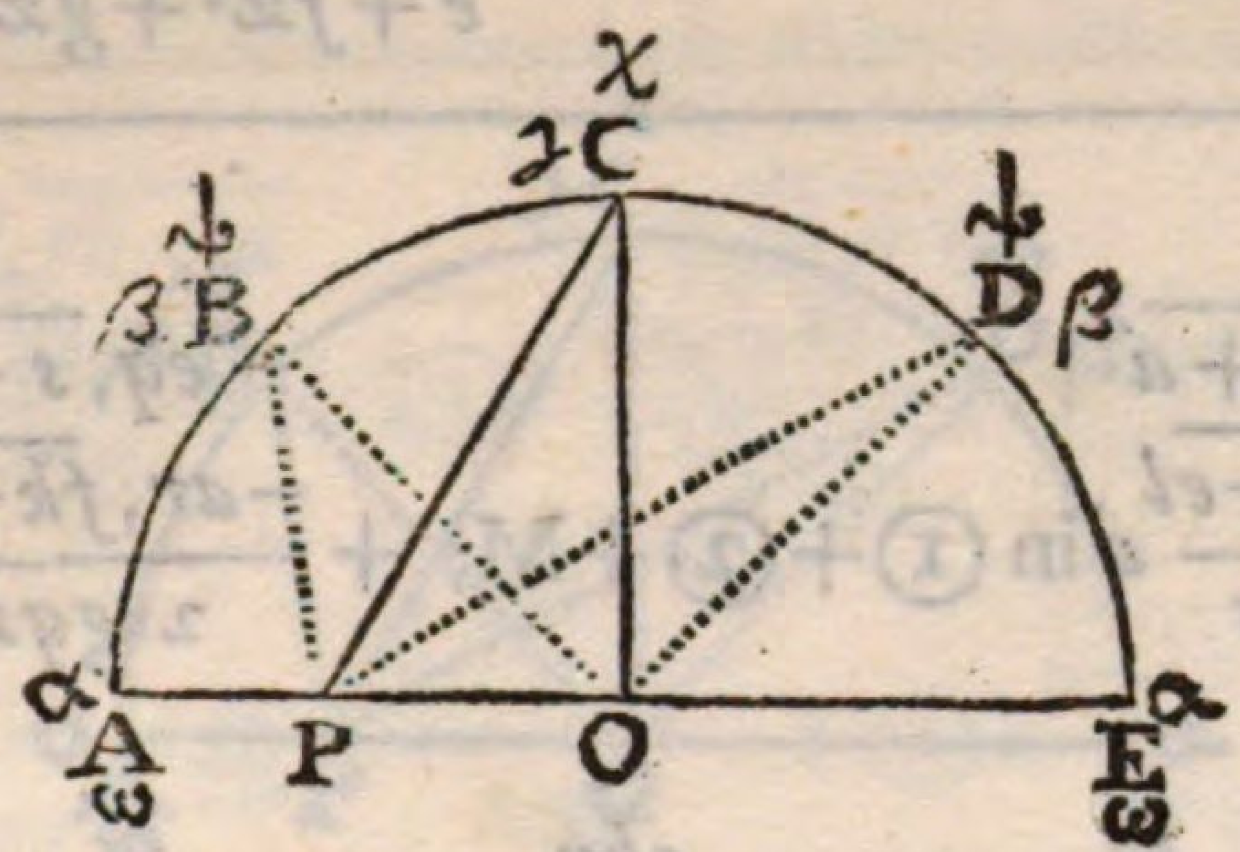
$$r=\sqrt{gq}.\quad s=\sqrt{-fgl+2ggk+2\sqrt{qg^{3}}}.\quad a=\sqrt[4]{qg^{3}}.$$

$$b=\sqrt{+\sqrt{-fg^{3}l+2g^{4}k+2\sqrt{qg^{7}}}+2\sqrt[4]{qg^{7}}}.$$

$$c=\sqrt{-\sqrt{-fg^{3}l+2g^{4}k+2\sqrt{qg^{7}}}+2\sqrt[4]{qg^{7}}}.$$

$$\mathrm{M}=\tfrac{1}{b}(a+b\mathrm{P}+g\mathrm{PP}:a-b\mathrm{P}+g\mathrm{PP}).$$

$$\mathrm{N}=\tfrac{1}{c}(a+c\mathrm{P}+g\mathrm{PP}:a-c\mathrm{P}+g\mathrm{PP}.)$$



$$\left\{\begin{array}{l} OP : OA :: \mathrm{P} : \sqrt[4]{k}\,; \quad \circledcirc = {}^{-+}\tfrac{1}{4}\sqrt[4]{\frac{1}{k}} \text{ in } -(AP : AO) \\ \quad +(EP : EO) - \chi(PCO). \end{array}\right.$$

Cas. 1.

$$\left\{\begin{array}{l} OP : OA :: {}^{-+}\tfrac{1}{2}c + g\,\mathrm{P} : \sqrt[2]{\tfrac{1}{4}cc - ag}\,; \; \textcircled{1} = {}^{-+}\tfrac{1}{2}\sqrt[2]{\frac{1}{\frac{1}{4}cc - ag}} \text{ in } (EP : AP). \\ OP : OA :: {}^{-}\tfrac{1}{2}c + g\,\mathrm{P} : \sqrt[2]{\tfrac{1}{4}cc - ag}\,; \; \textcircled{2} = {}^{-+}\tfrac{1}{2}\sqrt[2]{\frac{1}{\frac{1}{4}cc - ag}} \text{ in } (EP : AP). \\ OP : OA :: {}^{-+}\tfrac{1}{2}b + g\,\mathrm{P} : \sqrt[2]{\tfrac{1}{4}bb - ag}\,; \; \textcircled{3} = {}^{-+}\tfrac{1}{2}\sqrt[2]{\frac{1}{\frac{1}{4}bb - ag}} \text{ in } (EP : AP). \\ OP : OA :: {}^{-}\tfrac{1}{2}b + g\,\mathrm{P} : \sqrt[2]{\tfrac{1}{4}bb - ag}\,; \; \textcircled{4} = {}^{-+}\tfrac{1}{2}\sqrt[2]{\frac{1}{\frac{1}{4}bb - ag}} \text{ in } (EP : AP). \end{array}\right.$$

$$\left\{\begin{array}{l} OP : OA :: \mathrm{P} : \sqrt[4]{-k}\,; \quad \circledcirc = {}^{--}\tfrac{1}{4}\sqrt[4]{\frac{-1}{k}} \text{ in } +\beta(BP : BO) \\ \quad -\beta(DP : DO) + \psi(PBO) + \psi(PDO). \end{array}\right.$$

Cas. 2.

$$\left\{\begin{array}{l} OP : OA :: {}^{-+}\tfrac{1}{2}c + g\,\mathrm{P} : \sqrt[2]{ag - \tfrac{1}{4}cc}\,; \; \textcircled{1} = {}^{--}\tfrac{1}{2}\sqrt[2]{\frac{1}{ag - \frac{1}{4}cc}} \text{ in } \chi(PCO). \\ OP : OA :: {}^{-}\tfrac{1}{2}c + g\,\mathrm{P} : \sqrt[2]{ag - \tfrac{1}{4}cc}\,; \; \textcircled{2} = {}^{--}\tfrac{1}{2}\sqrt[2]{\frac{1}{ag - \frac{1}{4}cc}} \text{ in } \chi(PCO). \\ OP : OA :: {}^{-+}\tfrac{1}{2}b + g\,\mathrm{P} : \sqrt[2]{ag - \tfrac{1}{4}bb}\,; \; \textcircled{3} = {}^{--}\tfrac{1}{2}\sqrt[2]{\frac{1}{ag - \frac{1}{4}bb}} \text{ in } \chi(PCO). \\ OP : OA :: {}^{-}\tfrac{1}{2}b + g\,\mathrm{P} : \sqrt[2]{ag - \tfrac{1}{4}bb}\,; \; \textcircled{4} = {}^{--}\tfrac{1}{2}\sqrt[2]{\frac{1}{ag - \frac{1}{4}bb}} \text{ in } \chi(PCO). \end{array}\right.$$

θ	FORMÆ LXXX. $\dfrac{d\dot{z}z^{\theta\eta+\frac{1}{4}\eta-1}}{\overline{e+fz^{\eta}+gz^{2\eta}}\,\sqrt[4]{\overline{k+lz^{\eta}}}}$ *Caſ.* 2.
&c.	
2.	$\frac{4}{\eta g}d\textcircled{0}+\dfrac{\substack{-eq,\overline{s+a}\\ ar,\overline{fk-el}}}{2\eta gqs}d\,\text{in}\,\textcircled{1}+\textcircled{2}+\text{M};\;+\dfrac{\substack{-eq,\overline{s-a}\\ -ar,\overline{fk-el}}}{2\eta gqs}d\,\text{in}\,\textcircled{3}+\textcircled{4}+\text{N}.$
1.	$\dfrac{-akr}{2\eta qs}d\,\text{in}\,\textcircled{1}+\textcircled{2}+\text{M};\;+\dfrac{akr}{2\eta qs}d\,\text{in}\,\textcircled{3}+\textcircled{4}+\text{N}.$
0.	$\dfrac{\substack{+q,\overline{s+a}\\ +alr}}{2\eta qs}d\,\text{in}\,\textcircled{1}+\textcircled{2}+\text{M};\;+\dfrac{\substack{+q,\overline{s-a}\\ -alr}}{2\eta qs}d\,\text{in}\,\textcircled{3}+\textcircled{4}+\text{N}.$
&c.	

$$\text{P}=\sqrt[4]{\frac{k+lz^{\eta}}{z^{\eta}}}.\quad q=ell-fkl+gkk.$$

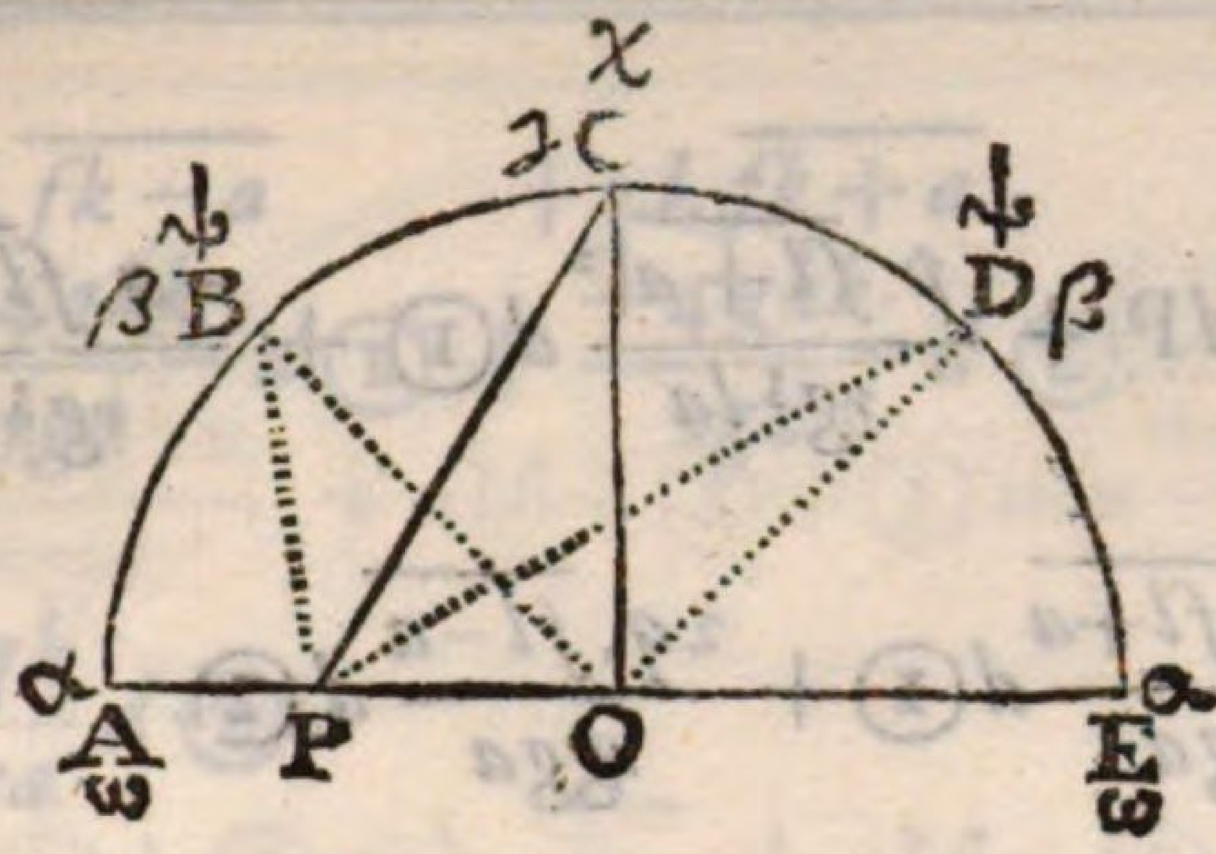
$$r=\sqrt{eq}.\quad s=\sqrt{-efk+2eel+2\sqrt{qe^3}}.\quad a=\sqrt[4]{qe^3}.$$

$$b=\sqrt{+\sqrt{-e^3fk+2e^4l+2\sqrt{qe^7}}+2\sqrt[4]{qe^7}}.$$

$$c=\sqrt{-\sqrt{-e^3fk+2e^4l+2\sqrt{qe^7}}+2\sqrt[4]{qe^7}}.$$

$$\text{M}=\tfrac{1}{b}(a+b\text{P}+e\text{PP}:a-b\text{P}+e\text{PP}).$$

$$\text{N}=\tfrac{1}{c}(a+c\text{P}+e\text{PP}:a-c\text{P}+e\text{PP}).$$



$$\left\{\begin{array}{l} OP:OA::P:\sqrt[4]{l}; \quad \circledcirc = \pm\tfrac{1}{4}\sqrt[4]{\tfrac{1}{l}} \text{ in } -(AP:AO) \\ \qquad +(EP:EO)-\chi(PCO). \end{array}\right.$$

Cas. 1.

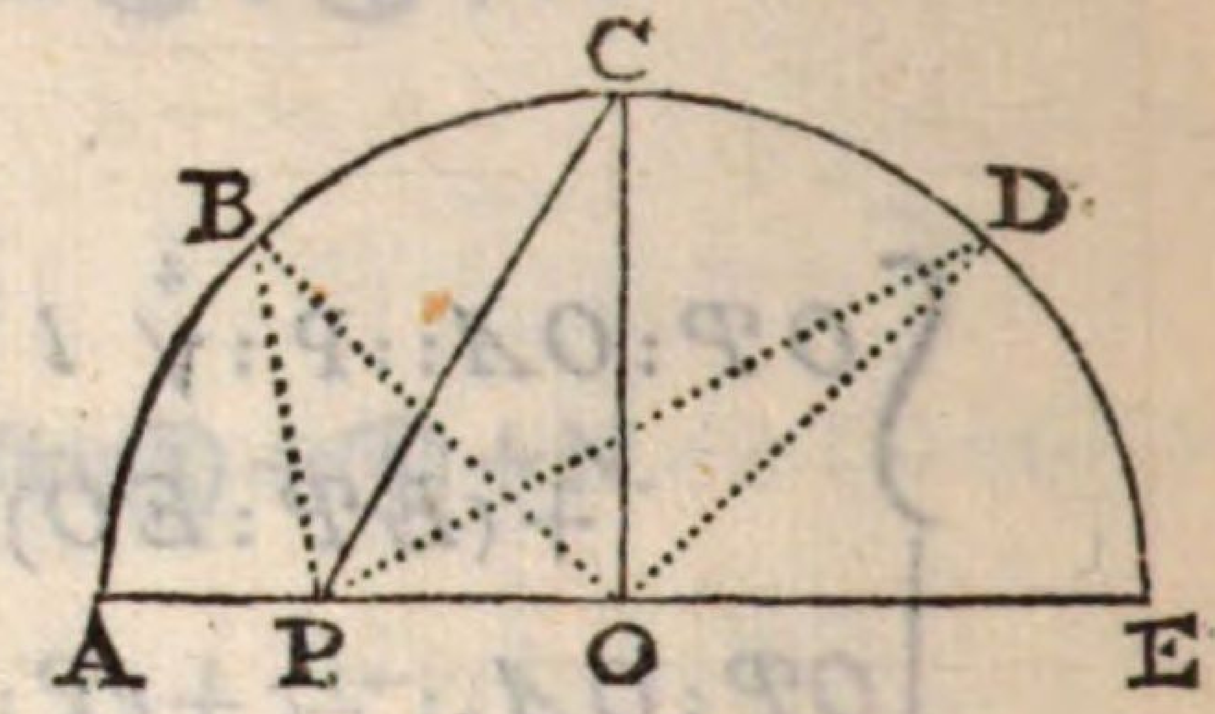
$$\left\{\begin{array}{l} OP:OA::\pm\tfrac{1}{2}c+eP:\sqrt[2]{\tfrac{1}{4}cc-ae};\ \textcircled{1}=\pm\tfrac{1}{2}\sqrt[2]{\dfrac{1}{\tfrac{1}{4}cc-ae}} \text{ in }(EP:AP). \\ OP:OA::-\tfrac{1}{2}c+eP:\sqrt[2]{\tfrac{1}{4}cc-ae};\ \textcircled{2}=\pm\tfrac{1}{2}\sqrt[2]{\dfrac{1}{\tfrac{1}{4}cc-ae}} \text{ in }(EP:AP). \\ OP:OA::\pm\tfrac{1}{2}b+eP:\sqrt[2]{\tfrac{1}{4}bb-ae};\ \textcircled{3}=\pm\tfrac{1}{2}\sqrt[2]{\dfrac{1}{\tfrac{1}{4}bb-ae}} \text{ in }(EP:AP). \\ OP:OA::-\tfrac{1}{2}b+eP:\sqrt[2]{\tfrac{1}{4}bb-ae};\ \textcircled{4}=\pm\tfrac{1}{2}\sqrt[2]{\dfrac{1}{\tfrac{1}{4}bb-ae}} \text{ in }(EP:AP). \end{array}\right.$$

$$\left\{\begin{array}{l} OP:OA::P:\sqrt[4]{-l}; \quad \circledcirc = -\tfrac{1}{4}\sqrt[4]{\tfrac{-1}{l}} \text{ in } +\beta(BP:BO) \\ \qquad -\beta(DP:DO)+\psi(PBO)+\psi(PDO). \end{array}\right.$$

Cas. 2.

$$\left\{\begin{array}{l} OP:OA::\pm\tfrac{1}{2}c+eP:\sqrt[2]{ae-\tfrac{1}{4}cc};\ \textcircled{1}=-\tfrac{1}{2}\sqrt[2]{\dfrac{1}{ae-\tfrac{1}{4}cc}} \text{ in }\chi(PCO). \\ OP:OA::-\tfrac{1}{2}c+eP:\sqrt[2]{ae-\tfrac{1}{4}cc};\ \textcircled{2}=-\tfrac{1}{2}\sqrt[2]{\dfrac{1}{ae-\tfrac{1}{4}cc}} \text{ in }\chi(PCO). \\ OP:OA::\pm\tfrac{1}{2}b+eP:\sqrt[2]{ae-\tfrac{1}{4}bb};\ \textcircled{3}=-\tfrac{1}{2}\sqrt[2]{\dfrac{1}{ae-\tfrac{1}{4}bb}} \text{ in }\chi(PCO). \\ OP:OA::-\tfrac{1}{2}b+eP:\sqrt[2]{ae-\tfrac{1}{4}bb};\ \textcircled{4}=-\tfrac{1}{2}\sqrt[2]{\dfrac{1}{ae-\tfrac{1}{4}bb}} \text{ in }\chi(PCO). \end{array}\right.$$

θ	FORMÆ LXXXI. $\frac{d\dot{z}z^{\theta n-1}\sqrt[4]{k+lz^n}}{e+fz^n+gz^{2n}}$ *Cas.* 1.
&c.	
3.	$\frac{-20fl+4gk+4glz^n}{5\eta ggl}dP+\frac{p.\overline{fl+a}\vert^2}{\eta g^3la}d\textcircled{1}+\frac{-q.\overline{fl-a}\vert^2}{\eta g^3la}d\textcircled{2}.$
2.	$\frac{+4}{\eta g}dP+\frac{-2p.\overline{fl+a}}{\eta gga}d\textcircled{1}+\frac{2q.\overline{fl-a}}{\eta gga}d\textcircled{2}.$
1.	$\frac{+4pl}{\eta ga}d\textcircled{1}-\frac{4ql}{\eta ga}d\textcircled{2}.$
0.	$\frac{-4k}{\eta e}d\textcircled{0}+\frac{2k.\overline{fl-a}\;-4ell}{\eta ea}d\textcircled{1}+\frac{-2k.\overline{fl+a}\;+4ell}{\eta ea}d\textcircled{2}.$
&c.	



$P=\sqrt[4]{k+lz^n}.\quad a=\sqrt{ffll-4egll}.\quad p=\frac{1}{2}fl-gk+\frac{1}{2}a.\quad q=\frac{1}{2}fl-gk-\frac{1}{2}a.$

Cas. 1.

$OP:OA::P:\sqrt[4]{k};\quad \textcircled{0}={}^{+}\frac{1}{4}\sqrt[4]{\frac{1}{k^3}}$ in $-(AP:AO)+(EP:EO)+\chi(PCO).$

$OP:OA::P:\sqrt[4]{\frac{p}{g}};\quad \textcircled{1}={}^{+}\frac{1}{4}\sqrt[4]{\frac{g^3}{p^3}}$ in $-\beta(BP:BO)+\beta(DP:DO)+\psi(PBO)+\psi(PDO).$

$OP:OA::P:\sqrt[4]{\frac{q}{g}};\quad \textcircled{2}={}^{+}\frac{1}{4}\sqrt[4]{\frac{g^3}{q^3}}$ in $-\beta(BP:BO)+\beta(DP:DO)+\psi(PBO)+\psi(PDO).$

Cas. 2.

$OP:OA::P:\sqrt[4]{-k};\quad \textcircled{0}={}^{-}\frac{1}{4}\sqrt[4]{\frac{-1}{k^3}}$ in $-\beta(BP:BO)+\beta(DP:DO)+\psi(PBO)+\psi(PDO).$

$OP:OA::P:\sqrt[4]{\frac{-p}{g}};\quad \textcircled{1}={}^{-}\frac{1}{4}\sqrt[4]{\frac{-g^3}{q^3}}$ in $-(AP:AO)+(EP:EO)+\chi(PCO).$

$OP:OA::P:\sqrt[4]{\frac{-q}{g}};\quad \textcircled{2}={}^{-}\frac{1}{4}\sqrt[4]{\frac{-g^3}{q^3}}$ in $-(AP:AO)+(EP:EO)+\chi(PCO).$

θ	FORMÆ LXXXII. $\dfrac{d\dot{z}z^{\theta n+\frac{1}{4}n-1}\sqrt[4]{k+lz^n}}{e+fz^n+gz^{2n}}$ *Cas.* I.
$\overline{}$ &c.	
1.	$\dfrac{+4l}{\eta g}d\circledcirc+\dfrac{\begin{matrix}-2l,\overline{fk-a}\\+4gkk\end{matrix}}{\eta ga}d\,\textcircled{1}+\dfrac{\begin{matrix}+2l,\overline{fk+a}\\-4gkk\end{matrix}}{\eta ga}d\,\textcircled{2}.$
0.	$\dfrac{-4pk}{\eta ea}d\,\textcircled{1}+\dfrac{4qk}{\eta ea}d\,\textcircled{2}.$
$\overline{1}.$	$\dfrac{-4}{\eta e}d\text{P}+\dfrac{2p,\overline{fk+a}}{\eta eea}d\,\textcircled{1}+\dfrac{-2q,\overline{fk-a}}{\eta eea}d\,\textcircled{2}.$
$\overline{2}.$	$\dfrac{\begin{matrix}+20fkz^n\\-4ek-4elz^n\end{matrix}}{5\eta eekz^n}d\text{P}+\dfrac{-p,\overline{fk+a}\vert^2}{\eta e^3ka}d\,\textcircled{1}+\dfrac{q,\overline{fk-a}\vert^2}{\eta e^3ka}d\,\textcircled{2}.$
&c.	

$\text{P}=\sqrt[4]{\dfrac{k+lz^n}{z^n}}.\ a=\sqrt[2]{ffkk-4egkk}.\ p=\tfrac{1}{2}fk-el+\tfrac{1}{2}a.\ q=\tfrac{1}{2}fk-el-\tfrac{1}{2}a.$

Cas. I.

$$\left\{\begin{aligned}&OP:OA::\text{P}:\sqrt[4]{l};\quad \circledcirc=+\tfrac{1}{4}\sqrt[4]{\tfrac{1}{l^3}}\ \text{in}-(AP:AO)\\&\quad+(EP:EO)+\chi(PCO).\end{aligned}\right.$$

$$\left\{\begin{aligned}&OP:OA::\text{P}:\sqrt[4]{\tfrac{p}{e}};\quad \textcircled{1}=+\tfrac{1}{4}\sqrt[4]{\tfrac{e^3}{p^3}}\ \text{in}-\beta(BP:BO)\\&\quad+\beta(DP:DO)+\psi(PBO)+\psi(PDO).\end{aligned}\right.$$

$$\left\{\begin{aligned}&OP:OA::\text{P}:\sqrt[4]{\tfrac{q}{e}};\quad \textcircled{2}=+\tfrac{1}{4}\sqrt[4]{\tfrac{e^3}{q^3}}\ \text{in}-\beta(BP:BO)\\&\quad+\beta(DP:DO)+\psi(PBO)+\psi(PDO).\end{aligned}\right.$$

Cas. 2.

$$\left\{\begin{aligned}&OP:OA::\text{P}:\sqrt[4]{-l};\quad \circledcirc=-\tfrac{1}{4}\sqrt[4]{\tfrac{-1}{l^3}}\ \text{in}-\beta(BP:BO)\\&\quad+\beta(DP:DO)+\psi(PBO)+\psi(PDO).\end{aligned}\right.$$

$$\left\{\begin{aligned}&OP:OA::\text{P}:\sqrt[4]{\tfrac{-p}{e}};\quad \textcircled{1}=-\tfrac{1}{4}\sqrt[4]{\tfrac{-e^3}{p^3}}\ \text{in}-(AP:AO)\\&\quad+(EP:EO)+\chi(PCO).\end{aligned}\right.$$

$$\left\{\begin{aligned}&OP:OA::\text{P}:\sqrt[4]{\tfrac{-q}{e}};\quad \textcircled{2}=-\tfrac{1}{4}\sqrt[4]{\tfrac{-e^3}{q^3}}\ \text{in}-(AP:AO)\\&\quad+(EP:EO)+\chi(PCO).\end{aligned}\right.$$

θ	FORMÆ LXXXI. $\frac{d\dot{z}z^{\theta n-1}\sqrt[4]{k+lz^{n}}}{e+fz^{n}+gz^{2n}}$ *Cas.* 2.
—	
&c.	
2.	$\frac{4}{\eta g}dP+\frac{\begin{matrix}-g,\overline{fl-gk}\\ as+gr\end{matrix}}{2\eta gs}d\,\text{in}\,\textcircled{1}+\textcircled{2}-M;+\frac{\begin{matrix}+g,\overline{fl-gk}\\ as-gr\end{matrix}}{2\eta gs}d\,\text{in}\,\textcircled{3}+\textcircled{4}-N.$
1.	$\frac{gl}{2\eta s}d\,\text{in}\,\textcircled{1}+\textcircled{2}-M;+\frac{-gl}{2\eta s}d\,\text{in}\,\textcircled{3}+\textcircled{4}-N.$
0.	$\frac{-4k}{\eta e}d\textcircled{0}+\frac{\begin{matrix}-gr-as\\ -ggk\end{matrix}}{2\eta es}d\,\text{in}\,\textcircled{1}+\textcircled{2}-M;+\frac{\begin{matrix}+gr-as\\ ggk\end{matrix}}{2\eta es}d\,\text{in}\,\textcircled{3}+\textcircled{4}-N.$
&c.	

$$P=\sqrt[4]{k+lz^{n}}.\quad q=ell-fkl+gkk.$$

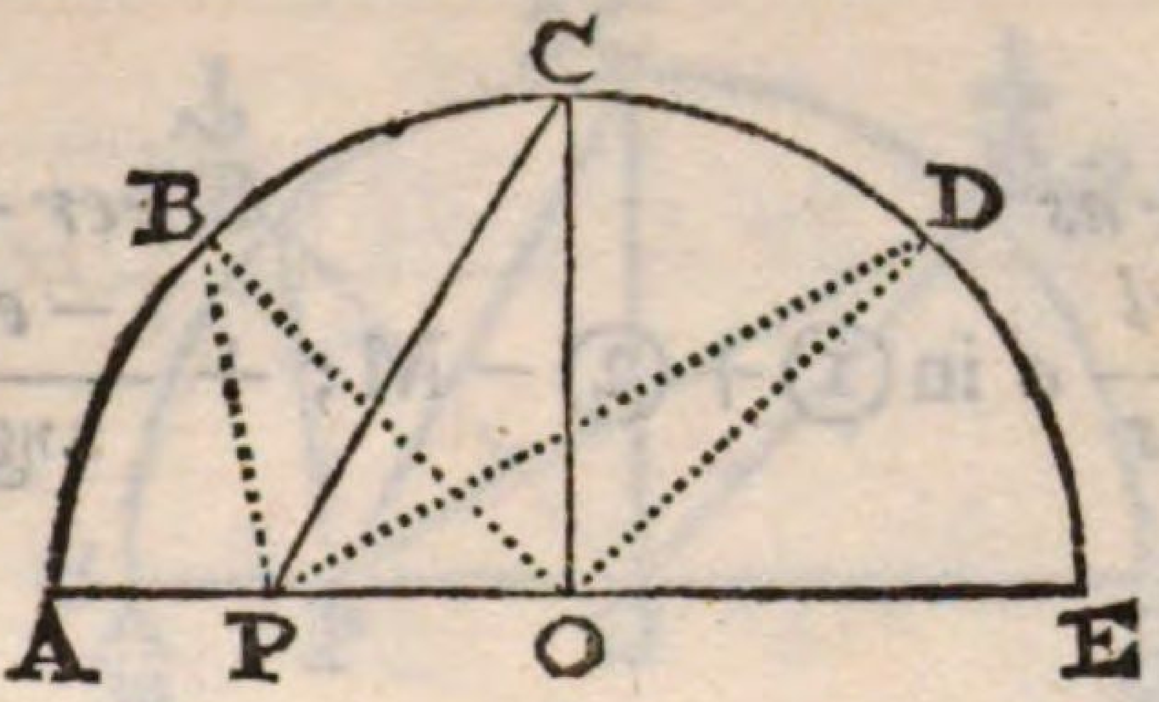
$$r=\sqrt{gq}.\quad s=\sqrt{-fgl+2ggk+2\sqrt{qg^{3}}}.\quad a=\sqrt[4]{qg^{3}}.$$

$$b=\sqrt{+\sqrt{-fg^{3}l+2g^{4}k+2\sqrt{qg^{7}}}+2\sqrt[4]{qg^{7}}}.$$

$$c=\sqrt{-\sqrt{-fg^{3}l+2g^{4}k+2\sqrt{qg^{7}}}+2\sqrt[4]{qg^{7}}}.$$

$$M=\tfrac{1}{b}(a+bP+gPP:a-bP+gPP).$$

$$N=\tfrac{1}{c}(a+cP+gPP:a-cP+gPP.)$$



$$\left\{\begin{array}{l} OP:OA::P:\sqrt[4]{k}; \quad \circledcirc = \pm\tfrac{1}{4}\sqrt[4]{\tfrac{1}{k^3}} \text{ in } -(AP:AO) \\ \quad +(EP:EO) + \chi(PCO). \end{array}\right.$$

Caſ. 1.
$$\left\{\begin{array}{l} OP:OA::\pm\tfrac{1}{2}c+gP:\sqrt[2]{\tfrac{1}{4}cc-ag}; \textcircled{1} = \pm\tfrac{1}{2}\sqrt[2]{\dfrac{1}{\tfrac{1}{4}cc-ag}} \text{ in } (EP:AP) \\ OP:OA::-\tfrac{1}{2}c+gP:\sqrt[2]{\tfrac{1}{4}cc-ag}; \textcircled{2} = \pm\tfrac{1}{2}\sqrt[2]{\dfrac{1}{\tfrac{1}{4}cc-ag}} \text{ in } (EP:AP) \\ OP:OA::\pm\tfrac{1}{2}b+gP:\sqrt[2]{\tfrac{1}{4}bb-ag}; \textcircled{3} = \pm\tfrac{1}{2}\sqrt[2]{\dfrac{1}{\tfrac{1}{4}bb-ag}} \text{ in } (EP:AP) \\ OP:OA::-\tfrac{1}{2}b+gP:\sqrt[2]{\tfrac{1}{4}bb-ag}; \textcircled{4} = \pm\tfrac{1}{2}\sqrt[2]{\dfrac{1}{\tfrac{1}{4}bb-ag}} \text{ in } (EP:AP) \end{array}\right.$$

$$\left\{\begin{array}{l} OP:OA::P:\sqrt[4]{-k}; \quad \circledcirc = -\tfrac{1}{4}\sqrt[4]{\tfrac{-1}{k^3}} \text{ in } -\beta(BP:BO) \\ \quad +\beta(DP:DO) + \psi(PBO) + \psi(PDO). \end{array}\right.$$

Caſ. 2.
$$\left\{\begin{array}{l} OP:OA::\pm\tfrac{1}{2}c+gP:\sqrt[2]{ag-\tfrac{1}{4}cc}; \textcircled{1} = -\tfrac{1}{2}\sqrt[2]{\dfrac{1}{ag-\tfrac{1}{4}cc}} \text{ in } \chi(PCO). \\ OP:OA::-\tfrac{1}{2}c+gP:\sqrt[2]{ag-\tfrac{1}{4}cc}; \textcircled{2} = -\tfrac{1}{2}\sqrt[2]{\dfrac{1}{ag-\tfrac{1}{4}cc}} \text{ in } \chi(PCO). \\ OP:OA::\pm\tfrac{1}{2}b+gP:\sqrt[2]{ag-\tfrac{1}{4}bb}; \textcircled{3} = -\tfrac{1}{2}\sqrt[2]{\dfrac{1}{ag-\tfrac{1}{4}bb}} \text{ in } \chi(PCO). \\ OP:OA::-\tfrac{1}{2}b+gP:\sqrt[2]{ag-\tfrac{1}{4}bb}; \textcircled{4} = -\tfrac{1}{2}\sqrt[2]{\dfrac{1}{ag-\tfrac{1}{4}bb}} \text{ in } \chi(PCO). \end{array}\right.$$

θ	FORMÆ LXXXII. $\frac{dz\,z^{\theta n+\frac{3}{4}n-1}\sqrt[4]{k+lz^n}}{e+fz^n+gz^{2n}}$ *Cas.* 2.
&c.	
1.	$\frac{4l}{\eta g}d\,\textcircled{0}+\frac{\substack{+er+as\\ +eel}}{2\eta gs}d$ in $\textcircled{1}+\textcircled{2}-\text{M};+\frac{\substack{-er+as\\ -eel}}{2\eta gs}d$ in $\textcircled{3}+\textcircled{4}-\text{N}$.
0.	$\frac{-ek}{2\eta s}d$ in $\textcircled{1}+\textcircled{2}-\text{M};+\frac{ek}{2\eta s}d$ in $\textcircled{3}+\textcircled{4}-\text{N}$.
$\overline{1}$.	$\frac{-4}{\eta e}d\,\text{P}+\frac{\substack{+e,\overline{fk-el}\\ -as-er}}{2\eta es}d$ in $\textcircled{1}+\textcircled{2}-\text{M};+\frac{\substack{-e,\overline{fk-el}\\ -as+er}}{2\eta es}d$ in $\textcircled{3}+\textcircled{4}-\text{N}$.
&c.	

$$\text{P}=\sqrt[4]{\frac{k+lz^n}{z^n}}.\quad q=ell-fkl+gkk.$$

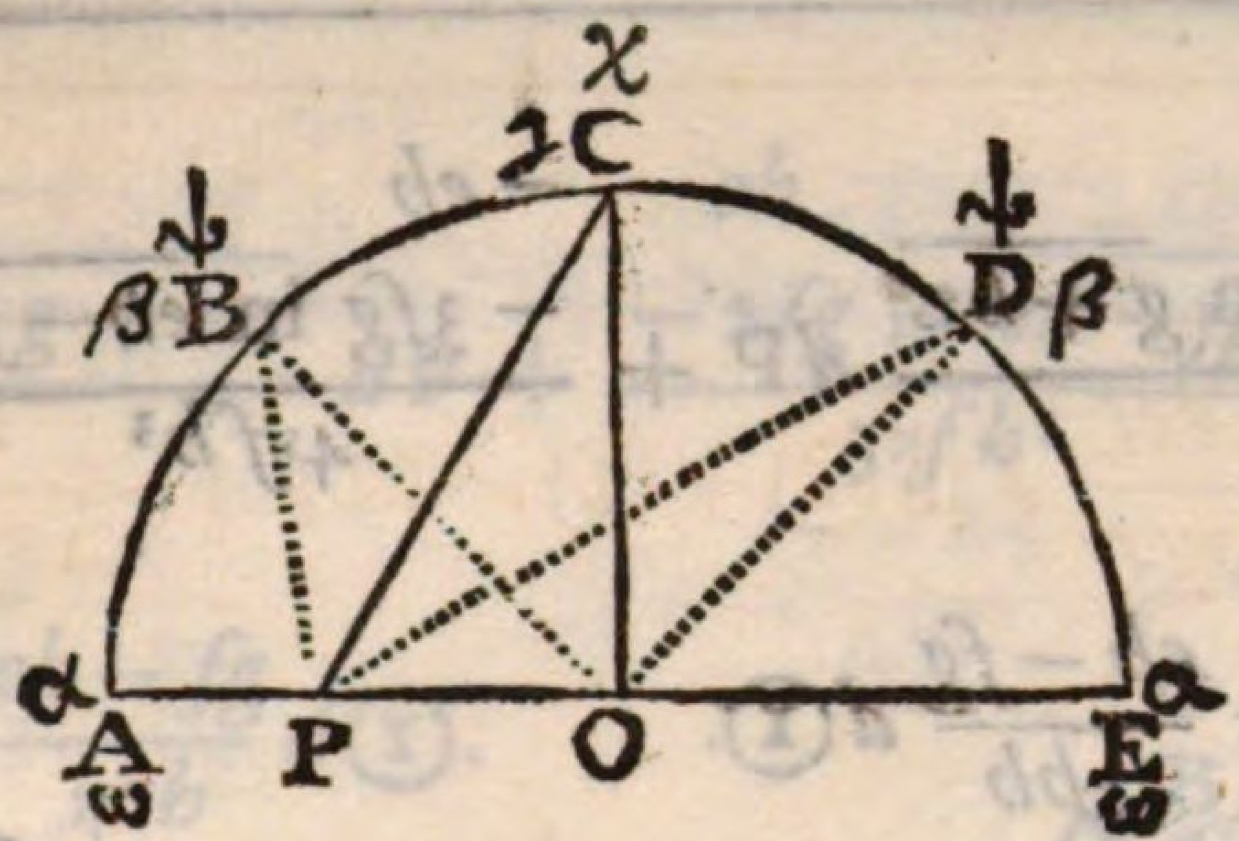
$$r=\sqrt{eq}.\quad s=\sqrt{-efk+2eel+2\sqrt{qe^3}}.\quad a=\sqrt[4]{qe^3}.$$

$$b=\sqrt{+\sqrt{-e^3fk+2e^4l+2\sqrt{qe^7}}+2\sqrt[4]{qe^7}}.$$

$$c=\sqrt{-\sqrt{-e^3fk+2e^4l+2\sqrt{qe^7}}+2\sqrt[4]{qe^7}}.$$

$$\text{M}=\tfrac{1}{b}(a+b\text{P}+e\text{PP}:a-b\text{P}+e\text{PP}).$$

$$\text{N}=\tfrac{1}{c}(a+c\text{P}+e\text{PP}:a-c\text{P}+e\text{PP}).$$



$$\left\{\begin{array}{l} OP:OA::P:\sqrt[4]{l}\,; \quad \circledcirc = {}^{+}\tfrac{1}{4}\sqrt[4]{\tfrac{1}{l^3}} \text{ in } -(AP:AO) \\ \qquad +(EP:EO)+\chi(PCO). \end{array}\right.$$

Caſ. 1.

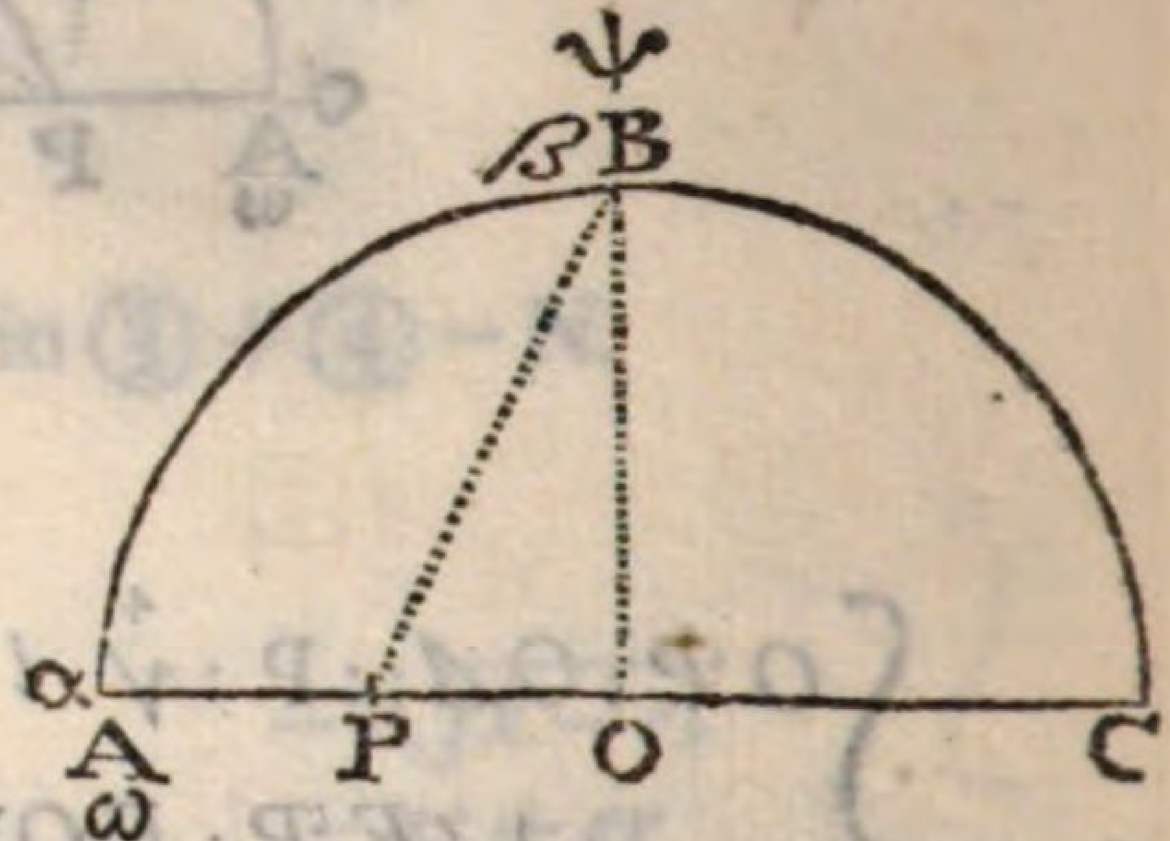
$$\left\{\begin{array}{l} OP:OA::{}^{+}\tfrac{1}{2}c+eP:\sqrt[2]{\tfrac{1}{4}cc-ae}\,;\ ① = {}^{+}\tfrac{1}{2}\sqrt[2]{\dfrac{1}{\frac{1}{4}cc-ae}} \text{ in } (EP:AP). \\ OP:OA::{}^{-}\tfrac{1}{2}c+eP:\sqrt[2]{\tfrac{1}{4}cc-ae}\,;\ ② = {}^{+}\tfrac{1}{2}\sqrt[2]{\dfrac{1}{\frac{1}{4}cc-ae}} \text{ in } (EP:AP). \\ OP:OA::{}^{+}\tfrac{1}{2}b+eP:\sqrt[2]{\tfrac{1}{4}bb-ae}\,;\ ③ = {}^{+}\tfrac{1}{2}\sqrt[2]{\dfrac{1}{\frac{1}{4}bb-ae}} \text{ in } (EP:AP). \\ OP:OA::{}^{-}\tfrac{1}{2}b+eP:\sqrt[2]{\tfrac{1}{4}bb-ae}\,;\ ④ = {}^{+}\tfrac{1}{2}\sqrt[2]{\dfrac{1}{\frac{1}{4}bb-ae}} \text{ in } (EP:AP). \end{array}\right.$$

$$\left\{\begin{array}{l} OP:OA::P:\sqrt[4]{-l}\,; \quad \circledcirc = {}^{-}\tfrac{1}{4}\sqrt[4]{\tfrac{-1}{l^3}} \text{ in } -\beta(BP:BO) \\ \qquad +\beta(DP:DO)+\psi(PBO)+\psi(PDO). \end{array}\right.$$

Caſ. 2.

$$\left\{\begin{array}{l} OP:OA::{}^{+}\tfrac{1}{2}c+eP:\sqrt[2]{ae-\tfrac{1}{4}cc}\,;\ ① = {}^{-}\tfrac{1}{2}\sqrt[2]{\dfrac{1}{ae-\frac{1}{4}cc}} \text{ in } \chi(PCO). \\ OP:OA::{}^{-}\tfrac{1}{2}c+eP:\sqrt[2]{ae-\tfrac{1}{4}cc}\,;\ ② = {}^{-}\tfrac{1}{2}\sqrt[2]{\dfrac{1}{ae-\frac{1}{4}cc}} \text{ in } \chi(PCO). \\ OP:OA::{}^{+}\tfrac{1}{2}b+eP:\sqrt[2]{ae-\tfrac{1}{4}bb}\,;\ ③ = {}^{-}\tfrac{1}{2}\sqrt[2]{\dfrac{1}{ae-\frac{1}{4}bb}} \text{ in } \chi(PCO). \\ OP:OA::{}^{-}\tfrac{1}{2}b+eP:\sqrt[2]{ae-\tfrac{1}{4}bb}\,;\ ④ = {}^{-}\tfrac{1}{2}\sqrt[2]{\dfrac{1}{ae-\frac{1}{4}bb}} \text{ in } \chi(PCO). \end{array}\right.$$

θ	FORMA LXXXIII. $\quad d\dot{z}z^{\theta n-1}\sqrt[2]{\dfrac{e+fz^n}{g+hz^n}}$
&c.	
2.	$\dfrac{\begin{array}{l}+eh\\ -3fg+2fhz^n \text{ in } \overline{g+hz^n}\end{array}}{4\eta fhh}\,d\mathrm{P}+\dfrac{\begin{array}{l}-eh\\ -3fg \text{ in } \overline{eh-fg}\end{array}}{4\eta fh^3}\,d\textcircled{1}.$
1.	$+\dfrac{g+hz^n}{\eta h}\,d\mathrm{P}+\dfrac{eh-fg}{\eta hh}\,d\textcircled{1}.$
0.	$+\dfrac{2f}{\eta h}\,d\textcircled{1}-\dfrac{2e}{\eta g}\,d\boxed{1}.$
$\overline{1}.$	$-\dfrac{+g+hz^n}{\eta gz^n}\,d\mathrm{P}-\dfrac{+fg-eh}{\eta gg}\,d\boxed{1}.$
$\overline{2}.$	$\dfrac{\begin{array}{l}-fgz^n\\ -2eg+3ehz^n \text{ in } \overline{g+hz^n}\end{array}}{4\eta eggz^{2n}}\,d\mathrm{P}+\dfrac{\begin{array}{l}+fg\\ +3eh \text{ in } \overline{fg-eh}\end{array}}{4\eta eg^3}\,d\boxed{1}.$
&c.	



$$\mathrm{P}=\sqrt[2]{\frac{e+fz^n}{g+hz^n}}$$

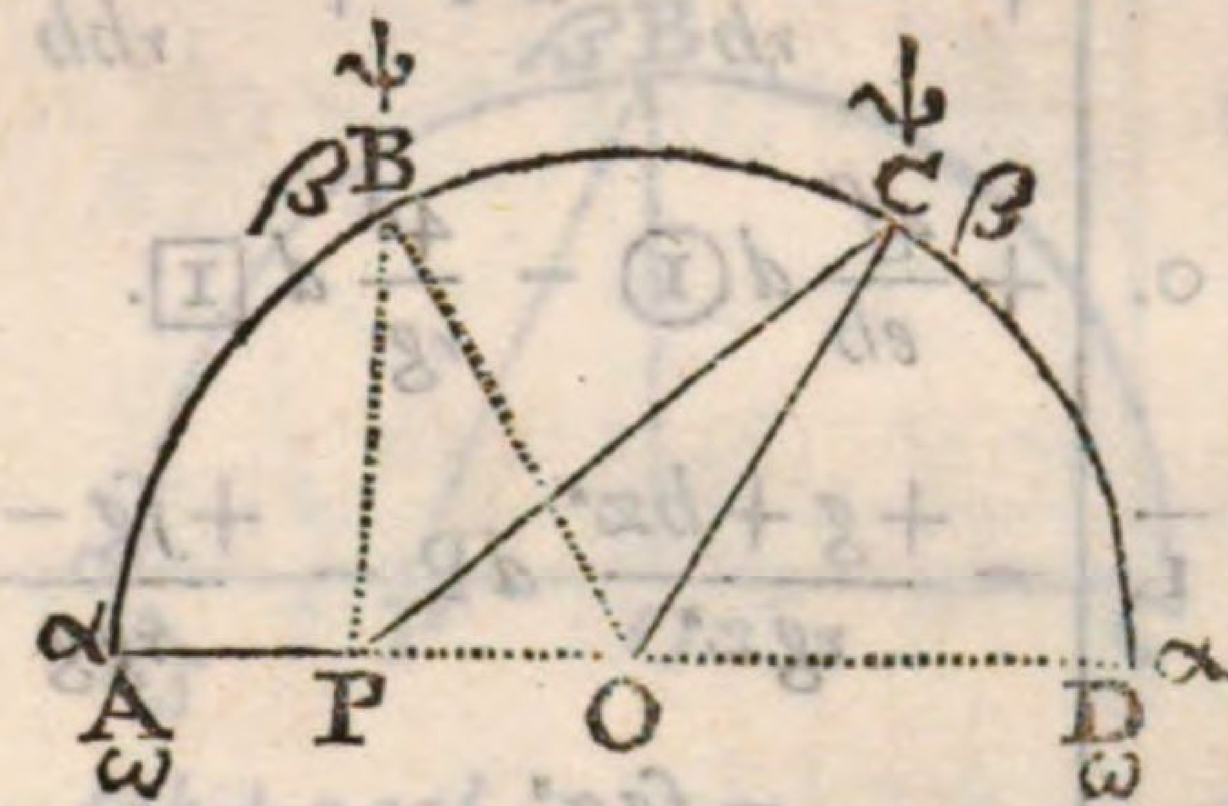
Cas. 1.

$$\left\{\begin{array}{l} OP:OA::\mathrm{P}:\sqrt[2]{\dfrac{f}{h}}\,;\quad \textcircled{1}={}^{+}\tfrac{1}{2}\sqrt[2]{\dfrac{h}{f}} \text{ in} -(AP:AO)+(CP:CO).\\ OP:OA::\mathrm{P}:\sqrt[2]{\dfrac{e}{g}}\,;\quad \boxed{1}={}^{+}\tfrac{1}{2}\sqrt[2]{\dfrac{g}{e}} \text{ in} -(AP:AO)+(CP:CO).\end{array}\right.$$

Cas. 2.

$$\left\{\begin{array}{l} OP:OA::\mathrm{P}:\sqrt[2]{\dfrac{-f}{h}}\,;\quad \textcircled{1}={}^{-}\tfrac{1}{2}\sqrt[2]{\dfrac{-h}{f}} \text{ in } \psi(PBO).\\ OP:OA::\mathrm{P}:\sqrt[2]{\dfrac{-e}{g}}\,;\quad \boxed{1}={}^{-}\tfrac{1}{2}\sqrt[2]{\dfrac{-g}{e}} \text{ in } \psi(PBO).\end{array}\right.$$

θ	FORMA LXXXIV. $d\dot{z}z^{\theta n-1}\sqrt[3]{\frac{e+fz^n}{g+hz^n}}$
&c.	
2.	$\frac{\begin{matrix}+eh\\ -4fg+3fhz^n\end{matrix} \text{ in } \overline{g+hz^n}}{6\eta fhh}dP + \frac{\begin{matrix}-eh\\ -2fg\end{matrix} \text{ in } \overline{eh-fg}}{3\eta fh^3}d\textcircled{I}.$
1.	$+\frac{g+hz^n}{\eta h}dP + \frac{eh-fg}{\eta hh}d\textcircled{I}.$
0.	$+\frac{3f}{\eta h}d\textcircled{I} - \frac{3e}{\eta g}d\boxed{I}.$
$\bar{1}.$	$-\frac{+g+hz^n}{\eta gz^n}dP - \frac{+fg-eh}{\eta gg}d\boxed{I}.$
$\bar{2}.$	$\frac{\begin{matrix}-fgz^n\\ -3eg+4ehz^n\end{matrix} \text{ in } \overline{g+hz^n}}{6\eta eggz^{2n}}dP + \frac{\begin{matrix}+fg\\ +2eh\end{matrix} \text{ in } \overline{fg-eh}}{3\eta eg^3}d\boxed{I}.$
&c.	



$$P=\sqrt[3]{\frac{e+fz^n}{g+hz^n}}$$

Cas. 1.

$$OP:OA::P:\sqrt[3]{\frac{f}{h}};\quad \textcircled{I}=\mp\tfrac{1}{3}\sqrt[3]{\frac{hh}{ff}} \text{ in } -(AP:AO) +\beta(CP:CO) +\psi(PCO).$$

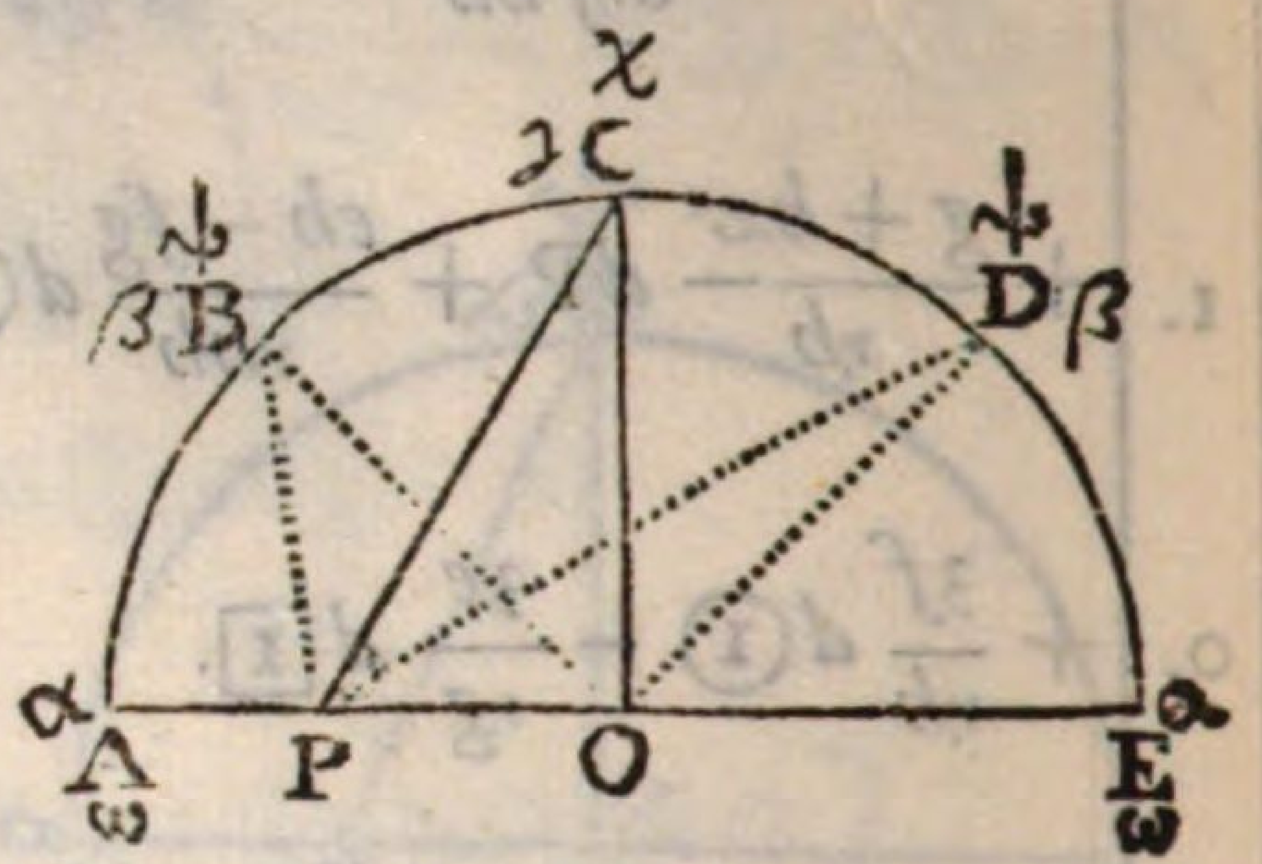
$$OP:OA::P:\sqrt[3]{\frac{e}{g}};\quad \boxed{I}=\mp\tfrac{1}{3}\sqrt[3]{\frac{gg}{ee}} \text{ in } -(AP:AO) +\beta(CP:CO) +\psi(PCO).$$

Cas. 2.

$$OP:OA::P:\sqrt[3]{\frac{-f}{h}};\quad \textcircled{I}=\mp\tfrac{1}{3}\sqrt[3]{\frac{hh}{ff}} \text{ in } -\beta(BP:BO) +(DP:DO) +\psi(PBO).$$

$$OP:OA::P:\sqrt[3]{\frac{-e}{g}};\quad \boxed{I}=\mp\tfrac{1}{3}\sqrt[3]{\frac{gg}{ee}} \text{ in } -\beta(BP:BO) +(DP:DO) +\psi(PBO).$$

θ	FORMA LXXXV. $d\dot{z}z^{\theta n-1}\sqrt[4]{\frac{e+fz^n}{g+hz^n}}$
&c.	
2.	$\frac{\begin{matrix}+eh\\-5fg+4fhz^n\end{matrix}\ \text{in}\ \overline{g+hz^n}}{8nfhh}\,dP+\frac{\begin{matrix}-3eh\\-5fg\end{matrix}\ \text{in}\ \overline{eh-fg}}{8nfh^3}\,d\textcircled{I}.$
1.	$+\frac{g+hz^n}{nh}\,dP+\frac{eh-fg}{nhh}\,d\textcircled{I}.$
0.	$+\frac{4f}{eh}\,d\textcircled{I}-\frac{4e}{ng}\,d\boxed{I}.$
$\overline{1}$.	$-\frac{+g+hz^n}{ngz^n}\,dP-\frac{+fg-eh}{ngg}\,d\boxed{I}.$
$\overline{2}$.	$\frac{\begin{matrix}-fgz^n\\-4eg+5ehz^n\end{matrix}\ \text{in}\ \overline{g+hz^n}}{8neggz^{2n}}\,dP+\frac{\begin{matrix}+3fg\\+5eh\end{matrix}\ \text{in}\ \overline{fg-eh}}{8neg^3}\,d\boxed{I}.$
&c.	

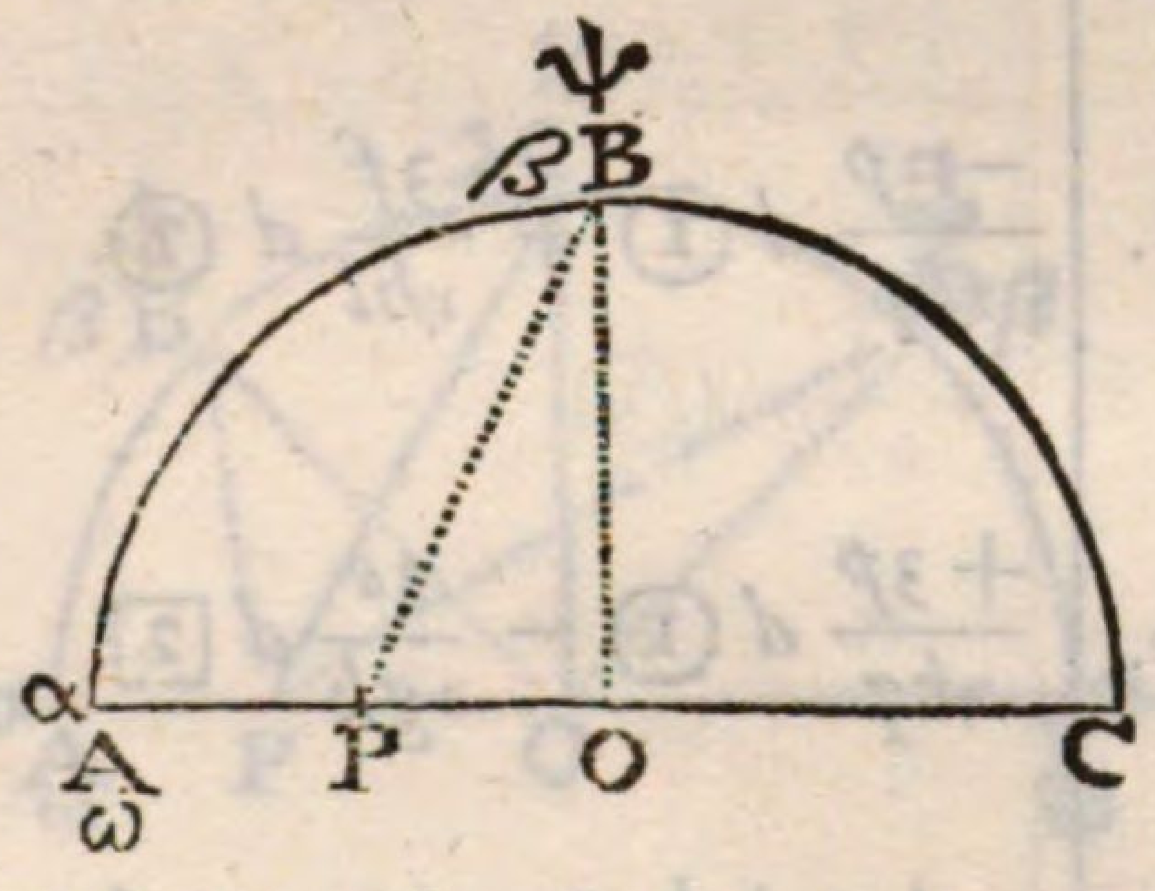


$$P=\sqrt[4]{\frac{e+fz^n}{g+hz^n}}$$

Cas. 1.
$$OP::OA::P:\sqrt[4]{\frac{f}{h}};\quad \textcircled{I}={}^{+}\tfrac{1}{4}\sqrt[4]{\frac{h^3}{f^3}}\ \text{in}\ -(AP:AO)+(EP:EO)+\chi(PCO).$$
$$OP::OA::P:\sqrt[4]{\frac{e}{g}};\quad \boxed{I}={}^{+}\tfrac{1}{4}\sqrt[4]{\frac{g^3}{e^3}}\ \text{in}\ -(AP:AO)+(EP:EO)+\chi(PCO).$$

Caf. 2.
$$OP:OA::P:\sqrt[4]{\frac{-f}{h}};\quad \textcircled{I}={}^{-}\tfrac{1}{4}\sqrt[4]{\frac{-h^3}{f^3}}\ \text{in}\ -\beta(BP:BO)+\beta(DP:DO)+\psi(PBO)+\psi(PDO).$$
$$OP:OA::P:\sqrt[4]{\frac{-e}{g}};\quad \boxed{I}={}^{-}\tfrac{1}{4}\sqrt[4]{\frac{-g^3}{e^3}}\ \text{in}\ -\beta(BP:BO)+\beta(DP:DO)+\psi(PBO)+\psi(PDO).$$

θ	FORMA LXXXVI. $\frac{dz\,z^{\theta n-1}}{k+lz^n}\sqrt[2]{\frac{e+fz^n}{g+hz^n}}$
&c.	
2.	$+\frac{g+hz^n}{nhl}d\,\mathrm{P}+\frac{2kp}{nllq}d\,\textcircled{1}+\frac{ehl-fgl-2fhk}{nhhll}d\,\textcircled{2}$.
1.	$\frac{-2p}{nlq}d\,\textcircled{1}+\frac{2f}{nhl}d\,\textcircled{2}$.
0.	$\frac{+2p}{nkq}d\,\textcircled{1}-\frac{2e}{ngk}d\,\boxed{2}$.
$\overline{1}$.	$-\frac{+g+hz^n}{ngkz^n}d\,\mathrm{P}-\frac{2lp}{nkkq}d\,\textcircled{1}+\frac{ehk-fgk+2egl}{nggkk}d\,\boxed{2}$.
&c.	



$$\mathrm{P}=\sqrt[2]{\frac{e+fz^n}{g+hz^n}}.\quad p=el-fk.\quad q=hk-gl.$$

Cas. 1.

$$OP:OA::\mathrm{P}:\sqrt[2]{\frac{p}{q}};\quad \textcircled{1}=+\tfrac{1}{2}\sqrt[2]{\frac{q}{p}}\ \text{in}\ \psi(PBO).$$

$$OP:OA::\mathrm{P}:\sqrt[2]{\frac{f}{h}};\quad \textcircled{2}=+\tfrac{1}{2}\sqrt[2]{\frac{h}{f}}\ \text{in}-(AP:AO)+(CP:CO).$$

$$OP:OA::\mathrm{P}:\sqrt[2]{\frac{e}{g}};\quad \boxed{2}=+\tfrac{1}{2}\sqrt[2]{\frac{g}{e}}\ \text{in}-(AP:AO)+(CP:CO).$$

Cas. 2.

$$OP:OA::\mathrm{P}:\sqrt[2]{\frac{-p}{q}};\quad \textcircled{1}=-\tfrac{1}{2}\sqrt[2]{\frac{-q}{p}}\ \text{in}-(AP:AO)+(CP:CO).$$

$$OP:OA::\mathrm{P}:\sqrt[2]{\frac{-f}{h}};\quad \textcircled{2}=-\tfrac{1}{2}\sqrt[2]{\frac{-h}{f}}\ \text{in}\ \psi(PBO).$$

$$OP:OA::\mathrm{P}:\sqrt[2]{\frac{-e}{g}};\quad \boxed{2}=-\tfrac{1}{2}\sqrt[2]{\frac{-g}{e}}\ \text{in}\ \psi(PBO).$$

θ	FORMA LXXXVII. $\frac{dzz^{\theta\eta-1}}{k+lz^{\eta}}\sqrt[3]{\frac{e+fz^{\eta}}{g+hz^{\eta}}}$
— &c.	
2.	$+\frac{g+hz^{\eta}}{\eta hl}\,d\,\mathrm{P}+\frac{3kp}{\eta llq}\,d\,\textcircled{1}+\frac{ehl-fgl-3fhk}{\eta hhll}\,d\,\textcircled{2}.$
1.	$\frac{-3p}{\eta lq}\,d\,\textcircled{1}+\frac{3f}{\eta hl}\,d\,\textcircled{2}.$ Vid. Fig. pag. 237.
0.	$\frac{+3p}{\eta kq}\,d\,\textcircled{1}-\frac{3e}{\eta gk}\,d\,\boxed{2}.$
$\overline{1}$. &c.	$-\frac{+g+hz^{\eta}}{\eta gkz^{\eta}}\,d\,\mathrm{P}-\frac{3lp}{\eta kkq}\,d\,\textcircled{1}+\frac{ehk-fgk+3egl}{\eta ggkk}\,d\,\boxed{2}.$

$$\mathrm{P}=\sqrt[3]{\frac{e+fz^{\eta}}{g+hz^{\eta}}}.\quad p=el-fk.\quad q=hk-gl.$$

Cas. 1.

$$\mathcal{OP}:\mathcal{OA}::\mathrm{P}:\sqrt[3]{\frac{p}{q}};\quad \textcircled{1}=+\tfrac{1}{3}\sqrt[3]{\frac{qq}{pp}}\ \text{in} -\beta(\mathcal{BP}:\mathcal{BO})+(\mathcal{DP}:\mathcal{DO})+\downarrow(\mathcal{PBO}).$$

$$\mathcal{OP}:\mathcal{OA}::\mathrm{P}:\sqrt[3]{\frac{f}{h}};\quad \textcircled{2}=+\tfrac{1}{3}\sqrt[3]{\frac{hh}{ff}}\ \text{in} -(\mathcal{AP}:\mathcal{AO})+\beta(\mathcal{CP}:\mathcal{CO})+\downarrow(\mathcal{PCO}).$$

$$\mathcal{OP}:\mathcal{OA}::\mathrm{P}:\sqrt[3]{\frac{e}{g}};\quad \boxed{2}=+\tfrac{1}{3}\sqrt[3]{\frac{gg}{ee}}\ \text{in} -(\mathcal{AP}:\mathcal{AO})+\beta(\mathcal{CP}:\mathcal{CO})+\downarrow(\mathcal{PCO}).$$

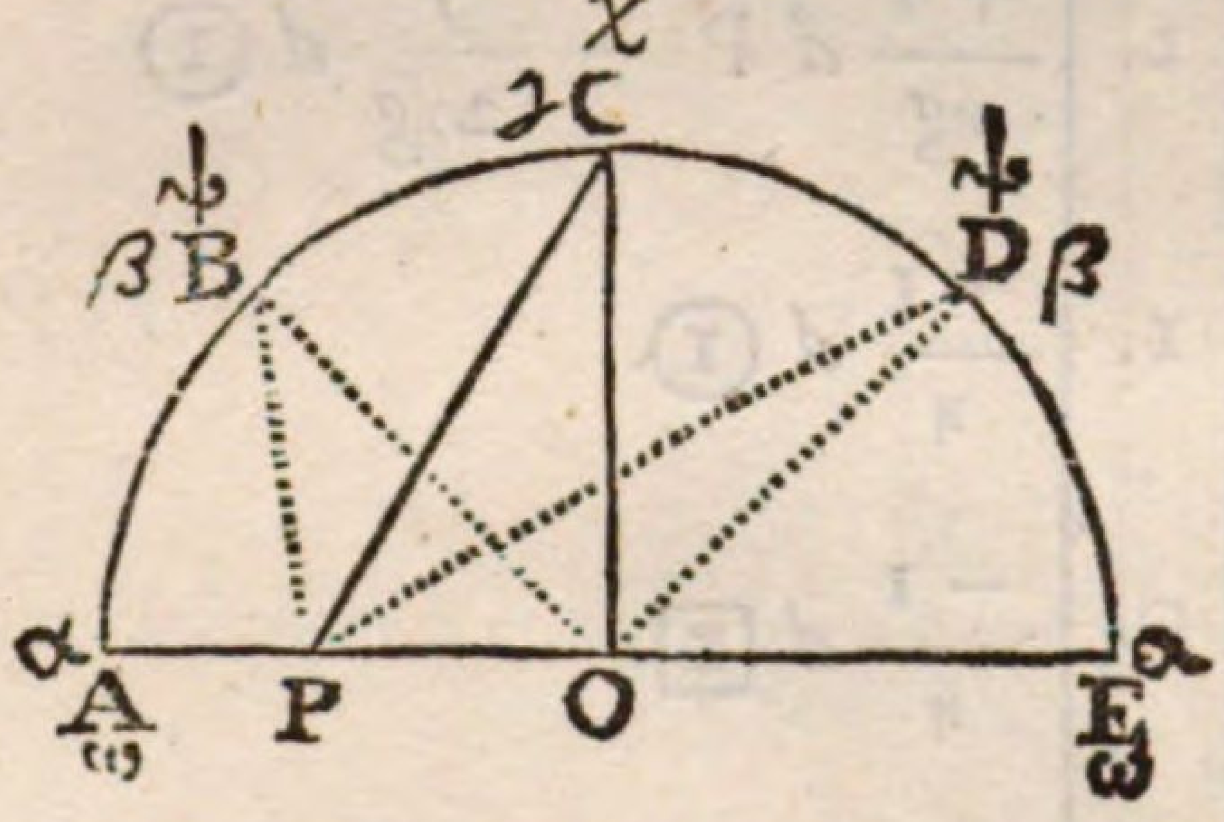
Cas. 2.

$$\mathcal{OP}:\mathcal{OA}::\mathrm{P}:\sqrt[3]{\frac{-p}{q}};\quad \textcircled{1}=-\tfrac{1}{3}\sqrt[3]{\frac{qq}{pp}}\ \text{in} -(\mathcal{AP}:\mathcal{AO})+\beta(\mathcal{CP}:\mathcal{CO})+\downarrow(\mathcal{PCO}).$$

$$\mathcal{OP}:\mathcal{OA}::\mathrm{P}:\sqrt[3]{\frac{-f}{h}};\quad \textcircled{2}=-\tfrac{1}{3}\sqrt[3]{\frac{hh}{ff}}\ \text{in} -\beta(\mathcal{BP}:\mathcal{BO})+(\mathcal{DP}:\mathcal{DO})+\downarrow(\mathcal{PBO}).$$

$$\mathcal{OP}:\mathcal{OA}::\mathrm{P}:\sqrt[3]{\frac{-e}{g}};\quad \boxed{2}=-\tfrac{1}{3}\sqrt[3]{\frac{gg}{ee}}\ \text{in} -\beta(\mathcal{BP}:\mathcal{BO})+(\mathcal{DP}:\mathcal{DO})+\downarrow(\mathcal{PBO}).$$

θ	FORMA LXXXVIII. $\frac{d\dot{z}z^{\theta n-1}}{k+lz^n}\sqrt[4]{\frac{e+fz^n}{g+hz^n}}$
&c.	
2.	$+\frac{g+hz^n}{nhl}d\mathrm{P}+\frac{4kp}{nllq}d\textcircled{1}+\frac{ehl-fgl-4fhk}{nhhll}d\textcircled{2}.$
1.	$\frac{-4p}{nlq}d\textcircled{1}+\frac{4f}{nhl}d\textcircled{2}.$
0.	$\frac{+4p}{nkq}d\textcircled{1}-\frac{4e}{ngk}d\boxed{2}.$
$\overline{1}$.	$-\frac{+g+hz^n}{ngkz^n}d\mathrm{P}-\frac{4lp}{nkkq}d\textcircled{1}+\frac{ehk-fgk+4egl}{nggkk}d\boxed{2}.$
&c.	



$$\mathrm{P}=\sqrt[4]{\frac{e+fz^n}{g+hz^n}}.\quad p=el-fk.\quad q=hk-gl.$$

Cas. 1.

$$OP:OA::\mathrm{P}:\sqrt[4]{\frac{p}{q}};\quad \textcircled{1}=\tfrac{+1}{4}\sqrt[4]{\frac{q^3}{p^3}}\text{ in }-\beta(BP:BO)+\beta(DP:DO)+\psi(PBO)+\psi(PDO).$$

$$OP:OA::\mathrm{P}:\sqrt[4]{\frac{f}{h}};\quad \textcircled{2}=\tfrac{+1}{4}\sqrt[4]{\frac{h^3}{f^3}}\text{ in }-(AP:AO)+(EP:EO)+\chi(PCO).$$

$$OP:OA::\mathrm{P}:\sqrt[4]{\frac{e}{g}};\quad \boxed{2}=\tfrac{+1}{4}\sqrt[4]{\frac{g^3}{e^3}}\text{ in }-(AP:AO)+(EP:EO)+\chi(PCO).$$

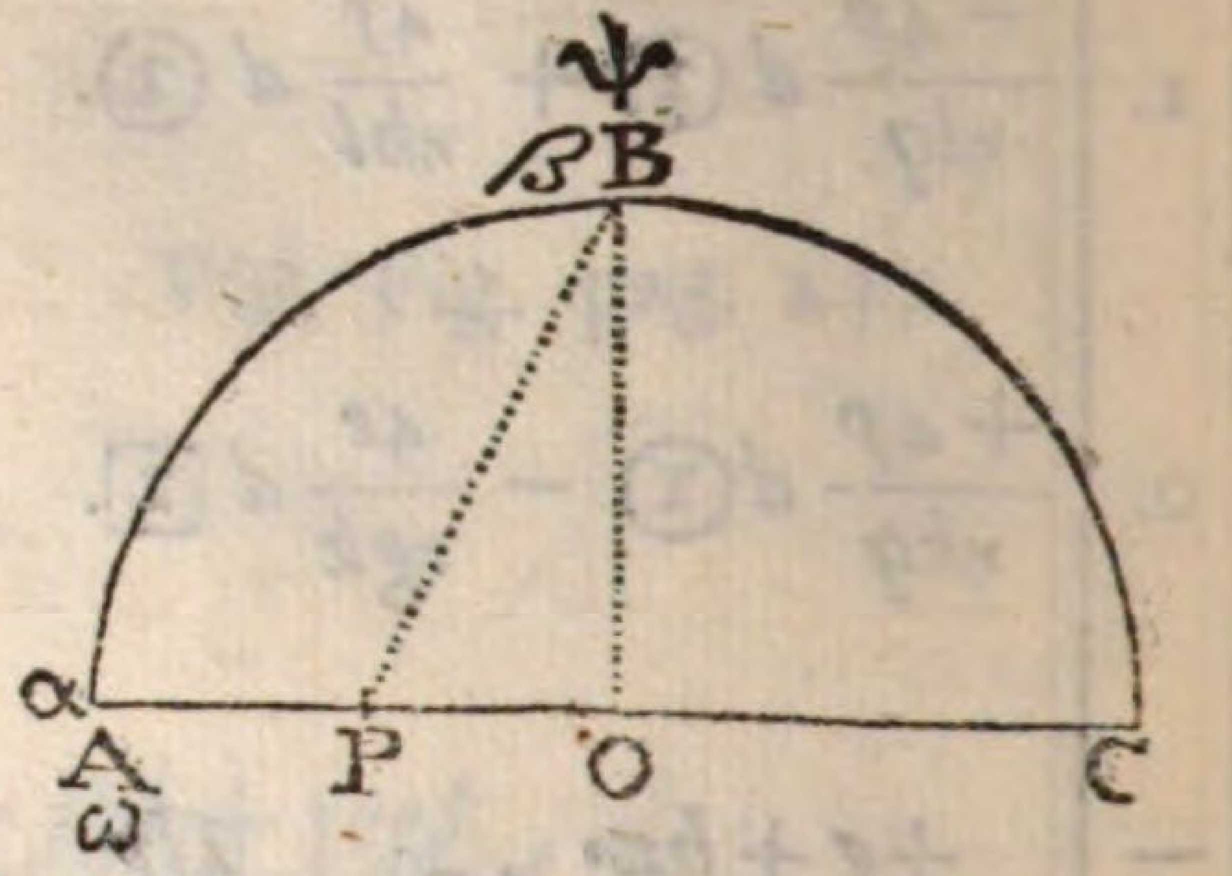
Cas. 2.

$$OP:OA::\mathrm{P}:\sqrt[4]{\frac{-p}{q}};\quad \textcircled{1}=\tfrac{-1}{4}\sqrt[4]{\frac{-q^3}{p^3}}\text{ in }-(AP:AO)+(EP:EO)+\chi(PCO).$$

$$OP:OA::\mathrm{P}:\sqrt[4]{\frac{-f}{h}};\quad \textcircled{2}=\tfrac{-1}{4}\sqrt[4]{\frac{-h^3}{f^3}}\text{ in }-\beta(BP:BO)+\beta(DP:DO)+\psi(PBO)+\psi(PDO).$$

$$OP:OA::\mathrm{P}:\sqrt[4]{\frac{-e}{g}};\quad \boxed{2}=\tfrac{-1}{4}\sqrt[4]{\frac{-g^3}{e^3}}\text{ in }-\beta(BP:BO)+\beta(DP:DO)+\psi(PBO)+\psi(PDO).$$

θ	FORMA LXXXIX. $\dfrac{d\dot{z}z^{\theta n-1}}{\sqrt{e+fz^{n}+gz^{2n}}}$
&c.	
3.	$\dfrac{-3f+2gz^{n}}{4\eta gg}\,d\,\mathrm{P}+\dfrac{3ff-4eg}{8\eta gg}\,d\,\textcircled{I}$.
2.	$\dfrac{+1}{\eta g}\,d\,\mathrm{P}-\dfrac{f}{2\eta g}\,d\,\textcircled{I}$
1.	$\dfrac{+1}{\eta}\,d\,\textcircled{I}$.
0.	$\dfrac{-1}{\eta}\,d\,\boxed{I}$.
$\overline{1}$.	$\dfrac{-1}{\eta e z^{n}}\,d\,\mathrm{P}+\dfrac{f}{2\eta e}\,d\,\boxed{I}$.
$\overline{2}$.	$\dfrac{-2e+3fz^{n}}{4\eta eez^{2n}}\,d\,\mathrm{P}+\dfrac{4eg-3ff}{8\eta ee}\,d\,\boxed{I}$.
&c.	



$$\mathrm{P}=\sqrt{e+fz^{n}+gz^{2n}}.$$

Caſ. 1. $\begin{cases} OP:OA::\dfrac{\frac{1}{2}f+gz^{n}}{\mathrm{P}}:\sqrt{g}\,; & \textcircled{I}=+\tfrac{1}{2}\sqrt{\dfrac{1}{g}}\ \text{in}\ (CP:AP). \\ OP:OA::\dfrac{e+\frac{1}{2}fz^{n}}{\mathrm{P}}:\sqrt{e}\,; & \boxed{I}=+\tfrac{1}{2}\sqrt{\dfrac{1}{e}}\ \text{in}\ (CP:AP). \end{cases}$

Caſ. 2. $\begin{cases} OP:OA::\dfrac{\frac{1}{2}f+gz^{n}}{\mathrm{P}}:\sqrt{-g}\,; & \textcircled{I}=-\tfrac{1}{2}\sqrt{\dfrac{-1}{g}}\ \text{in}\ \psi(PBO). \\ OP:OA::\dfrac{e+\frac{1}{2}fz^{n}}{\mathrm{P}}:\sqrt{-e}\,; & \boxed{I}=-\tfrac{1}{2}\sqrt{\dfrac{-1}{e}}\ \text{in}\ \psi(PBO). \end{cases}$

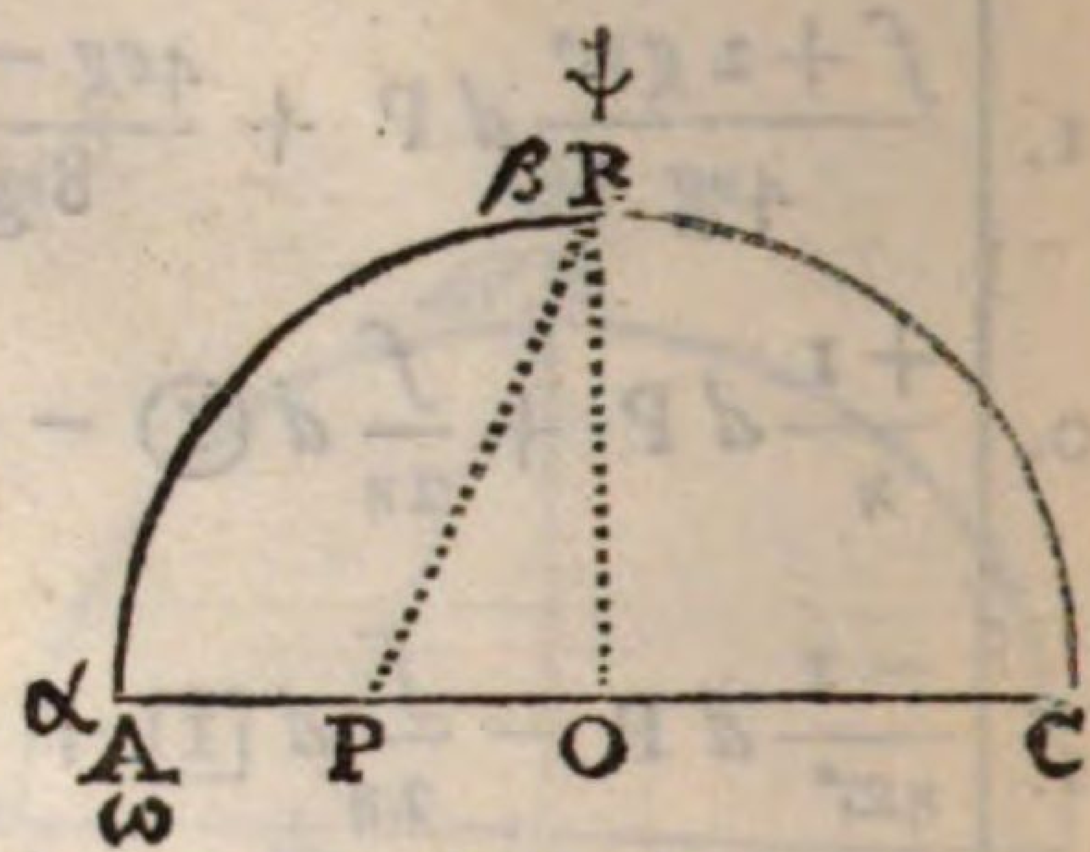
θ	FORMA XC. $dz z^{\theta n-1} \sqrt{e+fz^n+gz^{2n}}$.
&c.	
2.	$\dfrac{+8eg-3ff+2fgz^n+8ggz^{2n}}{24\eta gg}\, d\,\mathrm{P} + \dfrac{f^3-4efg}{16\eta gg}\, d\,\text{\textcircled{I}}$.
1.	$\dfrac{f+2gz^n}{4\eta g}\, d\,\mathrm{P} + \dfrac{4eg-ff}{8\eta g}\, d\,\text{\textcircled{I}}$.
0.	$\dfrac{+1}{\eta}\, d\,\mathrm{P} + \dfrac{f}{2\eta}\, d\,\text{\textcircled{I}} - \dfrac{e}{\eta}\, d\,\boxed{\mathrm{I}}$.
$\overline{1}$.	$\dfrac{-1}{\eta z^n}\, d\,\mathrm{P} - \dfrac{f}{2\eta}\, d\,\boxed{\mathrm{I}} + \dfrac{g}{\eta}\, d\,\text{\textcircled{I}}$.
$\overline{2}$.	$\dfrac{-2e-fz^n}{4\eta e z^{2n}}\, d\,\mathrm{P} + \dfrac{ff-4eg}{8\eta e}\, d\,\boxed{\mathrm{I}}$.
$\overline{3}$.	$\dfrac{-8ee-2efz^n+3ffz^{2n}-8egz^{2n}}{24\eta ee z^{3n}}\, d\,\mathrm{P} + \dfrac{4efg-f^3}{16\eta ee}\, d\,\boxed{\mathrm{I}}$.
&c.	

$$\mathrm{P} = \sqrt{e+fz^n+gz^{2n}}.$$

Caf. 1.
$$\begin{cases} OP:OA :: \dfrac{\frac{1}{2}f+gz^n}{\mathrm{P}} : \sqrt{g}\ ; & \text{\textcircled{I}} = \frac{1}{2}\sqrt{\dfrac{1}{g}}\ \text{in}\,(CP:AP). \\ OP:OA :: \dfrac{e+\frac{1}{2}fz^n}{\mathrm{P}} : \sqrt{e}\ ; & \boxed{\mathrm{I}} = \frac{1}{2}\sqrt{\dfrac{1}{e}}\ \text{in}\,(CP:AP). \end{cases}$$

Caf. 2.
$$\begin{cases} OP:OA :: \dfrac{\frac{1}{2}f+gz^n}{\mathrm{P}} : \sqrt{-g}\ ; & \text{\textcircled{I}} = \frac{1}{2}\sqrt{\dfrac{-1}{g}}\ \text{in}\downarrow(PBO). \\ OP:OA :: \dfrac{e+\frac{1}{2}fz^n}{\mathrm{P}} : \sqrt{-e}\ ; & \boxed{\mathrm{I}} = \frac{1}{2}\sqrt{\dfrac{-1}{e}}\ \text{in}\downarrow(PBO). \end{cases}$$

θ	FORMA XCI. $\dfrac{dz z^{\theta n-1}}{\overline{k+lz^n}\,\sqrt{e+fz^n+gz^{2n}}}$
→ &c.	
3.	$\frac{+1}{\eta gl} d\mathrm{P} - \frac{kk}{\eta ll} d\,\textcircled{1} + \frac{-fl-2gk}{2\eta gll} d\,\textcircled{2}$.
2.	$\frac{+k}{\eta l} d\,\textcircled{1} + \frac{1}{\eta l} d\,\textcircled{2}$.
1.	$\frac{-1}{\eta} d\,\textcircled{1}$.
0.	$\frac{+l}{\eta k} d\,\textcircled{1} - \frac{1}{\eta k} d\,\boxed{2}$.
$\overline{1}$.	$\frac{-1}{\eta e k z^n} d\mathrm{P} - \frac{ll}{\eta kk} d\,\textcircled{1} + \frac{fk+2el}{2\eta ekk} d\,\boxed{2}$.
&c.	



$$\mathrm{P} = \sqrt{e+fz^n+gz^{2n}}. \quad p = ell - fkl + gkk.$$

Caf. 1.

$$OP:OA :: \frac{l,\overline{e+\frac{1}{2}fz^n} - k,\overline{\frac{1}{2}f+gz^n}}{\mathrm{P}} : \sqrt{p};$$

$$\textcircled{1} = +\tfrac{1}{2}\sqrt{\frac{1}{p}} \text{ in } (CP:AP).$$

$$OP:OA :: \frac{\frac{1}{2}f+gz^n}{\mathrm{P}} : \sqrt{g}; \quad \textcircled{2} = +\tfrac{1}{2}\sqrt{\frac{1}{g}} \text{ in } (CP:AP).$$

$$OP:OA :: \frac{e+\frac{1}{2}fz^n}{\mathrm{P}} : \sqrt{e}; \quad \boxed{2} = +\tfrac{1}{2}\sqrt{\frac{1}{e}} \text{ in } (CP:AP).$$

Caf. 2.

$$OP:OA :: \frac{l,\overline{e+\frac{1}{2}fz^n} - k,\overline{\frac{1}{2}f+gz^n}}{\mathrm{P}} : \sqrt{-p};$$

$$\textcircled{1} = -\tfrac{1}{2}\sqrt{\frac{-1}{p}} \text{ in } \psi(PBO).$$

$$OP:OA :: \frac{\frac{1}{2}f+gz^n}{\mathrm{P}} : \sqrt{-g}; \quad \textcircled{2} = -\tfrac{1}{2}\sqrt{\frac{-1}{g}} \text{ in } \psi(PBO).$$

$$OP:OA :: \frac{e+\frac{1}{2}fz^n}{\mathrm{P}} : \sqrt{-e}; \quad \boxed{2} = -\tfrac{1}{2}\sqrt{\frac{-1}{e}} \text{ in } \psi(PBO).$$

θ	FORMA XCII. $\frac{dzz^{\theta n-1}\sqrt{e+fz^n+gz^{2n}}}{k+lz^n}$
&c.	
2.	$\frac{+fl \atop -4gk+2glz^n}{4\eta gll}\,d\mathrm{P}+\frac{kp}{\eta l^3}\,d\textcircled{1}+\frac{-ffll-4fgkl \atop +4egll+8ggkk}{8\eta gl^3}\,d\textcircled{2}.$
1.	$\frac{+1}{\eta l}\,d\mathrm{P}-\frac{p}{\eta ll}\,d\textcircled{1}+\frac{fl-2gk}{2\eta ll}\,d\textcircled{2}.$
0.	$\frac{+p}{\eta kl}\,d\textcircled{1}+\frac{g}{\eta l}\,d\textcircled{2}-\frac{e}{\eta k}\,d\boxed{2}.$
$\overline{1}$.	$\frac{-1}{\eta kz^n}\,d\mathrm{P}-\frac{p}{\eta kk}\,d\textcircled{1}+\frac{2el-fk}{2\eta kk}\,d\boxed{2}.$
$\overline{2}$.	$\frac{-fkz^n \atop -2ek+4elz^n}{4\eta ekkz^{2n}}\,d\mathrm{P}+\frac{lp}{\eta k^3}\,d\textcircled{1}+\frac{+ffkk+4efkl \atop -4egkk-8eell}{8\eta ek^3}\,d\boxed{2}.$
&c.	

$$\mathrm{P}=\sqrt{e+fz^n+gz^{2n}}.\quad p=ell-fkl+gkk.$$

Caf. 1.

$$OP:OA::\frac{l,\overline{e+\tfrac{1}{2}fz^n}-k,\overline{\tfrac{1}{2}f+gz^n}}{\mathrm{P}}:\sqrt{p};$$

$$\textcircled{1}=\pm\tfrac{1}{2}\sqrt{\frac{1}{p}}\ \text{in}\ (CP:AP).$$

$$OP:OA::\frac{\tfrac{1}{2}f+gz^n}{\mathrm{P}}:\sqrt{g};\quad \textcircled{2}=\pm\tfrac{1}{2}\sqrt{\frac{1}{g}}\ \text{in}\ (CP:AP).$$

$$OP:OA::\frac{e+\tfrac{1}{2}fz^n}{\mathrm{P}}:\sqrt{e};\quad \boxed{2}=\pm\tfrac{1}{2}\sqrt{\frac{1}{e}}\ \text{in}\ (CP:AP).$$

Caf. 2.

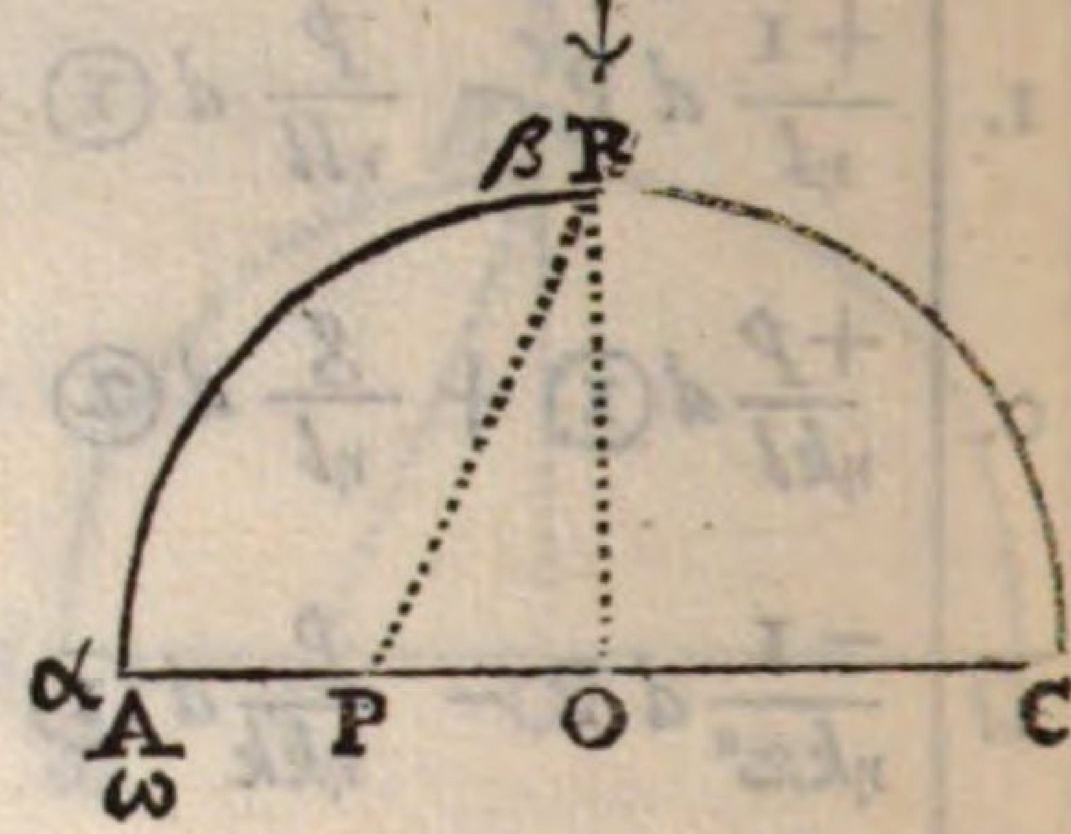
$$OP:OA::\frac{l,\overline{e+\tfrac{1}{2}fz^n}-k,\overline{\tfrac{1}{2}f+gz^n}}{\mathrm{P}}:\sqrt{-p};$$

$$\textcircled{1}=\mp\tfrac{1}{2}\sqrt{\frac{-1}{p}}\ \text{in}\ \psi(PBO).$$

$$OP:OA::\frac{\tfrac{1}{2}f+gz^n}{\mathrm{P}}:\sqrt{-g};\quad \textcircled{2}=\mp\tfrac{1}{2}\sqrt{\frac{-1}{g}}\ \text{in}\ \psi(PBO).$$

$$OP:OA::\frac{e+\tfrac{1}{2}fz^n}{\mathrm{P}}:\sqrt{-e};\quad \boxed{2}=\mp\tfrac{1}{2}\sqrt{\frac{-1}{e}}\ \text{in}\ \psi(PBO).$$

θ	FORMA XCIII. $\quad \dfrac{d\dot{z}z^{\theta n-1}}{\overline{k+lz^n+mz^{2n}}\sqrt{e+fz^n+gz^{2n}}}$
—	
&c.	
3.	$\dfrac{pp}{\eta am}d\textcircled{1}-\dfrac{qq}{\eta am}d\textcircled{2}+\dfrac{1}{\eta m}d\textcircled{3}.$
2.	$\dfrac{-p}{\eta a}d\textcircled{1}+\dfrac{q}{\eta a}d\textcircled{2}.$
1.	$\dfrac{m}{\eta a}d\textcircled{1}-\dfrac{m}{\eta a}d\textcircled{2}.$
0.	$\dfrac{-mm}{\eta ap}d\textcircled{1}+\dfrac{mm}{\eta aq}d\textcircled{2}-\dfrac{1}{\eta k}d\boxed{3}.$
&c.	



$$a=\sqrt{ll-4kl},\quad p=\tfrac{1}{2}l+\tfrac{1}{2}a,\quad q=\tfrac{1}{2}l-\tfrac{1}{2}a.\quad \mathrm{P}=\sqrt{e+fz^n+gz^{2n}}.$$

$$r=emm-fmp+gpp,\quad s=emm-fmq+gqq.$$

Cas. 1.

$$OP:OA::\frac{m,\overline{e+\tfrac{1}{2}fz^n}-p,\overline{\tfrac{1}{2}f+gz^n}}{\mathrm{P}}:\sqrt{r};$$

$$\textcircled{1}=+\tfrac{1}{2}\sqrt{\frac{1}{r}}\text{ in }(CP:AP).$$

$$OP:OA::\frac{m,\overline{e+\tfrac{1}{2}fz^n}-q,\overline{\tfrac{1}{2}f+gz^n}}{\mathrm{P}}:\sqrt{s};$$

$$\textcircled{2}=+\tfrac{1}{2}\sqrt{\frac{1}{s}}\text{ in }(CP:AP).$$

$$OP:OA::\frac{\tfrac{1}{2}f+gz^n}{\mathrm{P}}:\sqrt{g};\quad \textcircled{3}=+\tfrac{1}{2}\sqrt{\frac{1}{g}}\text{ in }(CP:AP).$$

$$OP:OA::\frac{e+\tfrac{1}{2}fz^n}{\mathrm{P}}:\sqrt{e};\quad \boxed{3}=+\tfrac{1}{2}\sqrt{\frac{1}{e}}\text{ in }(CP:AP).$$

θ	FORMA XCIV. $\frac{dz z^{\theta n-1}\sqrt{e+fz^n+gz^{2n}}}{k+lz^n+mz^{2n}}$
&c.	
2.	$\frac{1}{\eta m}dP-\frac{pr}{\eta amm}d\textcircled{1}+\frac{qs}{\eta amm}d\textcircled{2}+\frac{fm-2gl}{2\eta mm}d\textcircled{3}$.
1.	$\frac{r}{\eta am}d\textcircled{1}-\frac{s}{\eta am}d\textcircled{2}+\frac{g}{\eta m}d\textcircled{3}$.
0.	$\frac{-r}{\eta ap}d\textcircled{1}+\frac{s}{\eta aq}d\textcircled{2}-\frac{e}{\eta k}d\boxed{3}$.
$\bar{1}$.	$\frac{-1}{\eta kz^n}dP+\frac{rm}{\eta app}d\textcircled{1}-\frac{sm}{\eta aqq}d\textcircled{2}+\frac{2el-fk}{2\eta kk}d\boxed{3}$.
&c.	

Quantitates aſſumptæ eoſdem habent valores in utraque Forma.

Caſ. 2.

$$OP:OA::\frac{m,\overline{e+\tfrac{1}{2}fz^n}-p,\overline{\tfrac{1}{2}f+gz^n}}{P}:\sqrt{-r};$$

$$\textcircled{1}=\tfrac{-1}{2}\sqrt{\frac{-1}{r}}\ \text{in}\ \psi\,(PBO).$$

$$OP:OA::\frac{m,\overline{e+\tfrac{1}{2}fz^n}-q,\overline{\tfrac{1}{2}f+gz^n}}{P}:\sqrt{-s};$$

$$\textcircled{2}=\tfrac{-1}{2}\sqrt{\frac{-1}{s}}\ \text{in}\ \psi\,(PBO).$$

$$OP:OA::\frac{\tfrac{1}{2}f+gz^n}{P}:\sqrt{-g};\quad \textcircled{3}=\tfrac{-1}{2}\sqrt{\frac{-1}{g}}\ \text{in}\ \psi\,(PBO).$$

$$OP:OA::\frac{e+\tfrac{1}{2}fz^n}{P}:\sqrt{-e};\quad \boxed{3}=\tfrac{-1}{2}\sqrt{\frac{-1}{e}}\ \text{in}\ \psi\,(PBO).$$

SUPPLEMENTUM ad THEOREMATA PRO CONTINUATIONE FORMARUM ET FLUENTIUM AD INFINITUM.

THEOREMA VI.

Positis

$$Z = e + fz^{n} + gz^{2n} + hz^{3n}.$$

$$\dot{A} = d\dot{z}z^{\theta n - 1} Z^{\omega - 1} \qquad \dot{G} = d\dot{z}z^{\theta n - 1} Z^{\omega}$$
$$\dot{B} = d\dot{z}z^{\theta n + n - 1} Z^{\omega - 1} \qquad \dot{H} = d\dot{z}z^{\theta n + n - 1} Z^{\omega}$$
$$\dot{C} = d\dot{z}z^{\theta n + 2n - 1} Z^{\omega - 1} \qquad \dot{I} = d\dot{z}z^{\theta n + 2n - 1} Z^{\omega}.$$
$$\dot{D} = d\dot{z}z^{\theta n + 3n - 1} Z^{\omega - 1}$$
$$\dot{E} = d\dot{z}z^{\theta n + 4n - 1} Z^{\omega - 1}$$
$$\dot{F} = d\dot{z}z^{\theta n + 5n - 1} Z^{\omega - 1}$$

Erunt

$$\tfrac{1}{n} d z^{\theta n} Z^{\omega} = \theta e A + \overline{\theta + \omega},\, fB + \overline{\theta + 2\omega},\, gC + \overline{\theta + 3\omega},\, hD.$$

$$\tfrac{1}{n} d z^{\theta n + n} Z^{\omega} = \overline{\theta + 1},\, eB + \overline{\theta + 1 + \omega},\, fC + \overline{\theta + 1 + 2\omega},\, gD + \overline{\theta + 1 + 3\omega},\, hE.$$

$$\tfrac{1}{n} d z^{\theta n + 2n} Z^{\omega} = \overline{\theta + 2},\, eC + \overline{\theta + 2 + \omega},\, fD + \overline{\theta + 2 + 2\omega},\, gE + \overline{\theta + 2 + 3\omega},\, hF.$$

Erunt

$$G = eA + fB + gC + hD$$
$$H = eB + fC + gD + hE$$
$$I = eC + fD + gE + hF.$$

Atque inde datis G, H, I dabuntur A, B, C, D, E, F.

THEOREMA VII.

Positis

$$Z = e + fz^{\eta} + gz^{2\eta}$$

$$Y = k + lz^{\eta} + mz^{2\eta}$$

$$\dot{A} = d\dot{z}z^{\theta\eta-1}Z^{\omega-1}Y^{\psi-1}$$
$$\dot{B} = d\dot{z}z^{\theta\eta+\eta-1}Z^{\omega-1}Y^{\psi-1}$$
$$\dot{C} = d\dot{z}z^{\theta\eta+2\eta-1}Z^{\omega-1}Y^{\psi-1}$$
$$\dot{D} = d\dot{z}z^{\theta\eta+3\eta-1}Z^{\omega-1}Y^{\psi-1}$$
$$\dot{E} = d\dot{z}z^{\theta\eta+4\eta-1}Z^{\omega-1}Y^{\psi-1}$$
$$\dot{F} = d\dot{z}z^{\theta\eta+5\eta-1}Z^{\omega-1}Y^{\psi-1}$$

$$\dot{G} = d\dot{z}z^{\theta\eta-1}Z^{\omega}Y^{\psi-1}$$
$$\dot{H} = d\dot{z}z^{\theta\eta+\eta-1}Z^{\omega}Y^{\psi-1}$$
$$\dot{I} = d\dot{z}z^{\theta\eta+2\eta-1}Z^{\omega}Y^{\psi-1}$$
$$\dot{K} = d\dot{z}z^{\theta\eta+3\eta-1}Z^{\omega}Y^{\psi-1}.$$

$$\dot{L} = d\dot{z}z^{\theta\eta-1}Z^{\omega-1}Y^{\psi}$$
$$\dot{M} = d\dot{z}z^{\theta\eta+\eta-1}Z^{\omega-1}Y^{\psi}$$
$$\dot{N} = d\dot{z}z^{\theta\eta+2\eta-1}Z^{\omega-1}Y^{\psi}$$
$$\dot{O} = d\dot{z}z^{\theta\eta+3\eta-1}Z^{\omega-1}Y^{\psi}.$$

Erunt

$$\begin{aligned}
\tfrac{1}{\eta}dz^{\theta\eta}Z^{\omega}Y^{\psi} = {} & \theta ek\,A \\
& + \overline{\overline{\theta+\omega},\ fk + \overline{\theta+\psi},\ el},\ B \\
& + \overline{\overline{\theta+2\omega},\ gk + \overline{\theta+\omega+\psi},\ fl + \overline{\theta+2\psi},\ em},\ C \\
& + \overline{\overline{\theta+2\omega+\psi},\ gl + \overline{\theta+\omega+2\psi},\ fm},\ D \\
& + \overline{\theta+2\omega+2\psi},\ gm\,E.
\end{aligned}$$

$$\begin{aligned}
\tfrac{1}{\eta}dz^{\theta\eta+\eta}Z^{\omega}Y^{\psi} = {} & \overline{\theta+1},\ ek\,B \\
& + \overline{\overline{\theta+1+\omega},\ fk + \overline{\theta+1+\psi},\ el},\ C \\
& + \overline{\overline{\theta+1+2\omega},\ gk + \overline{\theta+1+\omega+\psi},\ fl + \overline{\theta+1+2\psi},\ em},\ D \\
& + \overline{\overline{\theta+1+2\omega+\psi},\ gl + \overline{\theta+1+\omega+2\psi},\ fm},\ E \\
& + \overline{\theta+1+2\omega+2\psi},\ gm\,F.
\end{aligned}$$

Erunt

$$G = eA + fB + gC \qquad L = kA + lB + mC$$
$$H = eB + fC + gD \qquad M = kB + lC + mD$$
$$I = eC + fD + gE \qquad N = kC + lD + mE$$
$$K = eD + fE + gF. \qquad O = kD + lE + mF.$$

Atque inde datis G, H, I, K vel L, M, N, O dabuntur A, B, C, D, E, F.

OPERA
MISCELLANEA
ROGERI COTES.

ÆSTIMATIO ERRORUM

IN

MIXTA MATHESI,

PER

VARIATIONES PARTIUM TRIANGULI PLANI ET SPHÆRICI.

AUCTORE

ROGERO COTES

INTRODUCTIO.

QUANDOQUIDEM universa pendet Astronomia ab idoneis Observationibus accurate & fideliter e Coelo deductis, Errores autem inter observandum penitus devitare non liceat per sensuum nostrorum finitam vim & varias Organorum etiam præstantissimorum imperfectiones: magni momenti fuerit ad Scientiæ nobilissimæ perfectionem, Errores istos intra terminos quam maxime fieri potest angustos concludere & coarctare, quos omnino tollere merito desperemus. Artificis igitur ingenium in eorum limitibus bene definiendis præsertim requiritur, ut cognitum habeat quibus modis alii ex aliis enascantur, ac probe prospiciat quales quantique sint ultimo evasuri. Ita demum non cæco fortunæ ductu, sed certo & tuto consilio istiusmodi Observationes instituet e quibus propositum suum Erroribus minimis affectum eliciet. Huc potissimum respiciens sequentia composui Theoremata quæ tamen & aliis insuper usibus peregregiis inservire poterunt. Continent vero determinationem proportionis quæ est inter Variationes minimas contemporaneas partium reliquarum Trianguli Plani & Sphærici dum binæ quævis invariatæ maneant. Nam cum omne Triangulum ex Senis constet partibus, tribus nempe lateribus & totidem angulis, poterunt harum duæ quævis magnitudinem suam servare, dum ceteris interim vel per augmentationem vel per imminutionem accedit aliqua mutatio. Isthæc porro incrementa & decrementa *Variationes* partium appello & in hoc Tractatu rationes earundem expenduntur. Cum vero quantitatum diversi generis nulla sit ad invicem ratio, pro Variationibus angulorum usurpandæ sunt eorundem *Mensuræ* lineares ut conferantur cum Variationibus laterum. Convenit itaque plenius definire quid intelligam per has Mensuras.

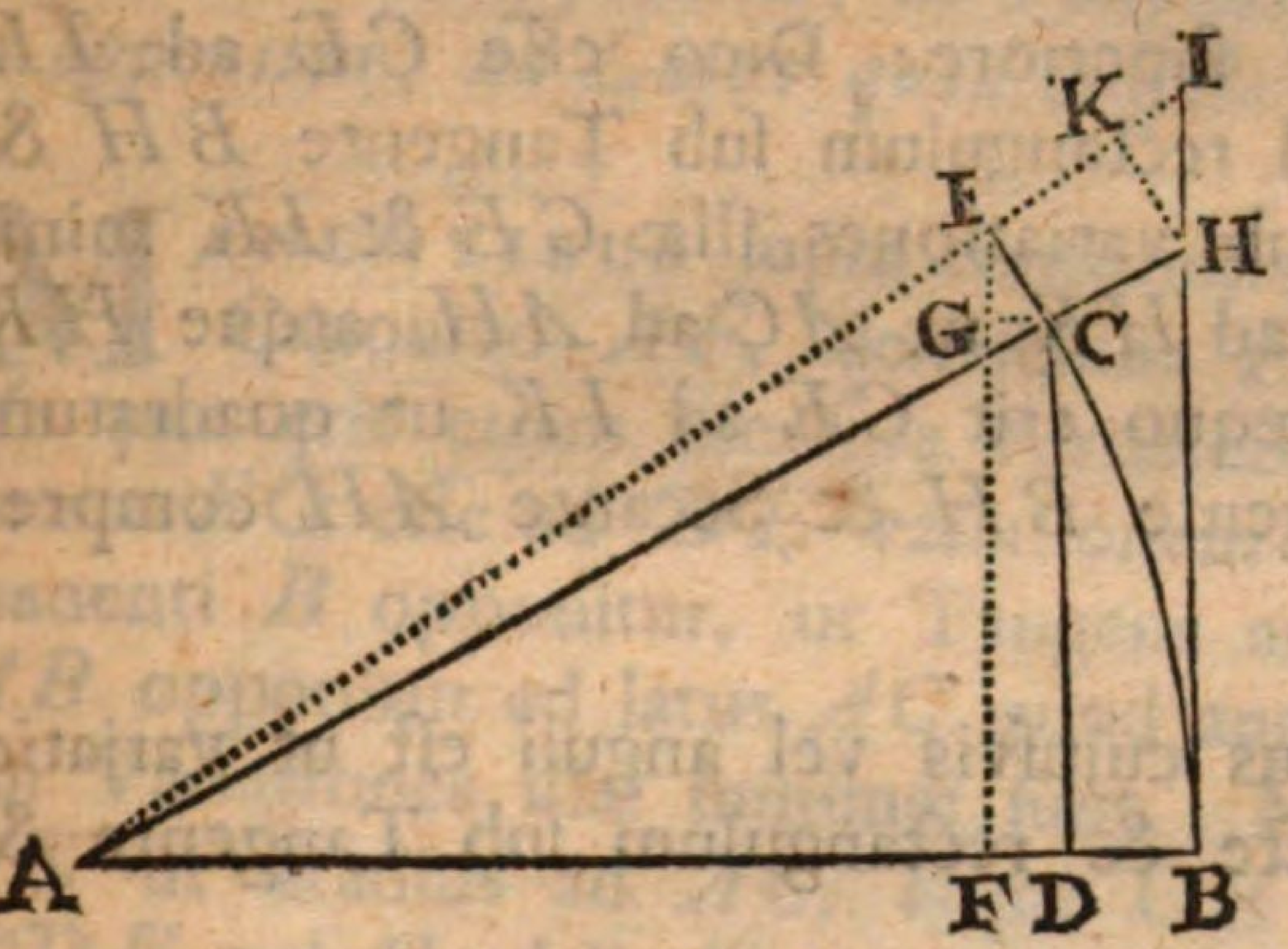
Solent Trigonometrici Sinus, Tangentes, reliquasque Circulo adscriptas lineas ad unum aliquem datum Radium referre & Canonem integrum numerosum ad eundem ordinare. *Mensuram* itaque Variationis anguli rectilinei appello partem illam circumferentiæ descriptæ per datum illum Radium Canonicum, cujus centrum est ipsum angulare punctum, & quæ subtenditur angulo Variationis. *Mensuram* autem Variationis anguli Sphærici appello partem illam circumferentiæ circuli maximi, cujus Polus est ipsum angulare punctum, & quæ subtenditur itidem Variationis angulo.

LEMMA

LEMMA I.

Variatio minima cujusvis arcus circularis est ad Variationem minimam Sinus ejusdem arcus ut Radius ad Sinum complementi.

CENTRO *A* radioque *AB* describatur circuli arcus *BC*, terminatus a radiis *AB*, *AC*. Sit ejus Sinus *CD* ad radium *AB* normalis, Sinus complementi *AD*; & mutetur arcus *BC* in arcum *BE*, simulque Sinus *CD* in Sinum *EF*, Sinus complementi *AD* in Sinum complementi *AF*, agaturque *CG* parallela radio *AB* quæ occurrat Sinui *EF* in *G*. Dico *CE* Variationem arcus *BC* esse ad *EG* Variationem Sinus *CD*, ut *AC* radius ad *AD* Sinum complementi arcus, ubi Variationes illæ minimæ sunt. Nam in hoc casu erunt Triangula rectangula *CEG*, *CAD* similia. Itaque *CE* est ad *EG* ut *AC* ad *AD* sive ut Radius ad Sinum complementi arcus. *Q.E.D.*



Corol. Variatio minima arcus vel anguli cujusvis est ut Variatio minima Sinus ejusdem directe utque Sinus complementi ejusdem inverse.

LEMMA II.

Variatio minima cujusvis arcus circularis est ad Variationem minimam Tangentis ejusdem arcus, ut quadratum Radii ad quadratum Secantis.

TANGAT enim recta *BI* circulum in *B* & occurrat productis *AC*, *AE* in *H* & *I*. Dico esse *CE* ad *HI* ut quadratum Radii ad quadratum Secantis *AH*, ubi variationes illæ *CE* & *HI* minimæ sunt. Sumatur enim *AK* æqualis ipsi *AH* & jungatur *HK*. Erunt itaque in hoc casu triangula *ACE*, *AHK* similia, ut & Triangula *ABH*, *HKI*. Unde erit *CE* ad *HK* ut *AC* ad *AH*, atque *HK* ad *HI* ut *AB* ad *AH*. Adeoque ex æquo erit *CE* ad *HI* ut quadratum Radii ad quadratum Secantis *AH*. *Q.E.D.*

Corol.

Corol. Variatio minima cujusvis arcus vel anguli est ut variatio minima Tangentis ejusdem directe & quadratum Secantis ejusdem inverse.

LEMMA III.

Variatio minima cujusvis arcus circularis est ad Variationem minimam Secantis ejusdem arcus, ut quadratum Radii ad rectangulum sub Tangente & Secante comprehensum.

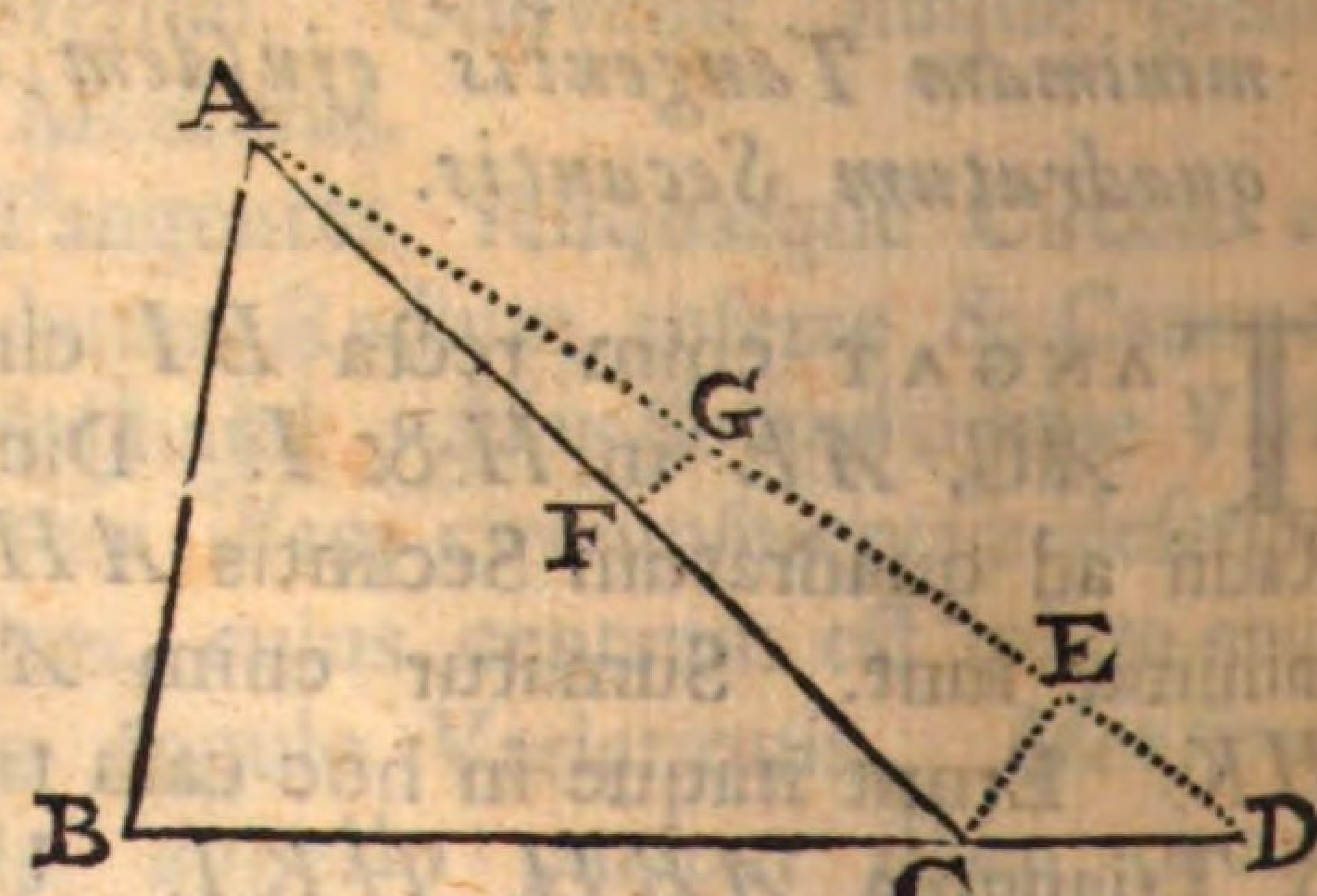
MANENTE constructione superiore: Dico esse *CE* ad *IK* ut quadratum Radii ad rectangulum sub Tangente *BH* & Secante *AH* comprehensum, ubi variationes illæ *CE* & *IK* minimæ sunt. Cum enim sit *CE* ad *HK* ut *AC* ad *AH*, atque *HK* ad *IK* ut *AB* ad *BH*: ex æquo erit *CE* ad *IK* ut quadratum Radii ad rectangulum sub Tangente *BH* & Secante *AH* comprehensum. *Q.E.D.*

Corol. Variatio minima arcus cujusvis vel anguli est ut variatio minima Secantis ejusdem directe & rectangulum sub Tangente & Secante ejusdem inverse.

THEOREMA I.

Trianguli rectilinei permanente uno quovis angulo atque uno laterum illi adjacentium: erit Variatio minima lateris reliqui adjacentis, ad Variationem minimam lateris oppositi, ut Radius, ad Sinum complementi illius anguli qui lateri permanenti opponitur.

TRIANGULUM *ABC*, permanente uno quovis angulo *B* atque uno laterum illi adjacentium *AB*, mutetur in triangulum *ABD*; & in *AD* sumatur *AE* ipsi *AC* æqualis, jungaturque *CE*. Dico lateris reliqui adjacentis variationem *CD* esse ad lateris oppositi *AC* variationem *DE* ut Radius ad Sinum complementi anguli illius *C* qui lateri permanenti *AB* opponitur, ubi Variationes istæ minimæ sunt. Nam in hoc



hoc casu habebuntur anguli recti *CED* & *ACE*. Itaque *CD* erit ad *DE*, ut Radius ad Sinum anguli *DCE* qui complementum est anguli *C*. *Q.E.D.*

THEOREMA II.

Iisdem permanentibus erit mensura minimæ Variationis anguli utriusvis reliqui, ad Variationem minimam lateris quod angulo permanenti opponitur, ut Tangens anguli qui lateri permanenti opponitur, ad Latus quod angulo permanenti opponitur.

MANENTIBUS supra constructis, centro *A* & Radio illo dato Canonico *AF*, describatur *FG* mensura Variationis anguli *A*, quæ erit etiam mensura Variationis anguli *C*. Dico hanc mensuram *FG* esse ad *DE* variationem lateris *AC* quod angulo permanenti *B* opponitur, ut Tangens anguli *C* qui lateri permanenti *AB* opponitur ad latus *AC* quod angulo permanenti *B* opponitur, ubi Variationes illæ minimæ sunt. Nam in hoc casu erit *FG* ad *CE* ut Radius ad *AC*, atque *CE* ad *DE* ut Tangens anguli *CDE* vel *C* ad Radium: ergo, ex æquo perturbate, erit *FG* ad *DE*, ut Tangens anguli *C* ad *AC*. *Q.E.D.*

THEOREMA III.

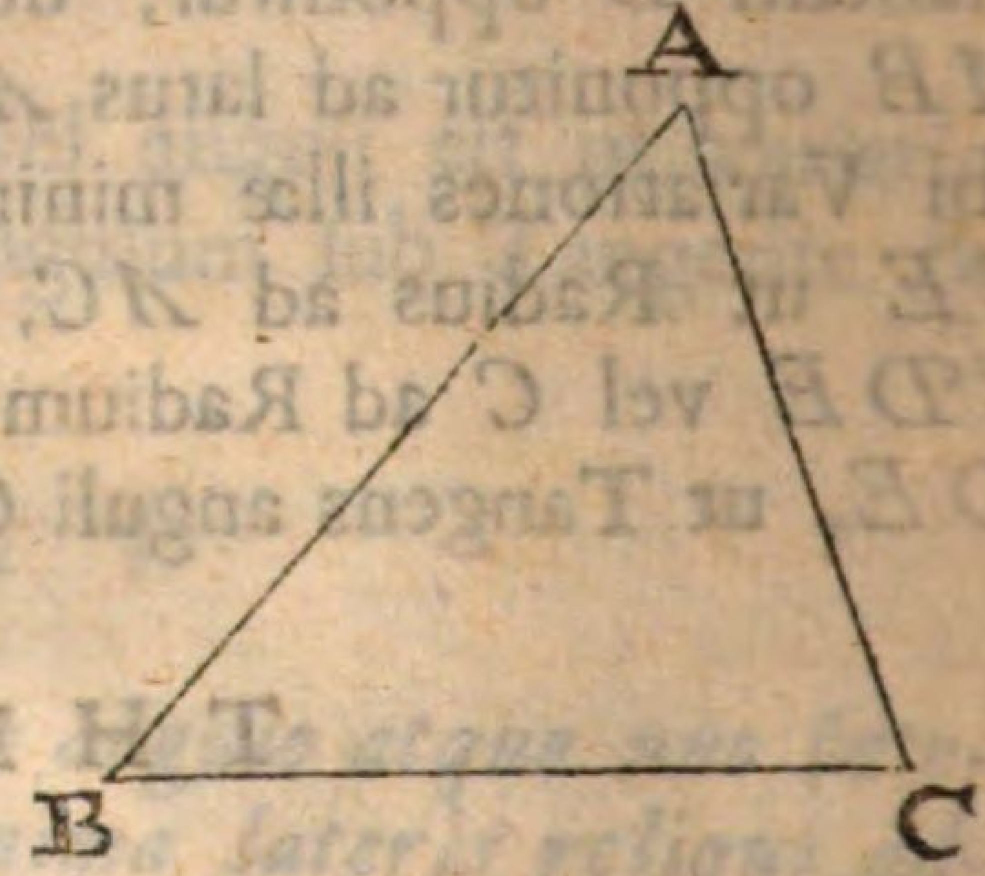
Iisdem permanentibus erit mensura minimæ Variationis anguli utriusvis reliqui, ad Variationem minimam lateris alterius adjacentium angulo permanenti, ut Sinus anguli qui lateri permanenti opponitur, ad latus quod angulo permanenti opponitur.

EADEM manente adhuc constructione: Dico *FG* mensuram variationis anguli *A* sive anguli *C*, esse ad *CD* variationem lateris alterius *BC* adjacentium angulo permanenti *B*, ut Sinus anguli *C* qui lateri permanenti *AB* opponitur ad latus *AC* quod angulo permanenti *B* opponitur; ubi Variationes illæ minimæ sunt. Nam in hoc casu erit *FG* ad *CE* ut Radius ad *AC* atque *CE* ad *CD* ut Sinus anguli *CDE* vel *C* ad Radium. Itaque, ex æquo perturbate, erit *FG* ad *CD*, ut Sinus anguli *C* ad *AC*. *Q.E.D.*

THEOREMA IV.

Trianguli rectilinei permanente uno quovis latere, atque angulo ipsi opposito: erit mensura minimæ Variationis anguli utriusvis reliqui, ad Variationem minimam lateris utriusvis reliqui; ut Tangens anguli qui lateri illi opponitur, ad illud latus.

TRIANGULI *ABC* permaneat unum quodvis latus *BC*, atque angulus *A* ipsi oppositus. Dico mensuram minimæ Variationis anguli utriusvis reliqui *C* vel *B* esse ad Variationem minimam lateris utriusvis reliqui *AB*, ut Tangens anguli *C*, qui lateri illi *AB* opponitur, ad latus *AB*. Est enim (per LEMMA I.) mensura Variationis anguli *C* ad Variationem minimam Sinus illius anguli, ut Radius ad Sinum complementi ejusdem. Præterea (cum ratio Sinus anguli *C* ad Latus *AB* eadem sit quæ est Sinus anguli *A* ad latus *BC*, adeoque permaneat) erit Variatio minima Sinus anguli *C* ad Variationem minimam lateris *AB*, ut Sinus anguli *C* ad latus *AB*. Unde mensura minimæ Variationis anguli *C* erit ad Variationem minimam lateris *AB* in ratione composita ex rationibus Radii ad Sinum complementi anguli *C*, & Sinus anguli *C* ad latus *AB*; sive ex rationibus Radii ad Latus *AB* & (Sinus anguli *C* ad Sinum complementi ejusdem vel) Tangentis anguli *C* ad Radium. Itaque mensura minimæ Variationis anguli *C* vel anguli *B* erit ad Variationem minimam lateris *AB*, ut Tangens anguli *C* ad latus *AB*. *Q.E.D.*

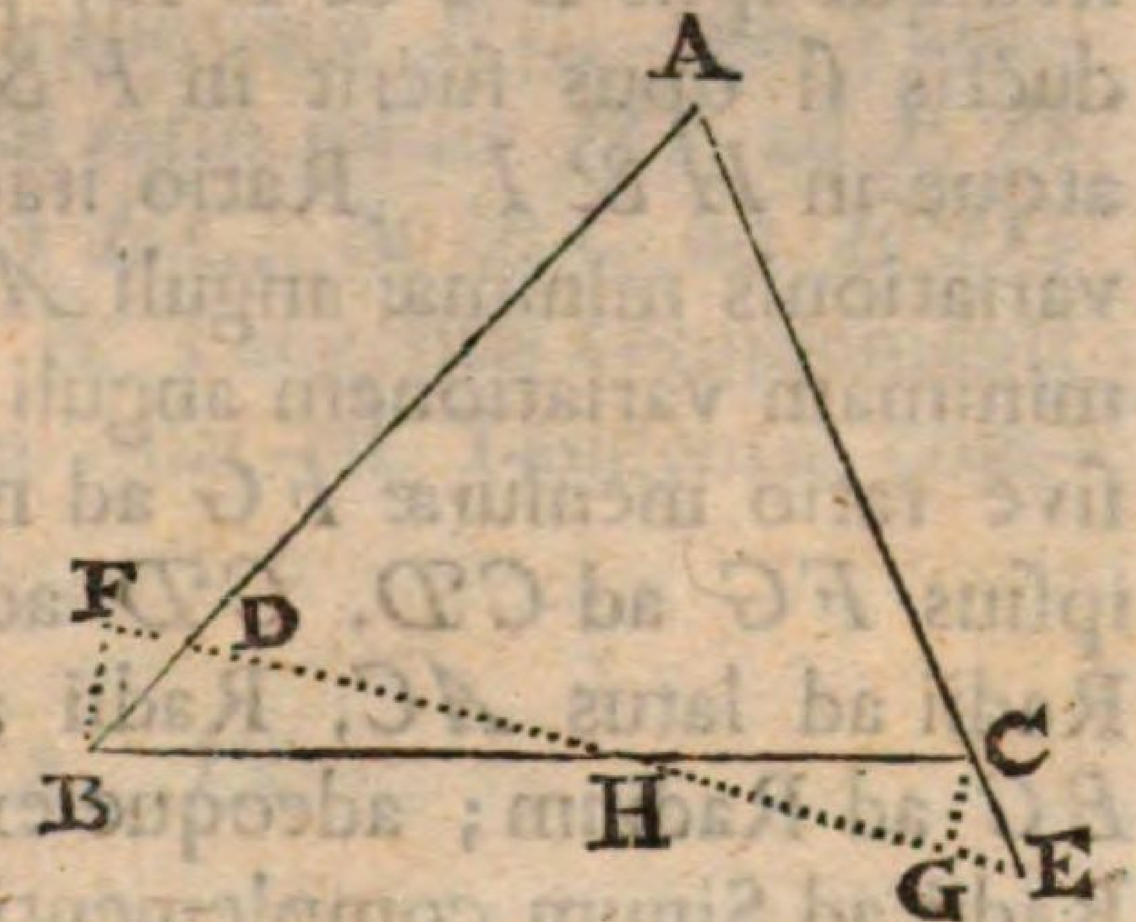


THEO-

THEOREMA V.

Iisdem permanentibus, erunt minimæ Variationes laterum reliquorum ad invicem ut Sinus complementorum angulorum qui lateribus istis opponuntur.

TRIANGULUM *ABC* permanente uno quovis angulo *A* atque uno latere *BC* ipsi opposito mutetur in triangulum *ADE*. Dico lateris *AB* variationem *BD* esse ad lateris *AC* variationem *CE*, ut Sinus complementi anguli *ACB*, qui lateri *AB* opponitur, ad Sinum complementi anguli *ABC* qui lateri *AC* opponitur, ubi Variationes illæ minimæ sunt. Secent enim latera *DE*, *BC*, producta si opus fuerit, se mutuo in *H*; & sumatur *HF* æqualis *HB*, itemque *HG* æqualis *HC*, & jungantur *BF*, *CG*. Erunt ergo *DF* & *EG* æquales. Est autem *BD* ad *DF*, ut Radius ad Sinum complementi anguli *ABC*; similiter *DF* sive *EG* est ad *CE*, ut Sinus complementi anguli *ACB* ad Radium. Itaque, ex æquo perturbate, *BD* est ad *CE*, ut Sinus complementi anguli *ACB* ad Sinum complementi anguli *ABC*. *Q.E.D.*



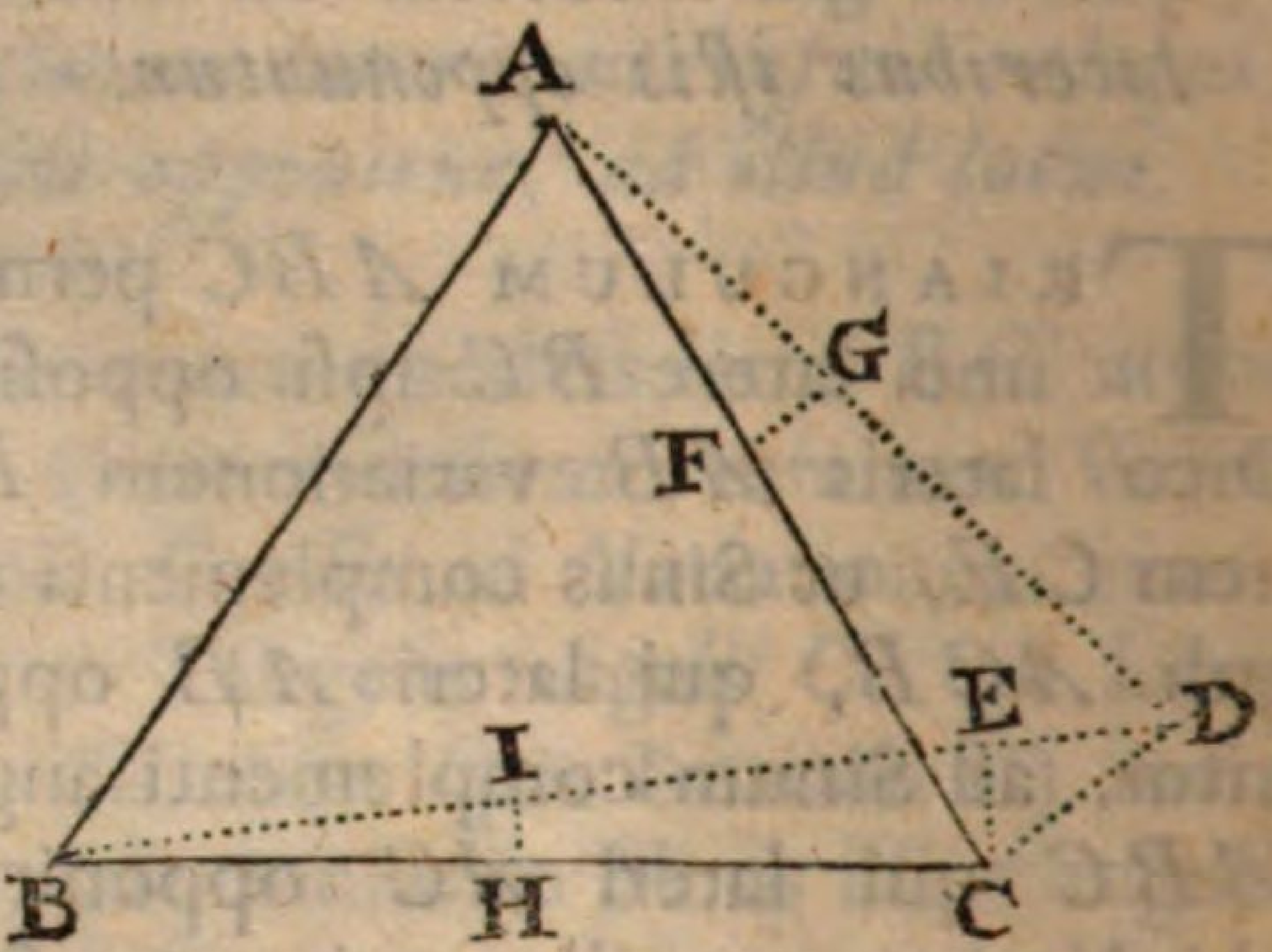
THEOREMA VI.

Trianguli rectilinei permanentibus duobus quibusvis lateribus: erit Variatio minima anguli lateribus permanentibus comprehensi, ad minimam Variationem anguli utriusvis reliqui in ratione composita ex ratione directa laterum quæ angulis illis opponuntur, & ex ratione Radii ad Sinum complementi anguli tertii.

TRIANGULI *ABC* permaneant duo quævis latera *AB* & *AC*: Dico minimam variationem anguli *A*, quem permanentia latera *AB* & *AC* comprehendunt, esse ad Variationem minimam anguli utriusvis reliqui *B* in ratione composita ex ratione lateris

BC

BC quod angulo *A* opponitur ad latus *AC* quod angulo *B* opponitur, & ex ratione Radii ad Sinum complementi anguli tertii *C*. Mutetur enim triangulum *ABC* in triangulum *ABD*, permanentibus laterum *AB* & *AC* vel *AD* magnitudinibus; & in latere *BD* ſumatur *BE* æqualis ipſi *BC*, & jungantur *CE*, *CD*. Centris inſuper *A* & *B* datoque illo Radio Canonico deſcribantur *FG*, *HI* menſuræ variationum angulorum *A* & *B*, occurrentes ipſis *AC* & *AD* itemque ipſis *BC* & *BD*, productis ſi opus fuerit in *F* & *G* atque in *H* & *I*. Ratio itaque variationis minimæ anguli *A* ad minimam variationem anguli *B*, ſive ratio menſuræ *FG* ad menſuram *HI* componetur ex rationibus ipſius *FG* ad *CD*, *CD* ad *CE*, *CE* ad *HI*; ſive ex rationibus Radii ad latus *AC*, Radii ad Sinum complementi anguli *C*, lateris *BC* ad Radium; adeoque ex rationibus lateris *BC* ad latus *AC*, & Radii ad Sinum complementi anguli *C*. *Q.E.D.*



THEOREMA VII.

Iiſdem permanentibus, erit menſura minimæ Variationis anguli lateribus permanentibus comprehenſi ad Variationem minimam lateris oppoſiti, ut Secans complementi anguli utriuſvis reliqui ad latus permanens ipſi conterminum.

EADEM manente conſtructione: Dico *FG* menſuram Variationis anguli *A* qui continetur lateribus permanentibus *AB* & *AC* eſſe ad *ED* variationem lateris oppoſiti *BC*, ut Secans complementi anguli utriuſvis reliqui *C* ad latus permanens ipſi conterminum *AC*, ubi variationes illæ minimæ ſunt. Nam in hoc caſu ratio ipſius *FG* ad *ED* (cum ſit compoſita ex rationibus *FG* ad *CD*, & *CD* ad *ED*) componetur ex rationibus Radii ad *AC*, & Secantis complementi anguli *C* ad Radium; adeoque erit eadem quæ eſt Secantis complementi anguli *C* ad latus *AC*. *Q.E.D.*

THEO-

THEOREMA VIII.

Iisdem permanentibus, erit mensura minimæ Variationis anguli utriusvis oppositorum lateribus permanentibus ad Variationem minimam lateris reliqui, ut Tangens complementi anguli reliqui oppositorum lateribus permanentibus ad ipsum latus reliquum.

EADEM adhuc manente constructione; Dico *HI* mensuram Variationis anguli utriusvis *B* oppositorum lateribus permanentibus *AC* & *AB* esse ad *ED* Variationem lateris reliqui *BC*, ut Tangens complementi anguli reliqui *C* oppositorum lateribus permanentibus *AC* & *AB* ad ipsum latus reliquum *BC*, ubi Variationes illæ minimæ sunt. Nam in hoc casu ratio ipsius *HI* ad *ED* (cum sit composita ex rationibus *HI* ad *EC*, & *EC* ad *ED*) componetur ex rationibus Radii ad *BC*, & Tangentis complementi anguli *C* ad Radium; adeoque erit eadem quæ est Tangentis complementi anguli *C* ad *BC*. *Q.E.D.*

THEOREMA IX.

Iisdem permanentibus, minimæ Variationes angulorum qui lateribus permanentibus opponuntur erunt ad invicem ut Tangentes eorundem angulorum.

QUONIAM permanent latera permanet ratio Sinuum angulorum qui ipsis opponuntur; itaque Sinuum illorum Variationes erunt ad invicem ut ipsi Sinus. Unde Variationes angulorum (cum sint, per Cor. LEM. I, ut Variationes Sinuum directe utque Sinus complementorum inverse) erunt ut ipsi Sinus directe utque Sinus complementorum inverse; hoc est, directe ut Tangentes eorundem angulorum. *Q.E.D.*

THEOREMA X.

Trianguli rectilinei permanentibus duobus quibusvis angulis permanet & tertius angulus; laterumque Variationes sunt ut ipsa latera.

NAM quoniam, ex Hypothesi, duo permanent anguli, atque omnium angulorum summa in triangulo rectilineo semper

eadem permanet; tertius etiam permanet angulus: adeoque Species trianguli & laterum ratio; laterum ergo Variationes sunt ut ipsa latera. *Q. E. D.*

LEMMA IV.

Si inter tres illos Polos laterum trianguli cujusvis Sphærici, qui sunt ipsi triangulo proximi, constituatur aliud triangulum Sphæricum: hujus latera & anguli erunt respective illius angulorum & laterum Supplementa ad Semicirculum. Quod si, mutato illo triangulo, simul & hoc ita mutetur ut hujus anguli semper sint ad Polos illos proximos laterum illius: erunt Variationes laterum hujus trianguli æquales mensuris Variationum angularium illius, & mensuræ Variationum angularium hujus trianguli æquales Variationibus laterum illius respective.

TRIANGULI Sphærici *ABC* sunto laterum Poli triangulo proximi *E*, *F* & *G*, nempe *E* Polus lateris *BC*, *F* Polus lateris *AC*, ac *G* Polus lateris *AB*; atque inter tres illos Polos constituatur triangulum aliud Sphæricum *EFG*. Dico latera *FG*, *EG* & *EF* esse Supplementa angulorum *A*, *B* & *C* respective; atque angulos *G*, *F* & *E* esse Supplementa laterum *AB*, *AC* & *BC* respective. Quod si, mutato utcunque triangulo *ABC*, simul mutetur triangulum *EFG* ita ut ejus anguli *E*, *F* & *G* semper sint ad Polos laterum *BC*, *AC* & *AB*: Erunt Variationes laterum *FG*, *EG* & *EF* æquales respective mensuris Variationum angulorum *A*, *B* & *C*, & mensuræ Variationum angulorum *G*, *F* & *E* æquales respective Variationibus laterum *AB*, *AC* & *BC*.



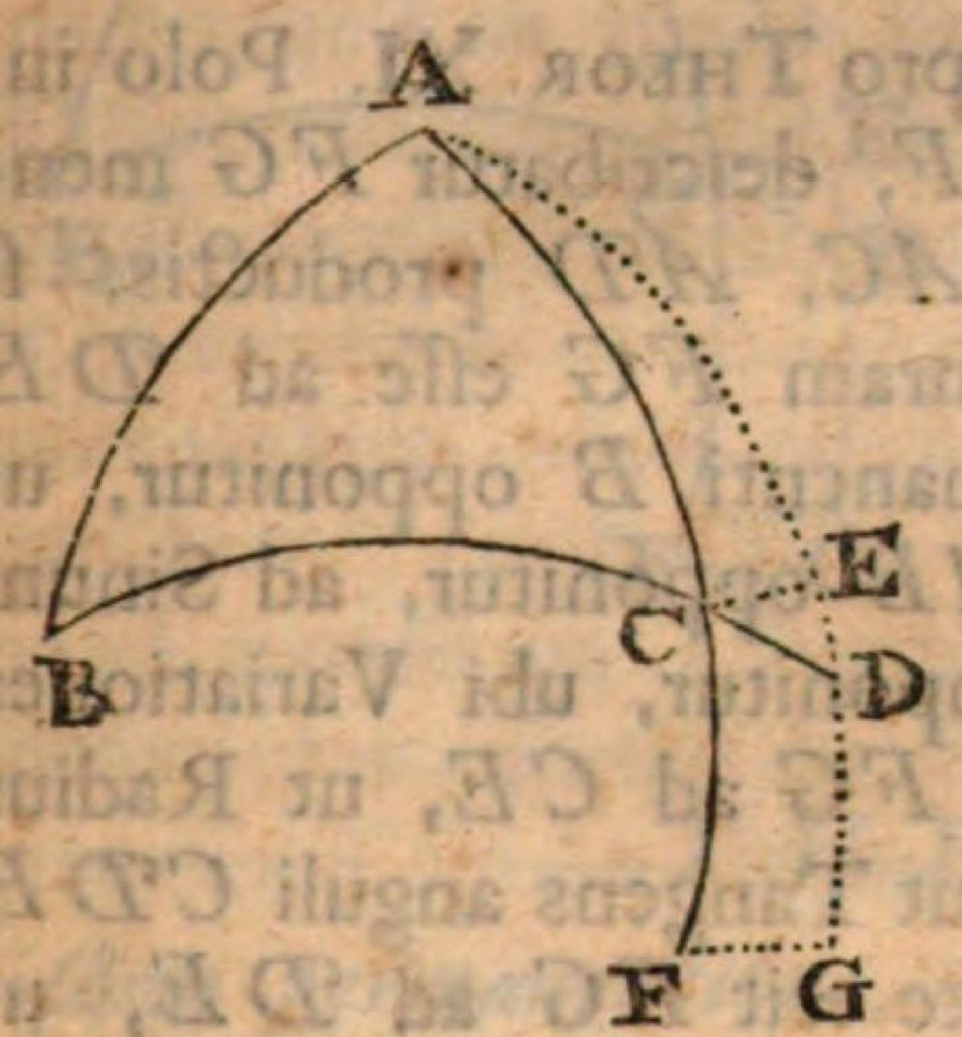
Pars prior hujus Lemmatis demonstratur a Scriptoribus Trigonometriæ Sphæricæ: posterior e priore facile colligitur.

THEO-

THEOREMA XI.

Trianguli Sphærici permanente uno quovis angulo atque uno laterum illi adjacentium: erit Variatio minima lateris reliqui adjacentis ad Variationem minimam lateris oppositi, ut Radius ad Sinum complementi illius anguli qui lateri permanenti opponitur.

TRIANGULUM *ABC*, permanente uno quovis angulo *B* atque uno laterum illi adjacentium *AB*, mutetur in triangulum *ABD*; & in *AD* sumatur *AE* ipsi *AC* æqualis jungaturque *CE*. Dico lateris reliqui adjacentis *BC* Variationem *CD* esse ad lateris oppositi *AC* Variationem *DE*, ut Radius ad Sinum complementi illius anguli *C* qui lateri permanenti *AB* opponitur. Nam, ubi Variationes illæ minimæ sunt, habebuntur anguli recti *CED* & *ACE*. Itaque *CD* erit ad *DE* ut Radius ad Sinum complementi anguli *C*. *Q.E.D.*



THEOREMA XII.

Iisdem permanentibus, erit minima Variatio anguli reliqui adjacentium lateri permanenti, ad Variationem minimam anguli oppositi, ut Radius ad Sinum complementi illius lateris quod angulo permanenti opponitur. (Vide Fig. LEM. IV.)

TRIANGULI *ABC* permanente uno quovis latere *AB* atque uno angulorum illi adjacentium *B*: Dico Variationem minimam anguli *A*, qui est reliquus adjacentium lateri permanenti *AB*, esse ad Variationem minimam anguli oppositi *C*, ut Radius ad Sinum complementi illius lateris *AC* quod angulo permanenti *B* opponitur. Nam, positis & constructis iisdem quæ supra fuerunt in LEMMATE IV constructa & posita, permanebunt in triangulo Polari *EFG* angulus *G* atque latus *EG*. Unde (per THEOR. XI) Variatio minima lateris *FG* erit ad Variationem minimam lateris *FE*, ut Radius

dius ad Sinum complementi anguli *F*: Adeoque minima Variatio anguli *A* ad Variationem minimam anguli *C*, ut Radius ad Sinum complementi lateris *AC*. *Q. E. D.*

THEOREMA XIII.

Iisdem permanentibus erit mensura minimæ Variationis anguli reliqui adjacentium lateri permanenti, ad Variationem minimam lateris quod angulo permanenti opponitur, ut Tangens anguli qui lateri permanenti opponitur ad Sinum lateris quod permanenti angulo opponitur.

MANENTIBUS quæ erant constructa pro THEOR. XI. Polo insuper *A*, ad distantiam quadrantis *AF*, describatur *FG* mensura Variationis anguli *A* occurrens ipsis *AC*, *AD* productis, si opus fuerit, in *F* & *G*. Dico hanc mensuram *FG* esse ad *DE* Variationem lateris *AC*, quod angulo permanenti *B* opponitur, ut Tangens anguli *C*, qui lateri permanenti *AB* opponitur, ad Sinum lateris *AC* quod angulo permanenti *B* opponitur, ubi Variationes illæ minimæ sunt. Nam in hoc casu erit *FG* ad *CE*, ut Radius ad Sinum lateris *AC*; atque *CE* ad *DE*, ut Tangens anguli *CDE* vel *C* ad Radium: Ergo ex æquo perturbate erit *FG* ad *DE*, ut Tangens anguli *C* ad Sinum lateris *AC*. *Q. E. D.*

THEOREMA XIV.

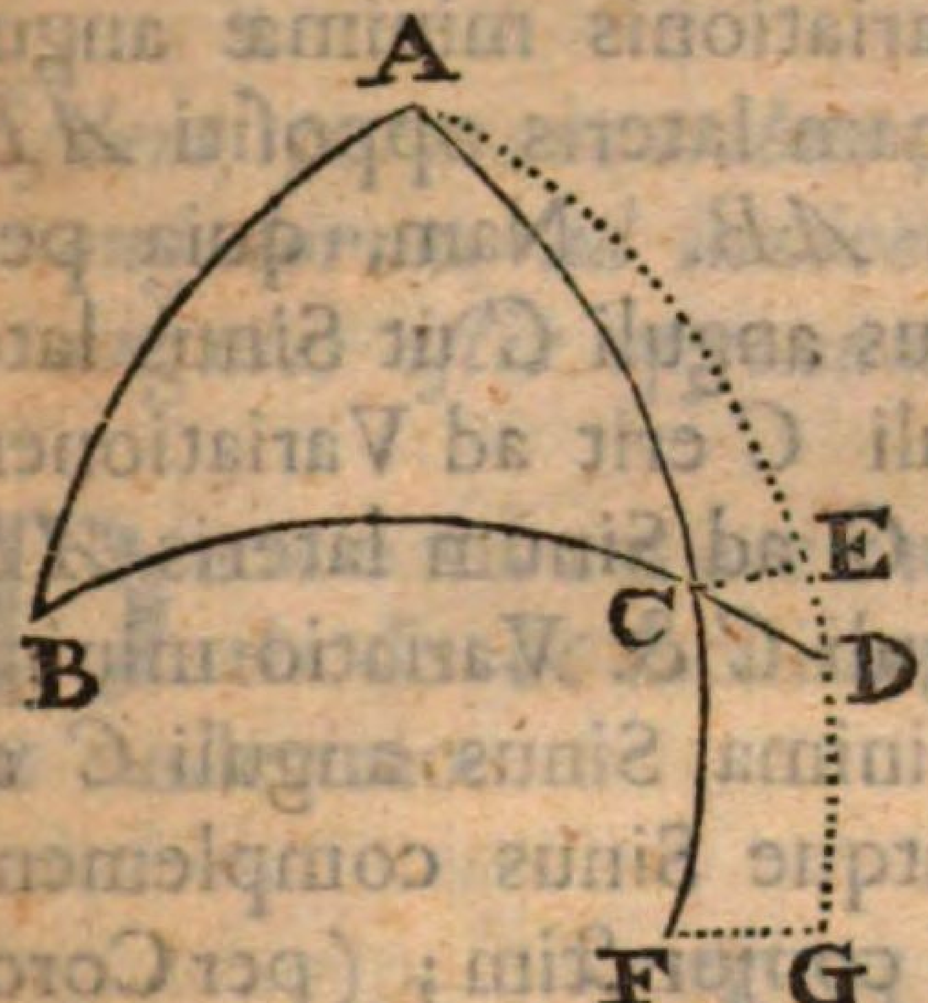
Iisdem permanentibus, erit Variatio minima lateris reliqui adjacentium angulo permanenti, ad mensuram minimæ Variationis anguli illius qui lateri permanenti opponitur, ut Tangens lateris quod angulo permanenti opponitur ad Sinum anguli qui lateri permanenti opponitur.

DEMONSTRATUR ope Lemmatis IV e Theoremate XIII eodem modo quo supra demonstratum est Theor. XII e Theor. XI.

THEO-

THEOREMA XV.

Iisdem permanentibus, erit mensura minimæ Variationis anguli reliqui adjacentium lateri permanenti ad Variationem minimam lateris reliqui adjacentium angulo permanenti, ut Sinus anguli qui lateri permanenti opponitur ad Sinum lateris quod angulo permanenti opponitur.



MANENTIBUS supra constructis; Dico *FG* mensuram Variationis anguli *A* esse ad *CD* Varitionem lateris *BC*, ut Sinus anguli *C* ad Sinum lateris *AC*, ubi Variationes illæ minimæ sunt. Nam in hoc casu erit *FG* ad *CE*, ut Radius ad Sinum lateris *AC*; atque *CE* ad *CD*, ut Sinus anguli *C* ad Radium. Itaque, ex æquo perturbate, erit *FG* ad *CD* ut Sinus anguli *C* ad Sinum lateris *AC*. *Q.E.D.*

THEOREMA XVI.

Iisdem permanentibus, erit mensura minimæ Variationis anguli qui lateri permanenti opponitur ad Variationem minimam lateris quod angulo permanenti opponitur, ut Tangens anguli ad Tangentem lateris.

TRIANGULI *ABC* permanentibus angulo *B* atque latere *AB*; Dico mensuram minimæ Variationis anguli *C* esse ad Variationem minimam lateris *AC*, ut Tangens anguli *C* ad Tangentem lateris *AC*. Nam, quia permanent angulus *B* & latus *AB*, erit Sinus anguli *C* ut Sinus lateris *AC* inverse. Ergo Variatio minima Sinus anguli *C* erit ad Variationem minimam Sinus lateris *AC*, ut Sinus anguli *C* ad Sinum lateris *AC*. Cum itaque mensura minimæ Variationis anguli *C* sit ad Variationem minimam lateris *AC*, ut Variatio minima Sinus anguli *C* ad Variationem minimam Sinus lateris *AC*, utque Sinus complementi lateris *AC* ad Sinum complementi anguli *C* conjunctim; (per Corol. LEM. I) erunt etiam ad invicem ut Sinus anguli *C* ad Sinum lateris *AC*, utque Sinus complementi lateris *AC* ad Sinum complementi anguli *C* conjunctim; hoc est, ut Tangens anguli *C* ad Tangentem lateris *AC*. *Q.E.D.*

THEOREMA XVII.

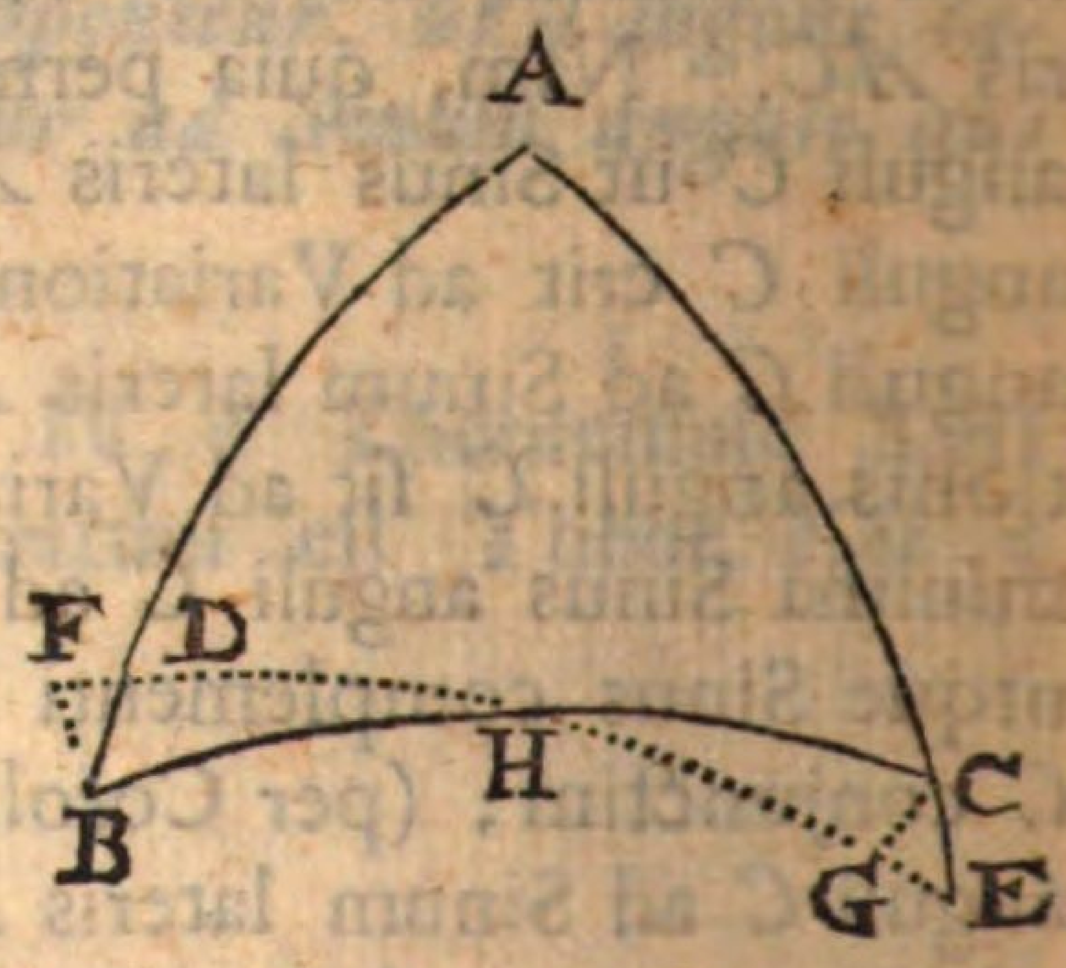
Trianguli Sphærici permanente uno quovis latere atque angulo ipsi opposito; erit mensura minimæ Variationis anguli utriusvis reliqui ad Variationem minimam lateris oppositi, ut Tangens anguli ad Tangentem lateris.

TRIANGULI *ABC* permanentibus latere *BC*, atque angulo *A* ipsi opposito: Dico mensuram Variationis minimæ anguli utriusvis reliqui *C* esse ad Variationem minimam lateris oppositi *AB*, ut Tangens anguli *C* ad Tangentem lateris *AB*. Nam, quia permanent latus *BC* atque angulus *A*, erit Sinus anguli *C* ut Sinus lateris *AB*. Ergo Variatio minima Sinus anguli *C* erit ad Variationem minimam Sinus lateris *AB*, ut Sinus anguli *C* ad Sinum lateris *AB*. Itaque cum mensura minimæ Variationis anguli *C* & Variatio minima lateris *AB* sint ad invicem ut Variatio minima Sinus anguli *C* ad Variationem minimam Sinus lateris *AB*, utque Sinus complementi lateris *AB* ad Sinum complementi anguli *C* conjunctim; (per Corol. LEM. I) erunt etiam ad invicem ut Sinus anguli *C* ad Sinum lateris *AB*, utque Sinus complementi lateris *AB* ad Sinum complementi anguli *C* conjunctim; hoc est, ut Tangens anguli *C* ad Tangentem lateris *AB*. *Q.E.D.*

THEOREMA XVIII.

Iisdem permanentibus, erunt Variationes minimæ laterum reliquorum ad invicem, ut Sinus complementorum angulorum qui lateribus istis opponuntur.

TRIANGULUM *ABC*, permanente uno quovis angulo *A* atque latere ipsi opposito *BC*, mutetur in triangulum *ADE*. Dico lateris *AB* Variationem *BD* esse ad lateris *AC* Variationem *CE*, ut Sinus complementi anguli *ACB* qui lateri *AB* opponitur ad Sinum complementi anguli *ABC* qui lateri *AC* opponitur, ubi Variationes illæ minimæ sunt. Secent enim latera *DE* & *BC* producta, si opus fuerit, se mutuo in *H*; & sumatur *HF* æqualis ipsi *HB*,



itemque

itemque *HG* æqualis ipſi *HC*, & jungantur *BF* & *CG*: erunt ergo *DF* & *EG* æquales. Eſt autem *BD* ad *DF*, ut Radius ad Sinum complementi anguli *ABC*; atque ſimiliter *DF* ſive *EG* eſt ad *CE*, ut Sinus complementi anguli *ACB* ad Radium. Itaque, ex æquo perturbate, *BD* eſt ad *CE*, ut Sinus complementi anguli *ACB* ad Sinum complementi anguli *ABC*. *Q.E.D.*

THEOREMA XIX.

Iiſdem permanentibus, erunt Variationes minimæ reliquorum angulorum ad invicem, ut Sinus complementorum laterum qui angulis iſtis opponuntur.

DEMONSTRATUR e Theoremate præcedente, eadem methodo & facilitate qua ſupra demonſtratum eſt duodecimum ex undecimo.

THEOREMA XX.

Iiſdem permanentibus, erit Variatio minima lateris utriuſvis reliqui ad menſuram Variationis minimæ anguli reliqui adjacentis, in ratione compoſita ex rationibus Tangentis ejuſdem lateris ad Tangentem anguli oppoſiti, & Sinus complementi ejuſdem lateris ad Sinum complementi lateris tertii; ſive ex rationibus Tangentis lateris tertii ad Tangentem anguli, qui lateri tertio opponitur, & Sinus complementi anguli tertii ad Sinum complementi ejuſdem anguli oppoſiti tertio lateri.

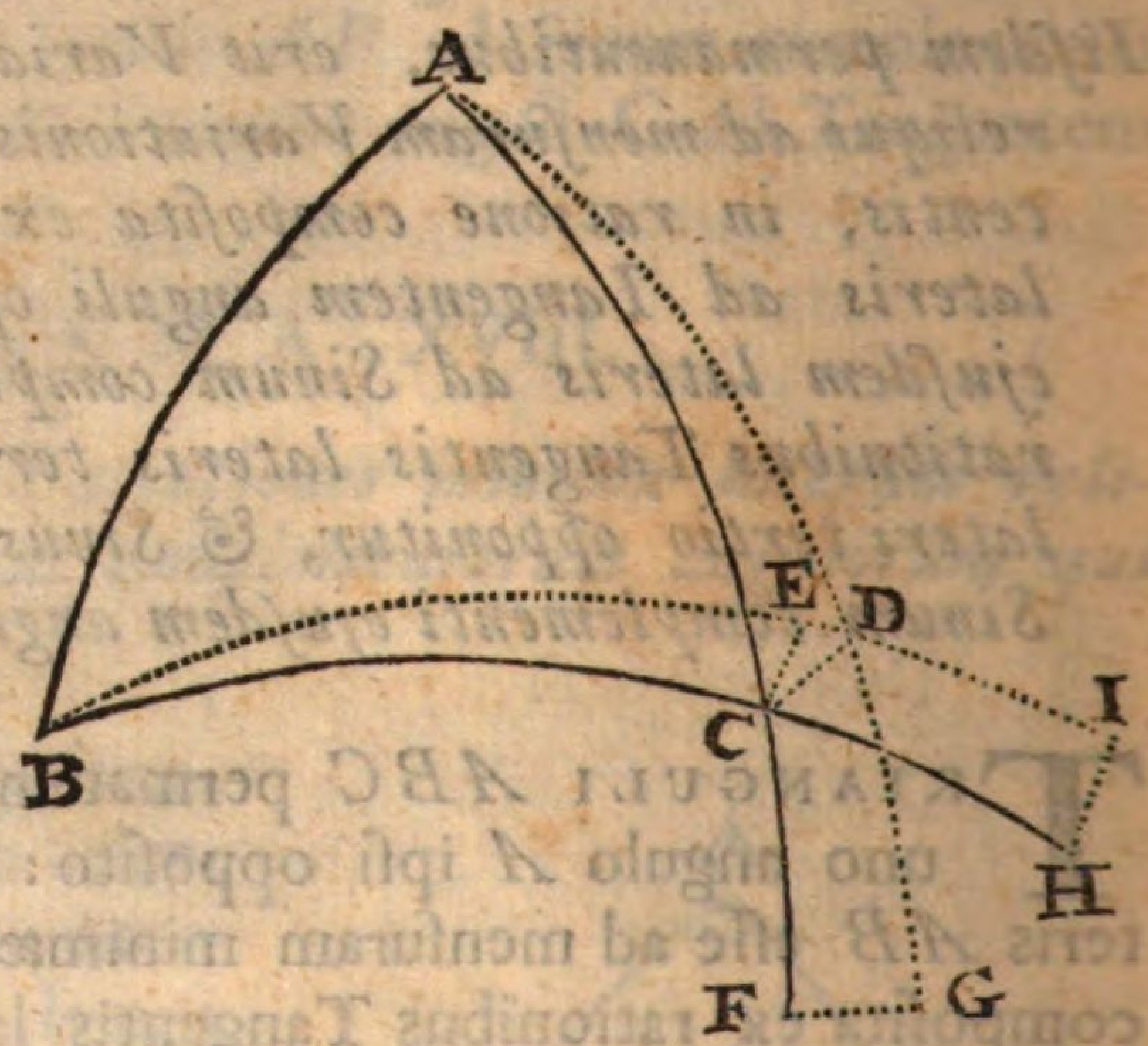
TRIANGULI *ABC* permanente uno quovis latere *BC* atque uno angulo *A* ipſi oppoſito: Dico Variationem minimam lateris *AB* eſſe ad menſuram minimæ Variationis anguli *B* in ratione compoſita ex rationibus Tangentis lateris *AB* ad Tangentem anguli *C*, & Sinus complementi lateris *AB* ad Sinum complementi lateris *AC*; ſive ex rationibus Tangentis lateris *AC* ad Tangentem anguli *B*, & Sinus complementi anguli *C* ad Sinum complementi anguli *B*. Nam quia ratio minimæ Variationis lateris *AB* ad menſuram minimæ Variationis anguli *B* componitur ex rationibus Variationis minimæ lateris *AB* ad menſuram minimæ Variationis anguli *C*, & menſuræ Variationis minimæ anguli *C* ad menſuram Variationis minimæ anguli *B*; ſive ex rationibus Variationis minimæ lateris *AB* ad Varia-

Variationem minimam lateris *AC*, & Variationis minimæ lateris *AC* ad mensuram Variationis minimæ anguli *B*: componetur utique ex rationibus prædictis per Theoremata tria novissima. *Q.E.D.*

THEOREMA XXI.

Trianguli Sphærici permanentibus duobus quibusvis lateribus; erit anguli a lateribus permanentibus comprehensi Variatio minima ad Variationem minimam anguli utriusvis reliqui in ratione composita ex ratione Sinuum laterum quæ angulis istis opponuntur, & ex ratione Radii ad Sinum complementi anguli tertii.

TRIANGULI *ABC* permaneant duo quævis latera *AB* & *AC*: Dico minimam Variationem anguli *A*, quem permanentia latera *AB* & *AC* comprehendunt, esse ad minimam Variationem anguli utriusvis reliqui *B* in ratione composita ex ratione Sinus lateris *BC*, quod angulo *A* opponitur, ad Sinum lateris *AC* quod angulo *B* opponitur, & ex ratione Radii ad Sinum complementi anguli tertii *C*. Mutetur enim triangulum *ABC* in triangulum *ABD*, permanentibus laterum *AB* & *AC* vel *AD* magnitudinibus; & in latere *BD* sumatur *BE* æqualis ipsi *BC* & jungantur *CE*, *CD*. Polis insuper *A* & *B* ad distantiam quadrantis describantur *FG*, *HI* mensuræ Variationum angulorum *A* & *B*, occurrentes ipsis *AC* & *AD* itemque ipsis *BC* & *BD* productis, si opus fuerit, in *F* & *G* atque in *H* & *I*. Ratio itaque minimæ Variationis anguli *A* ad Variationem minimam anguli *B*, sive ratio mensuræ *FG* ad mensuram *HI* componitur ex rationibus ipsius *FG* ad *CD*, *CD* ad *CE*, *CE* ad *HI*; sive ex rationibus Radii ad Sinum lateris *AC*, Radii ad Sinum complementi anguli *C*, Sinus lateris *BC* ad Radium. Adeoque ex rationibus Sinus lateris *BC* ad Sinum lateris *AC*, & Radii ad Sinum complementi anguli *C*. *Q.E.D.*



THEO-

THEOREMA XXII.

Iisdem permanentibus, erit mensura minimæ Variationis anguli qui lateribus permanentibus continetur ad Variationem minimam lateris oppositi, ut Secans complementi anguli utriusvis reliqui ad Sinum lateris permanentis & contermini.

EADEM manente constructione: Dico FG mensuram Variationis anguli A qui continetur a lateribus permanentibus AB & AC esse ad ED Variationem lateris oppositi BC, ut Secans complementi anguli utriusvis reliqui C ad Sinum lateris AC permanentis & contermini, ubi Variationes illæ minimæ sunt. Nam, in hoc casu, ratio ipsius FG ad ED (cum sit composita ex rationibus FG ad CD, & CD ad ED) componetur ex rationibus Radii ad Sinum lateris AC, & Secantis complementi anguli C ad Radium. Adeoque erit eadem quæ est Secantis complementi anguli C ad Sinum lateris AC. *Q.E.D.*

THEOREMA XXIII.

Iisdem permanentibus, erit mensura minimæ Variationis anguli utriusvis oppositorum lateribus permanentibus ad Variationem minimam lateris reliqui, ut Tangens complementi anguli reliqui oppositorum lateribus permanentibus ad Sinum ipsius lateris reliqui.

EADEM adhuc manente constructione; Dico HI mensuram Variationis anguli utriusvis B oppositorum lateribus permanentibus AC & AB esse ad ED Variationem lateris reliqui BC, ut Tangens complementi anguli reliqui C oppositorum lateribus permanentibus ad Sinum ipsius lateris reliqui BC, ubi Variationes illæ minimæ sunt. Nam, in hoc casu, ratio ipsius HI ad ED (cum sit composita ex rationibus HI ad EC, & EC ad ED) componetur ex rationibus Radii ad Sinum lateris BC, & Tangentis complementi anguli C ad Radium. Adeoque erit eadem quæ est Tangentis complementi anguli C ad Sinum lateris BC. *Q.E.D.*

THEOREMA XXIV.

Iisdem permanentibus, Variationes minimæ angulorum qui lateribus permanentibus opponuntur erunt ad invicem ut Tangentes eorundem angulorum.

QUONIAM permanent latera permanet ratio Sinuum angulorum qui ipsis opponuntur. Itaque Sinuum illorum Variationes erunt ad invicem ut ipsi Sinus. Unde Variationes angulorum (cum sint ut Variationes Sinuum directe utque Sinus complementorum inverse per Corol. LEM. I) erunt ut Sinus ipsi directe utque Sinus complementorum inverse; hoc est, directe ut Tangentes eorundem angulorum. *Q. E. D.*

THEOREMA XXV.

Trianguli Sphærici permanentibus duobus quibusvis angulis; erit Variatio minima lateris interjecti ad Variationem minimam lateris utriusvis reliqui, in ratione composita ex ratione directa Sinuum angulorum qui lateribus illis opponuntur, & ex ratione Radii ad Sinum complementi lateris tertii.

THEOREMA XXVI.

Iisdem permanentibus, erit Variatio minima lateris interjecti ad mensuram Variationis minimæ anguli oppositi, ut Secans complementi lateris utriusvis reliqui ad Sinum anguli permanentis & contermini.

THEOREMA XXVII.

Iisdem permanentibus, erit Variatio minima lateris utriusvis oppositorum angulis permanentibus ad mensuram Variationis minimæ anguli reliqui, ut Tangens complementi lateris reliqui oppositorum angulis permanentibus ad Sinum ipsius anguli reliqui.

THEOREMA XXVIII.

Iisdem permanentibus, erunt Variationes minimæ laterum quæ angulis permanentibus opponuntur ad invicem ut Tangentes eorundem laterum.

DEMONSTRANTUR hæc ultima quatuor Theoremata, ope Lemmatis IV, e totidem præcedentibus, eadem methodo qua supra demonstratum est duodecimum ex undecimo.

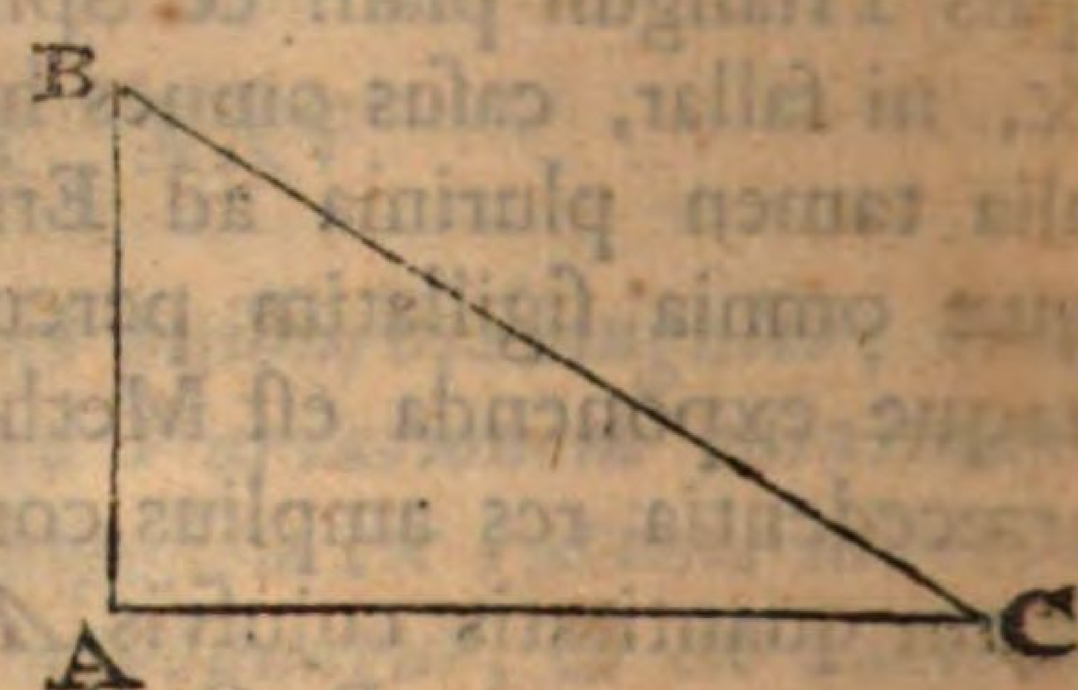
Scholium I.

Hactenus de Variationibus istis egimus quæ fiunt in partibus reliquis Trianguli plani & Sphærici dum binæ quævis non mutantur; &, ni fallar, casus omnes harum Variationum expendimus. Sunt & alia tamen plurima ad Errorum limites determinandos spectantia, quæ omnia sigillatim percurrere nimii forsan esset operis. Breviter itaque exponenda est Methodus qua uti licebit ubi per Theoremata præcedentia res amplius confici non poterit. Ponatur Errorem quæsitum quantitatis cujusvis *A* genitum esse ex Erroribus datis quantitatum quarumvis *B*, *C*, *D*. Collige ergo valorem quantitatis illius *A* ex valoribus quantitatum *B*, *C*, *D*; deinde hujus valoris Fluxionem computabis per methodum *Newtonianam*, & positis erroribus illis datis quantitatum *B*, *C*, *D* signo suo Affirmativo vel Negativo affectis pro earum Fluxionibus, habebis errorem desideratum quantitatis *A*. Sunt enim errores quantitatum, de quibus hic agimus, quam proxime ut earundem Fluxiones, ubi exigui sunt. Ceterum quoniam in Trigonometricis non semper ipsæ quantitates *A*, *B*, *C*, *D* inter se immediate conferantur, sed earum plerumque Sinus, Tangentes & Secantes, pro Fluxionibus Sinuum, Tangentium & Secantium substituendi sunt earum valores qui dantur ex datis arcuum Fluxionibus per Lemmata tria prima.

Scholium

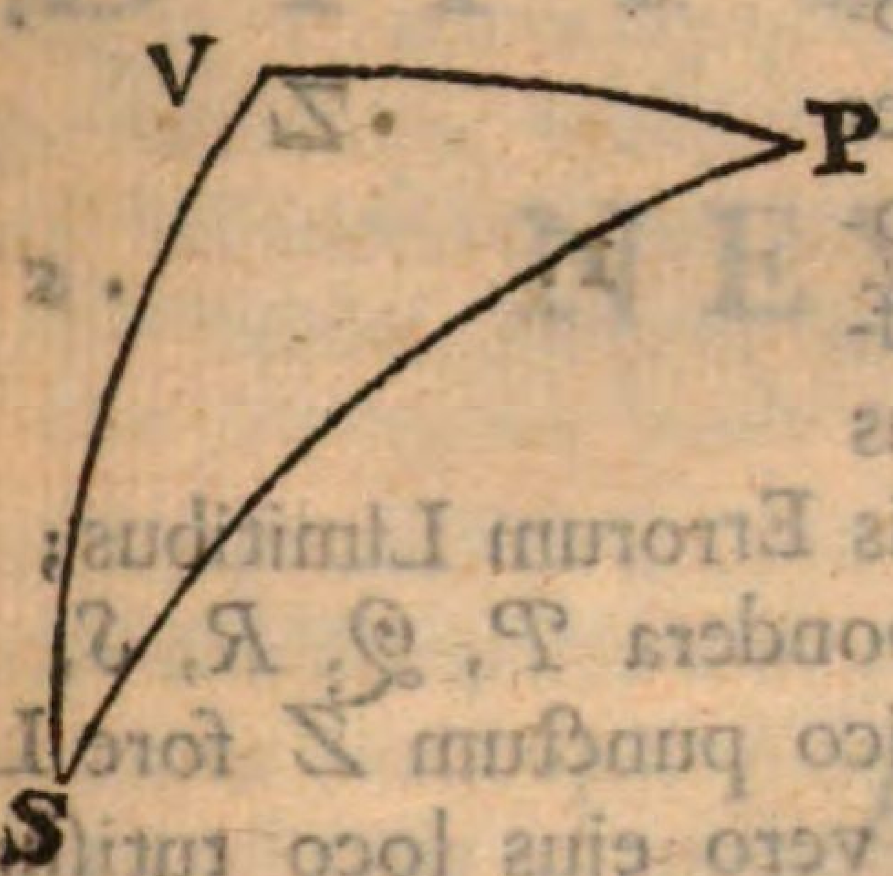
Scholium II.

Diximus Theoremata præcedentia ad Errorum præcipue Theoriam concinnata fuisse: opportunum itaque fuerit exponere per unum vel alterum e facilioribus Exemplis, qua ratione huic usui accommodari poterunt. Data sit igitur longitudo rectæ AC, cui ad angulos rectos insistat recta AB. Notum est hujus longitudinem quantumlibet accuratam inveniri per Trigonometriam, si modo accurate datus fuerit angulus ACB quem facit recta AC cum recta BC quæ conjungit puncta B & C. Angulus autem iste per Instrumentum aliquod exploretur ut locus sit Errori; & ex dato hujus Erroris limite propositum sit nobis invenire limitem Erroris maximi qui inde derivari possit in longitudinem AB. Manentibus igitur angulo recto BAC & longitudine lateris AC, erit (per THEOR. III) Variatio lateris AB ad mensuram Variationis anguli BCA ut latus BC ad Sinum anguli ABC, hoc est, ob angulum rectum BAC, ut duplicatum latus AB ad Sinum duplicati anguli ACB. Error adeo lateris AB est ad mensuram Erroris anguli ACB ut duplicatum latus AB ad Sinum duplicati anguli ACB, ac proinde lateris Error eandem habet rationem ad latus quam habet mensura duplicata Erroris anguli ad Sinum duplicati anguli. Error itaque lateris (ceteris paribus) fit minimus ubi Sinus duplicati anguli fit maximus, hoc est, ubi angulus fit semirectus. Unde si invenienda sit longitudo lateris AB ex observata magnitudine anguli ACB, commodissimum erit ad eam distantiam AC observationem instituere ut angulus ACB fiat graduum 45 quamproxime. Videamus jam quantus sit futurus error lateris in hac positione ubi anguli error minutum unum primum adæquat. Ponendo radium habere partes 10000000, mensura hujus Erroris, sive arcus minuti unius primi, habebit partes 2909, & mensura duplicata partes 5818. Jam hæc mensura duplicata 5818 est ad Sinum anguli semirecti ACB duplicati sive ad radium 10000000, ut 1 ad 1719: atque adeo error lateris est ipsius lateris pars $\frac{1}{1719}$. Si error anguli, ceteris manentibus, augeatur vel minuatur, augebitur lateris error vel minuetur in eadem ratione. Si angulus ACB, cæteris manentibus, minor sit vel major quam graduum 45, lateris error augebitur in ratione Radii ad Sinum anguli duplicati.

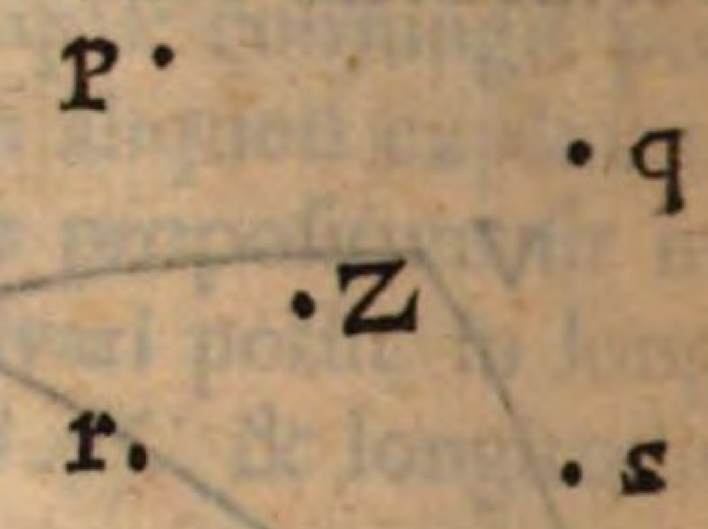


In

In praxi Astronomica sæpissime exquirenda venit diei vel noctis Hora per Sideris alicujus altitudinem. Observatæ altitudinis Errore dato, proponatur Errorem temporis definire. Designet igitur *PV* Meridianum, *P* Polum, *V* verticem loci, *VS* circulum verticalem & *PS* circulum horarium, utrumque transeuntem per Sidus. Itaque latera *PV*, *PS*, *VS* erunt respective complementa ad quadrantem Elevationis Poli, Declinationis Sideris, Altitudinis ejusdem supra Horizontem loci. Permanentibus autem lateribus *PV*, *PS*; erit mensura Variationis anguli *VPS* ad Variationem lateris *VS* ut Secans complementi anguli *PVS* ad Sinum lateris *PV*, per THEOREMA XXII. Est igitur mensura Variationis anguli *VPS* ut Variatio lateris *VS* directe & Secans complementi anguli *PVS* directe utque Sinus lateris *PV* inverse, hoc est, ut Variatio lateris *VS* directe ut Sinus anguli *PVS* inverse utque Sinus lateris *PV* inverse. Unde cum Error admissus in tempore definiendo sit ut Variatio anguli *VPS*, & Error in capienda Sideris altitudine sit ipsa lateris *VS* Variatio: erit Error temporis ut Error altitudinis directe, ut Sinus anguli *PVS* inverse utque Sinus complementi Elevationis Poli inverse. Hisce constitutis si detur altitudinis Error, sub data Poli Elevatione: Error temporis erit ut Sinus anguli *PVS* inverse. Eandem igitur quantitatem habebit, quæcunque fuerit in dato verticali Sideris altitudo; minimus autem erit ubi verticalis fecerit angulos rectos cum Meridiano. Absolute vero minimus erit Observatori sub Æquatore degenti, ubi verticalis fecerit angulos rectos cum Meridiano: quo in casu si Error altitudinis fuerit minuti unius primi, Error temporis erit quatuor scrupulorum secundorum. Si Observator recedat ab Æquatore versus alterutrum Polum, augebitur Error temporis in ratione Radii ad Sinum complementi Latitudinis. Unde in Latitudine graduum 45, erit scrupulorum secundorum $5\frac{2}{3}$; in Latitudine graduum 50, erit scrupulorum secundorum $6\frac{2}{9}$; in Latitudine graduum 55, erit scrupulorum secundorum $6\frac{37}{38}$. Si Sidus constitutum sit in alio quovis verticali, qui sit ad Meridianum obliquus, adhuc augebitur Error in ratione Radii ad Sinum illius obliqui anguli. Denique si altitudinis Error sit vel major vel minor quam minuti unius primi: Error temporis simul augebitur vel minuetur in eadem ratione.



Ad eundem fere modum in aliis casibus Limites inveniuntur Errorum qui ex minus accuratis observationibus ortum ducunt, quin & Positiones ad Observandum commodissimæ deprehenduntur: ut mihi vix quidquam ulterius desiderari videatur postquam ostensum fuerit qua ratione Probabilitas maxima in his rebus haberi possit, ubi diversæ Observationes, in eundem finem institutæ, paullulum diversas ab invicem conclusiones exhibent. Id autem fiet ad modum sequentis Exempli. Sit p locus Objecti alicujus ex Observatione prima definitus, q, r, s ejusdem Objecti loca ex Observationibus subsequentibus; sint insuper P, Q, R, S pondera reciproce proportionalia spatiis Evagationum, per quæ se diffundere possint Errores ex Observationibus singulis prodeuntes, quæque dantur ex datis Errorum Limitibus; & ad puncta p, q, r, s posita intelligantur pondera P, Q, R, S, & inveniatur eorum gravitatis centrum Z: dico punctum Z fore Locum Objecti maxime probabilem, qui pro vero ejus loco tutissime haberi potest.



DE
METHODO
DIFFERENTIALI
NEWTONIANA.

AUCTORE

ROGERO COTES.

ILLUSTRISSIMUS *Newtonus* noster, sub finem Libri tertii *Principiorum Philosophiæ*, ex locis aliquot Cometæ observatis, Locum ejus ad tempus quodvis intermedium invenire docet. Ea vero data occasione describit egregiam illam & maxime generalem Methodum *ducendi Curvam Geometricam per data quotcunque puncta*, quam olim ad *Leibnitium* scribens merito inter pulcherrima numerabat. Hanc itaque, cum permagni sit momenti, præsertim in rebus Astronomicis, & a nemine quod sciam sit hactenus demonstrata operandi ratio, paullo fusius in sequentibus exponam & nonnulla insuper de meis adjiciam.

PRO-

PROPOSITIO I. THEOREMA

Proposita qualibet serie Quantitatum; si facta omnia cujusvis dimensionis quæ ex omnibus præter Ultimam componi possunt in unam summam colligantur, similiterque facta omnia ejusdem dimensionis quæ ex omnibus præter primam componi possunt in unam summam colligantur, & subducatur summa posterior e priore, residuumque dividatur per differentiam quæ fit subducendo ultimam quantitatum propositarum e prima: Dico quantitatem quæ oritur ex ista divisione æqualem esse summæ factorum omnium dimensionis proxime inferioris, quæ componi possunt ex tota serie quantitatum propositarum.

Exempl. 1. Si constet series proposita ex duabus tantum quantitatibus a & b, & sint facta ex his componenda quatuor dimensionum: Factum unicum ex omnibus præter ultimam, nempe factum e prima, est a^4; & factum unicum ex omnibus præter primam, nempe factum ex ultima, est b^4; subducatur hoc e priore, residuum erit $a^4 - b^4$, quod si dividas per $a - b$ habebis $a^3 + aab + abb + b^3$, summam utique factorum omnium trium dimenmensionum, quæ ex tota serie quantitatum a & b componi possunt.

Exempl. 2. Constet jam series proposita ex tribus quantitatibus a, b, c, & sint facta ex his componenda trium dimensionum. Summa ergo factorum ex omnibus præter ultimam erit $a^3 + aab + abb + b^3$, & summa factorum ex omnibus præter primam erit $b^3 + bbc + bcc + c^3$. Subducatur hæc summa e priore, residuum erit $a^3 + aab + abb - bbc - bcc - c^3$, quod si dividas per $a - c$, habebis $aa + ab + ac + bb + bc + cc$, summam nempe factorum omnium duarum dimensionum, quæ ex tota serie quantitatum a, b, c componi possunt.

Exempl. 3. Proponatur series quatuor quantitatum a, b, c, d; & sint facta ex his componenda duarum dimensionum. Summa ergo factorum ex omnibus præter ultimam erit $aa + ab + ac + bb + bc + cc$, summa vero factorum ex omnibus præter primam erit $bb + bc + bd + cc + cd + dd$. Subducatur hæc summa e priore, residuum erit $aa + ab + ac - bd - cd - dd$, quod si dividas per $a - d$ habebis $a + b + c + d$, summam scilicet factorum omnium unius dimensionis (si ita dicere liceat) quæ ex tota serie componi possunt.

Exempl.

Exempl. 4. Si series proposita sit quinque quantitatum a, b, c, d, e & sint facta ex his componenda unius dimensionis: Summa factorum ex omnibus præter ultimam erit $a+b+c+d$, atque summa factorum ex omnibus præter primam erit $b+c+d+e$. Subducatur summa posterior e priore, residuum erit $a-e$. Divide hoc per $a-e$, & habebis 1, unicum nempe factum nullius dimensionis quod ex tota serie componi potest.

PROPOSITIO II. THEOREMA.

Si Æquatio indefinita $y=f+gx+hxx+kx^3+$&c. *cujus dentur omnes coefficientes* f, g, h, k, &c. *terminetur ita ut ad datum quemlibet Ordinem Analyticum pertineat. Et pro* x *ponantur successive valores quilibet diversi* a, b, c, d, e, &c. *adeoque pro* y *emergant respective valores* $\alpha, \beta, \gamma, \delta, \varepsilon$, &c. *atque hi valores respectivi in tabella e regione invicem collocentur,*

			x	y							
			a	α							
		λ'			r'	ρ'					
	μ'		b	β			s'	σ'			
ν'		λ''			r''	ρ''			t'	τ'	
	μ''		c	γ			s''	σ''			&c.
ν''		λ'''			r'''	ρ'''			t''	τ''	
	μ'''		d	δ			s'''	σ'''			
		λ''''			r''''	ρ''''					
			e	ε							

differentiæque adjungantur sequenti modo; ita ut sit $\lambda'=a-b$, $\lambda''=b-c$, $\lambda'''=c-d$, $\lambda''''=d-e$, &c. $\mu'=a-c$, $\mu''=b-d$, $\mu'''=c-e$, &c. $\nu'=a-d$, $\nu''=b-e$, &c. *Deinde* $r'=\alpha-\beta$, $r''=\beta-\gamma$, $r'''=\gamma-\delta$, $r''''\delta-\varepsilon$, &c. *Porro* $\rho'=\frac{r'}{\lambda'}$, $\rho''=\frac{r''}{\lambda''}$, $\rho'''=\frac{r'''}{\lambda'''}$, &c. *Iterumque* $s'=\rho'-\rho''$, $s''=\rho''-\rho'''$, &c. *Tum* $\sigma'=\frac{s'}{\mu'}$, $\sigma''=\frac{s''}{\mu''}$, &c. *atque ita deinceps: Differentiæ sic divisæ Primæ quidem* ρ', ρ'', ρ''', &c. *inter se & altissimi gradus coefficienti* g *æquales erunt, si fuerit æquatio ordinis primi* $y=f+gx$; *Secundæ* $\sigma', \sigma'', \sigma'''$, &c. *inter se & altissimi gradus coefficienti* h *æquales erunt, si fuerit æquatio* $y=f+gx+hxx$ *ordinis*

ordinis secundi; *Tertiæ* τ', τ'', τ''', &c. *inter se & altissimi gradus coefficienti* k *æquales erunt*, *si fuerit æquatio* $y = f + gx + hxx + kx^3$ *ordinis tertii*; *atque ita deinceps, cujuscunque ordinis fuerit æquatio, progredi licebit.*

Cas. 1. DESINT termini omnes æquationis propositæ præter eum qui est altissimi gradus, sitque hujus coefficiens æqualis unitati; & erit y æqualis puræ alicui potestati ipsius x. Igitur ad quamcunque dimensionem ipsarum a, b, c, d, e ascendant α, β, γ, δ, ε patet (ex Prop. I) fore ϱ' summam factorum omnium dimensionis proxime inferioris quæ ex quantitatibus a & b componi possunt, ϱ'' summam omnium ex b & c, ϱ''' summam omnium ex c & d, ϱ'''' summam omnium ex d & e. Similiter σ' erit summa factorum omnium dimensionis proxime inferioris quæ ex quantitatibus ternis a, b, c componi possunt; σ'' summa omnium ex b, c, d; σ''' summa omnium ex c, d, e. Iterum τ', τ'', erunt summæ omnium dimensionis adhuc inferioris ex quaternis; atque ita deinceps, donec ad Unitates tandem deventum fuerit, ut in Exemplo quarto Propositionis I; quod contingere oportet in differentiis divisis vel primis vel secundis vel tertiis *&c.* prout y fuerit æqualis vel ipsi x vel ejus quadrato vel ejus cubo *&c.*

Cas. 2. Iisdem positis, sit jam alius quilibet valor coefficientis termini illius altissimi: & erit y æqualis alicui potestati ipsius x in illum valorem ductæ. Itaque erunt α, β, γ, δ, ε æquales respective eidem potestati ipsarum a, b, c, d, e in eundem coefficientem ductæ adeoque coefficiens ille differentias primas & subsequentes omnes afficiet. Unde illæ ultimæ differentiæ, quæ in Casu priore erant singulæ unitati æquales, in hoc Casu erunt huic coefficienti singulæ æquales.

Cas. 3. Adsint jam quotcunque termini æquationis. Patet illas differentiarum partes primo æquales evadere atque evanescere, quæ ex inferioribus terminis oriuntur, ac deinde illas quæ ex superioribus, atque ultimo manere solas illas quæ ex altissimo proveniunt. Erunt itaque differentiæ illæ ultimæ, in hoc etiam Casu, æquales singulæ coefficienti termini altissimi. *Q. E. D.*

PRO-

PROPOSITIO III. PROBLEMA.

Iisdem positis, dari intelligantur a, b, c, d, &c. *tot minimum valores indefinitæ quantitatis* x *quot habere potest terminos propositus Æquationis ordo. Dentur præterea* α, β, γ, δ, &c. *totidem inde emergentes valores indefinitæ quantitatis* y; *&* *incognita adhuc Æquatione, propositum sit invenire* ζ *valorem ipsius* y, *ubi* z *ponitur pro valore ipsius* x.

PERAGANTUR omnia ad Normam propositionis superioris, donec tandem contigerit differentias aliquas divisas, puta τ', τ'', &c. æquales esse; tum cape $\lambda = z - a$, $\mu = z - b$, $\nu = z - c$, &c.

			z	ζ						
		λ			r	ϱ				
	μ		a	α			s	σ		
ν		λ'			r'	ϱ'			t	τ
	μ'		b	β			s'	σ'		
ν'		λ''			r''	ϱ''			t'	τ'
	μ''		c	γ			s''	σ''		
ν''		λ'''			r'''	ϱ'''			t''	τ''
	μ'''		d	δ			s'''	σ'''		
		λ''''			r''''	ϱ''''				
			e	ε						

eo usque dum tot fuerint λ, μ, ν, &c. quot sunt ϱ', σ', τ', &c. Et erit $\zeta = \alpha + \varrho'\lambda + \sigma'\lambda\mu + \tau'\lambda\mu\nu +$ &c. *Q. E. I.*

Cogita enim $r = \zeta - \alpha$, $\varrho = \frac{r}{\lambda}$; $s = \varrho - \varrho'$, $\sigma = \frac{s}{\mu}$; $t = \sigma - \sigma'$, $\tau = \frac{t}{\nu}$; & erunt (per Prop. II) τ, τ', τ'', simul omnes inter se æquales. Itaque $t = \tau\nu = \tau'\nu$, & $\sigma = \sigma' + t = \sigma' + \tau'\nu$, & $s = \sigma\mu = \sigma'\mu + \tau'\mu\nu$, & $\varrho = \varrho' + s = \varrho' + \sigma'\mu + \tau'\mu\nu$, & $r = \varrho\lambda = \varrho'\lambda + \sigma'\lambda\mu + \tau'\lambda\mu\nu$. Denique $\zeta = a + r = \alpha + \varrho'\lambda + \sigma'\lambda\mu + \tau'\lambda\mu\nu$. *Q. E. D.*

PROPOSITIO IV. PROBLEMA.

Iisdem positis datisque, proponatur Æquationem incognitam eruere, determinando singulorum terminorum coefficientes f, g, h, k, &c.

PONE $z = 0$, eritque ζ (per Problema superius reperienda) æqualis quantitati f, evanescente enim z sive x simul evanescunt termini $gx + hxx + kx^3 +$ &c. manetque f æqualis ipsi y sive ζ.

Subdu-

Subducatur jam cognita f ab utraque parte æquationis, reſiduaque dividantur per x, & emerget $\frac{y-f}{x}=g+hx+kxx+$ &c. Pone $y'=\frac{y-f}{x}$ & erit æquatio jam depreſſior $y'=g+hx+kxx+$ &c. Similiter in tabella pro $\alpha, \beta, \gamma, \delta, \varepsilon$, &c. ſcribe $\alpha', \beta', \gamma', \delta', \varepsilon'$, &c. ita ut ſit $\alpha'=\frac{\alpha-f}{a}$, $\beta'=\frac{\beta-f}{b}$, &c. Pone denuo $z=0$, eritque ζ (per Problema ſuperius reperienda) æqualis quantitati g, propter rationes ſupra allatas. Iterata operatione deprimes perpetuo æquationem, omneſque tandem determinabis coefficientes. *Q. E. F.*

Idem aliter.

Conſtat, ex Propoſitione II, differentias ultimas diviſas æquales eſſe coefficienti termini altiſſimi k. Innoteſcet itaque coefficiens ille k capiendo differentias per Methodum ſupra deſcriptam. Subducatur jam terminus altiſſimus kx^3 ab utraque parte Æquationis & habebis $y-kx^3=f+gx+hxx$. Pone $y'=y-kx^3$ & erit æquatio jam depreſſior $y'=f+gx+hxx$. Similiter in tabella pro $\alpha, \beta, \gamma, \delta$, &c. ſcribe $\alpha', \beta', \gamma', \delta'$, &c. ita ut ſit $\alpha'=\alpha-ka^3$, $\beta'=\beta-kb^3$, &c. Cape iterum differentias & habebis coefficientem h termini altiſſimi hujus novæ Æquationis. Iterata operatione deprimes perpetuo æquationem, omneſque tandem determinabis coefficientes. *Q. E F.*

PROPOSITIO V. THEOREMA

Quæcunque ſit quantitatum quotcunque datarum a, b, c, d, &c. *quarum nullæ ſint æquales inter ſe, ad quantitates totidem datas quaſcunque* $\alpha, \beta, \gamma, \delta$, &c. *ſingularum ad ſingulas relatio reſpectiva; ſi e prioribus quantitatibus unaquæque indefinite vocetur* x, *atque illi correſpondens e poſterioribus indefinite vocetur* y, *poterit ſemper ea relatio exprimi per Æquationem hujus quæ ſequitur formæ* $y=f+gx+hxx+kx^3+$ &c. *tot habentem terminos ex altera parte æquales ipſi* y *quot fuerint datæ quantitates* a, b, c, d, &c. *vel* $\alpha, \beta, \gamma, \delta$, &c.

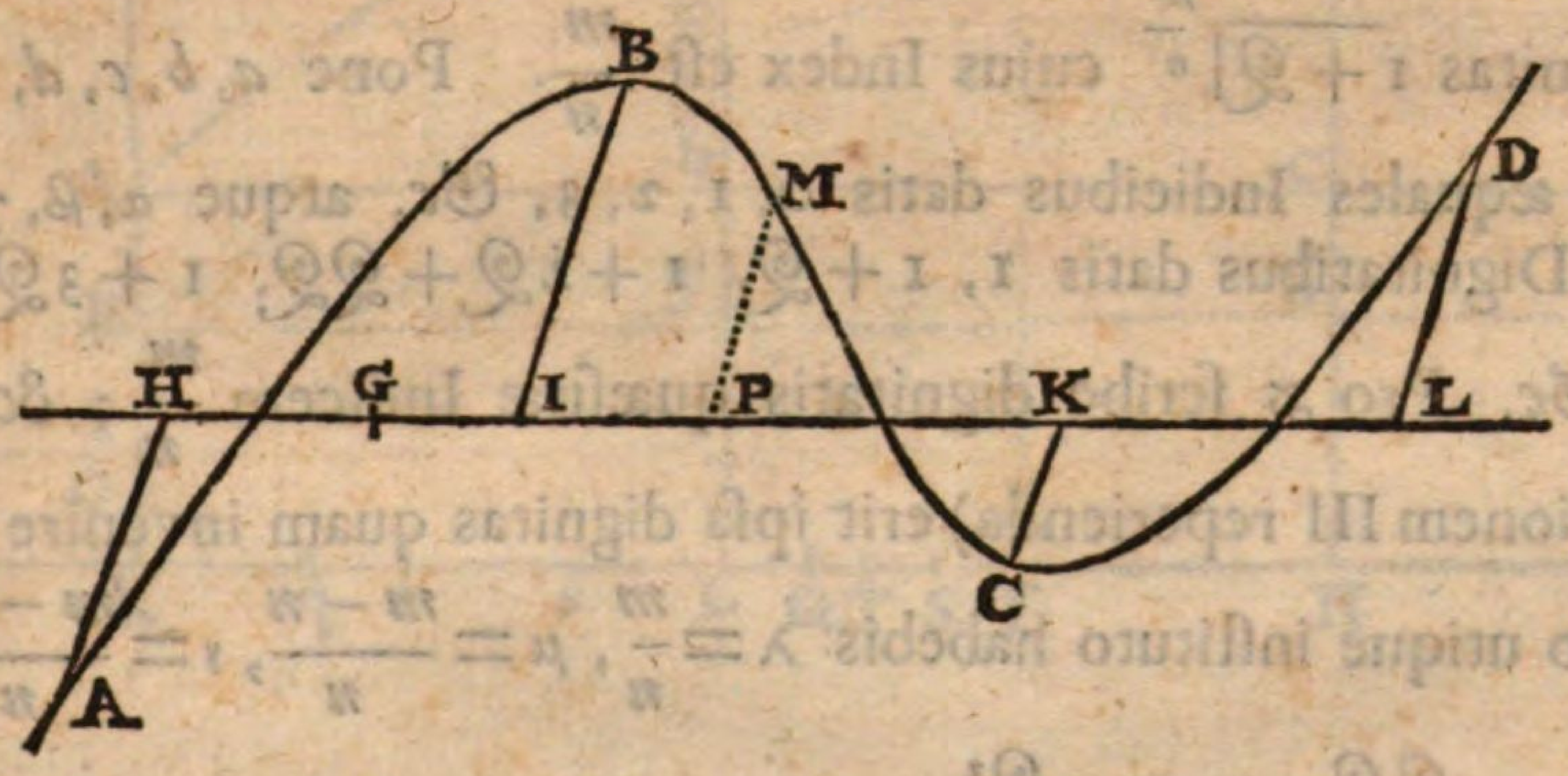
DATÆ ſint tres quantitates a, b, c; itemque tres aliæ α, β, γ: Dico eos valores coefficientium f, g, h inveniri poſſe qui Propoſitioni ſatisfacient. Dantur enim ad horum trium inventionem

nem tres diversæ æquationes $\alpha = f + ga + haa$, $\beta = f + gb + hbb$, $\gamma = f + gc + hcc$. Itaque per notas Analyseos leges inveniri possunt f, g, h. Simili Methodo constabit Propositio si datus fuerit alius quilibet numerus quantitatum. *Q. E. D.*

PROPOSITIO VI. PROBLEMA.

Invenire lineam Curvam generis Parabolici, quæ per data quotcunque puncta transibit.

SUNTO puncta illa data A, B, C, D, & ab iisdem ad rectam quamvis lineam HL ducantur rectæ AH, BI, CK, DL, in ea quavis data inclinatione ut harum unica tantum in idem illius punctum incidat. Puta rectas AH, BI, CK, DL ordinatas esse,



quæ insistant abscissis GH, GI, GK, GL a dato puncto G dimensis. Illarum quævis indefinite vocetur y, harum quævis x. Liquet ex præcedente propositione relationem inter ordinatam & abscissam, quæcunque illa fuerit, exprimi semper posse per hanc æquationem generalem $y = f + gx + hxx + kx^3 + \&c.$ in qua literæ f, g, h, k, *&c.* quantitates invariabiles denotant nondum tamen determinatas. Unde, cum illa Æquatio sit ad Curvam generis Parabolici, constat hujusmodi Curvam semper transire posse per data puncta.

Solutio prior.

Jam si quæratur longitudo cujusvis ordinatæ novæ PM, datæ abscissæ GP insistentis: scribe $\alpha, \beta, \gamma, \delta$ respective pro $-HA$, $+IB$, $-KC$, $+LD$; itemque a, b, c, d pro $-GH$, $+GI$, $+GK$, $+GL$, atque z pro $+GP$: eritque ζ, per Propositionem tertiam reperienda, æqualis ordinatæ quæsitæ PM. *Q. E F.*

Solutio posterior.

Quod si Æquatio ad Curvam desideretur; illa quoque, iisdem positis, per Propositionem IV facile determinabitur. *Q. E. F.*

Corollarium. Hinc datis quibuslibet quantitatibus totidemque aliis a quibus pendent, poterit desiderata quævis interpolari congruenter: exponendo nimirum datas illas quantitates per ordinatas *AH*, *BI*, *CK*, *DL*, &c. alias vero illas, a quibus reliquæ pendent, per abscissas *GH*, *GI*, *GK*, *GL* & quantitatem interserendam per Ordinatam *PM*.

Hujus Methodi exemplum subjungere non pigebit. Quantitatis $1+Q$ dentur Dignitates $1, 1+Q, 1+2Q+QQ, 1+3Q+3QQ+Q^3$, &c. ut & earum Indices $0, 1, 2, 3$, &c. & invenienda sit ejusdem quantitatis Dignitas $\overline{1+Q}|^{\frac{m}{n}}$ cujus Index est $\frac{m}{n}$. Pone a, b, c, d, &c. respective æquales Indicibus datis $0, 1, 2, 3$, &c. atque $\alpha, \beta, \gamma, \delta$, &c. æquales Dignitatibus datis $1, 1+Q; 1+2Q+QQ; 1+3Q+3QQ+Q^3$, &c. Pro z scribe dignitatis quæsitæ Indicem $\frac{m}{n}$; & ζ (per Propositionem III reperienda) erit ipsa dignitas quam invenire oportet. Computo utique instituto habebis $\lambda=\frac{m}{n}$, $\mu=\frac{m-n}{n}$, $\nu=\frac{m-2n}{n}$, &c. $\varsigma'=Q$; $\sigma'=\frac{QQ}{1\times 2}$, $\tau'=\frac{Q^3}{1\times 2\times 3}$, &c. Unde colligetur $\overline{1+Q}|^{\frac{m}{n}}=\zeta=$ $\alpha+\varsigma'\lambda+\sigma'\lambda\mu+\tau'\lambda\mu\nu+$ &c. $=1+\frac{m}{n}Q+\frac{m}{n}\times\frac{m-n}{2n}QQ$ $+\frac{m}{n}\times\frac{m-n}{2n}\times\frac{m-2n}{3n}Q^3+$ &c.

PROPOSITIO VII. PROBLEMA.

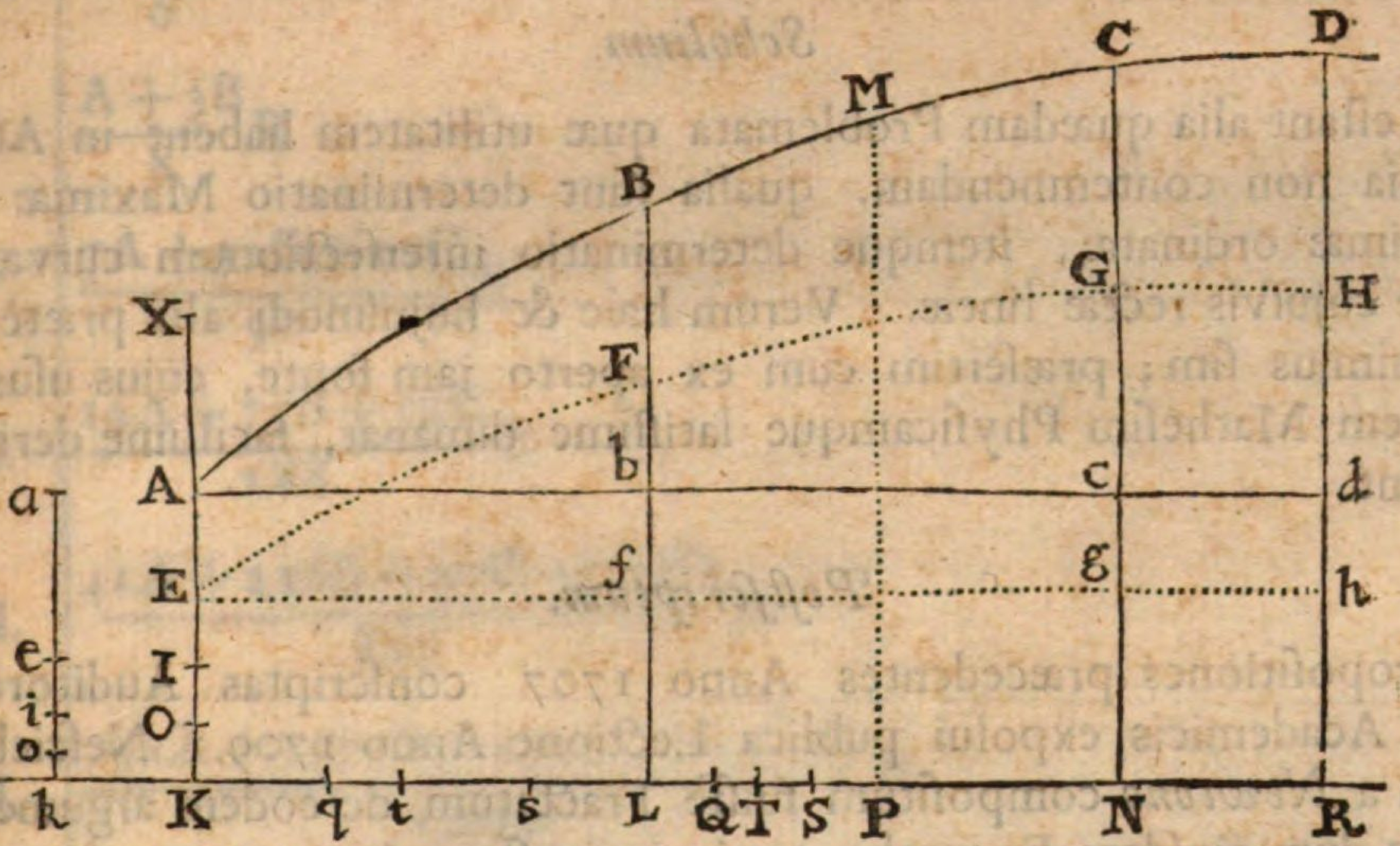
Curvarum omnium Areas quam proxime determinare.

Per aliquot Curvæ propositæ puncta transire intelligatur Curva Parabolica per Problema novissimum definienda; hujus area (quæ semper dabitur per Methodos notissimas) erit eadem quam proxime atque area Curvæ propositæ.

Oblata

Oblata sit Curva *ABCD*. Ad rectam *KR* referantur quot libuerit ordinatæ *AK, BL, CN, DR*, &c. & ex hisce earumque respectivis abscissis, a puncto *K* dimensis, determinetur Æquatio naturam Curvæ quam proxime exhibens $y=f+gx+hxx+kx^3+$ &c. Eritque abscissæ *KP* sive x atque ordinatæ *PM* sive y adjacens Area $AKPM=fx+\frac{1}{2}gxx+\frac{1}{3}hx^3+\frac{1}{4}kx^4+$ &c.

Quod si Geometrica potius quam Arithmetica Quadratura desideretur: erit ea quæ sequitur satis concinna.



Age rectam *Ad* parallelam rectæ *KR*, intersecantem ordinatas *LB*, *NC*, *RD* (si opus fuerit productas) in *b*, *c*, *d*. Cape *LF*, *NG*, *RH* ad *Bb*, *Cc*, *Dd* respective, ut est longitudo quævis *KS* (pro lubitu sumpto puncto *S*) ad *KL*, *KN*, *KR*, & ponantur *LF*, *NG*, *RH* ad easdem partes respective a punctis *L*, *N*, *R* ad quas jacent a punctis *b*, *c*, *d* lineæ *Bb*, *Cc*, *Dd*. Transeat deinde linea per puncta *F*, *G*, *H* secetque *AK* in *E*, actaque denuo *Eh* parallela rectæ *KR* iterataque operatione ducatur alia linea quæ secet *AK* in *I*, atque alia quæ secet eandem in *O*, donec ultima linearum evadat & recta & parallela rectæ *KR*. Jam factis *KP*, *KS*, *KT*, *KQ* continue proportionalibus, atque *Ks* subdupla abscissæ *KP*, *Kt* subtripla ejusdem, *Kq* subquadrupla; Cape rectam *ka* æqualem rectæ *KA*, itemque *ke* ad *KE* ut est *Ks* ad *KS*, atque *ki* ad *KI* ut est *Kt* ad *KT*, denique *ko* ad *KO* ut est *Kq* ad *KQ*, pariterque deinceps; positisque singulis *ka*, *ke*, *ki*, *ko* ad easdem partes a puncto

cto k ad quas positæ sunt respective KA, KE, KI, KO a puncto K, si capiatur KX æqualis summæ omnium consistentium ad easdem partes a puncto k, vel differentiæ consistentium ad contrarias: erit rectangulum $XK \times KP$ æquale areæ $AKPM$ quamproxime. *Q. E. F.*

Si puncta S & P coire velis; constructio paullo simplicior evadet. Etenim recta ka æqualis erit, uti prius, rectæ KA, recta vero ke subdupla erit rectæ KE, recta ki subtripla rectæ KI, recta ko subquadrupla rectæ KO, atque ita deinceps.

Scholium.

Restant alia quædam Problemata quæ utilitatem habent in Astronomia non contemnendam, qualia sunt determinatio Maximæ vel Minimæ ordinatæ, itemque determinatio intersectionum curvæ & datæ cujusvis rectæ lineæ. Verum hæc & hujusmodi alia prætereo, ne nimius sim; præsertim cum ex aperto jam fonte, cujus usus in omnem Mathesim Physicamque latissime dimanat, facillime derivari possint.

Postscriptum.

Propositiones præcedentes Anno 1707 conscriptas Auditoribus meis Academicis exposui publica Lectione Anno 1709. Nesciebam enim a *Newtono* compositum fuisse Tractatum de eodem argumento usque dum ejusdem Exemplar typis impressum Anno 1711 dono accepissem a doctissimo Editore Domino *Jones*, cui etiam mearum Apographum ejus humanitate permotus statim transmisi. Perlectis autem Viri celeberrimi Propositionibus intellexi me in plerisque diversa incessisse via, id quod patebit conferenti Propositionem illius secundam & mearum quartam. Pulcherrima vero in primis & utilissima visa est Regula qua ex datis quatuor Ordinatis ad æqualia intervalla sitis metitur aream inter primam & ultimam. Hujusmodi Regularum Seriem aliquantulum productam hic subjungo. Harum prima exhibet aream interjectam extremis trium æquidistantium Ordinatarum, secunda dat aream inter quatuor æquidistantium extremas, & *Newtono* debetur, per tertiam invenitur area ex quinque similiter datis, &c. In hisce vero omnibus A ponitur pro summa Ordinatarum extimarum, hoc est, pro summa primæ & ultimæ; B pro summa proximarum extimis, hoc est, pro summa secundæ & penultimæ; C pro summa sequentium, hoc est, pro summa tertiæ & antepenultimæ; atque ita deinceps continuatur ordo literarum D, E, F, &c. donec

vel

vel ad mediam ordinatam perventum fuerit, ubi numerus earum eſt impar, vel ad ſummam binarum mediarum ubi numerus eſt par. Intervallum denique inter extimas Ordinatas, hoc eſt, areæ quadrandæ Baſis deſignatur per literam R.

Numerus Ordinatarum.	Areæ
III.	$\frac{A+4B}{6}R$
IV.	$\frac{A+3B}{8}R$
V.	$\frac{7A+32B+12C}{90}R$
VI.	$\frac{19A+75B+50C}{288}R$
VII.	$\frac{41A+216B+27C+272D}{840}R$
VIII.	$\frac{751A+3577B+1323C+2989D}{17280}R$
IX.	$\frac{989A+5888B-928C+10496D-4540E}{28350}R$
X.	$\frac{2857A+15741B+1080C+19344D+5778E}{89600}R$
XI.	$\frac{16067A+106300B-48525C+272400D-260550E+427368F}{598752}R$
&c.	

CANONOTECHNIA

SIVE

CONSTRUCTIO TABULARUM

PER

DIFFERENTIAS.

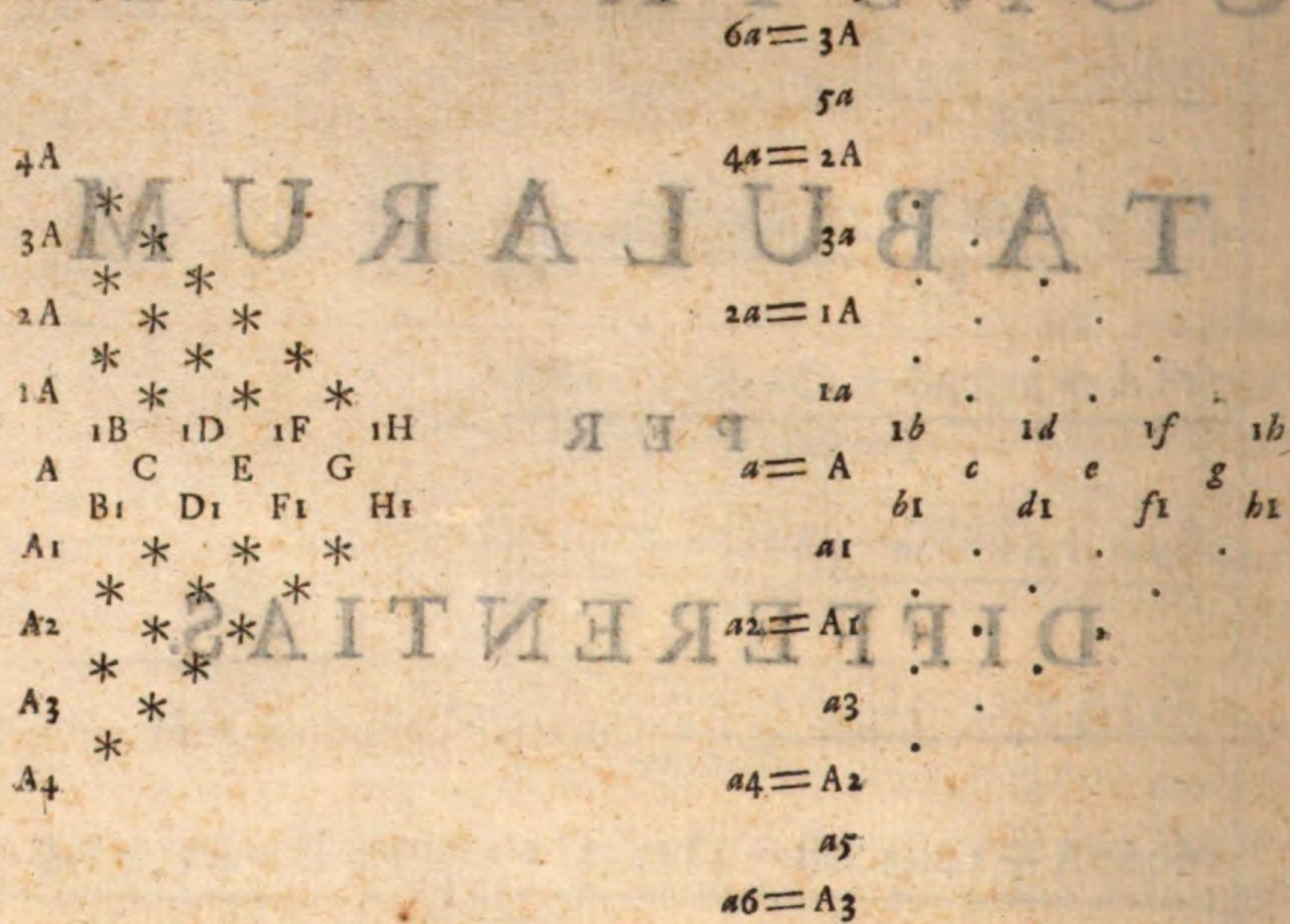
AUCTORE

ROGERO COTES.

PROPOSITIO I.

Datis Seriei cujuscunque terminis aliquot æquidistantibus, quorum unus est inter reliquos medius; propositum sit eorundem intervalla bifariam secare, terminum unum inter datorum binos quosque proximos interserendo.

SUNTO termini illi dati & æquidistantes &c, 4A, 3A, 2A, 1A, A, A1, A2, A3, A4, &c. quorum unus A est inter reliquos medius: ex singulis æqualibus horum terminorum intervallis facienda sunt duo æqualia intervalla, inveniendo terminos &c, *5a*, *3a*, *1a*, *a1*, *a3*, *a5*, &c. quos interserere oportet ad complendam Seriem &c, 3A, *5a*, 2A, *3a*, 1A, *1a*, A, *a1*, A1, *a3*, A2, *a5*, A3, &c.



Terminorum datorum capiantur differentiæ primæ, ponendo 1B = 1A − A, 2B = 2A − 1A, 3B = 3A − 2A, &c. B1 = A − A1, B2 = A1 − A2, B3 = A2 − A3, &c. secundæ, ponendo 1C = 2B − 1B, C = 1B − B1, C1 = B1 − B2, &c. tertiæ, ponendo 1D = 1C − C, D1 = C − C1, &c. & sic deinceps continuanda erit operatio, signis + & − probe observatis, donec ad ultimas differentias perventum fuerit.

Deinde excerpantur differentiæ mediæ

1B	1D	1F	1H	
A	C	E	G	&c.
B1	D1	F1	H1	

& scribendo B pro semisumma ipsarum 1B & B1, D pro semisumma ipsarum

rum 1D & D1, F pro ſemiſumma ipſarum 1F & F1, H pro ſemiſumma ipſarum 1H & H1, &c. habebitur differentiarum primarius ordo Medialis A, B, C, D, E, F, G, H, &c. Notetur autem hujuſmodi differentias ut & ipſas terminorum Series ex quibus oriuntur *Mediales* appellari vel *Bimediales* prout Series illæ vel unicum terminum medium vel binos medios habeant; atque ille quidem unicus Medius primum obtinet locum in differentiarum ordine Mediali, ſemiſumma vero binorum mediorum in ordine Bimediali.

Ordinis primarii differentiæ diviſæ per numeros in ratione dupli creſcentes dant alium differentiarum ordinem qui *Rotundus* appellari poteſt, & ſic denotari $\textcircled{0}, \textcircled{1}, \textcircled{2}, \textcircled{3}, \textcircled{4}, \textcircled{5}$, &c. ſumendo nempe $\textcircled{0} = A$, $\textcircled{1} = \frac{1}{2}B$, $\textcircled{2} = \frac{1}{4}C$, $\textcircled{3} = \frac{1}{8}D$, $\textcircled{4} = \frac{1}{16}E$, $\textcircled{5} = \frac{1}{32}F$, &c.

Ex ordine rotundo colligendus eſt differentiarum ordo tertius qui non incommode *Quadratus* appellari poteſt & ſic denotari $\boxed{0}, \boxed{1}, \boxed{2}, \boxed{3}, \boxed{4}, \boxed{5}$, &c. Hujus autem ordinis differentiæ inveniuntur per hæcce Theoremata *priora*, quorum continuatio petenda eſt ex Propoſitione ſeptima.

&c.

$\boxed{10} = \textcircled{10} -$ &c.

$\boxed{8} = \textcircled{8} - \boxed{10} -$ &c.

$\boxed{6} = \textcircled{6} - \frac{3}{4}\boxed{8} - \frac{1}{16}\boxed{10} -$ &c.

$\boxed{4} = \textcircled{4} - \frac{1}{2}\boxed{6} - \frac{1}{16}\boxed{8}$ *

$\boxed{2} = \textcircled{2} - \frac{1}{4}\boxed{4}$ *

$\boxed{0} = \textcircled{0}$ *

&c.

$\boxed{9} = \textcircled{9} -$ &c.

$\boxed{7} = \textcircled{7} - \frac{5}{4}\boxed{9} -$ &c.

$\boxed{5} = \textcircled{5} - \boxed{7} - \frac{5}{16}\boxed{9} -$ &c.

$\boxed{3} = \textcircled{3} - \frac{3}{4}\boxed{5} - \frac{1}{8}\boxed{7}$ *

$\boxed{1} = \textcircled{1} - \frac{1}{2}\boxed{3}$ *

Denique inventis differentiis quadratis, dabuntur terminorum interſerendorum ſemiſummæ & ſemidifferentiæ atque ipſi adeo termini interſerendi per ea quæ ſequuntur Theoremata *poſteriora*, quorum continuatio facillima eſt & ex Propoſitione decima petenda.

I. ſemiſum. $\frac{1a + a1}{2} = \boxed{0} + \frac{1}{2}\boxed{2}$ *
ſemidiff. $\frac{1a - a1}{2} = \boxed{1}$ *

III. ſemiſum. $\frac{3a + a3}{2} = \boxed{0} + 4\frac{1}{2}\boxed{2} + 3\boxed{4} + \frac{1}{2}\boxed{6}$ *
ſemidiff. $\frac{3a - a3}{2} = 3\boxed{1} + 4\boxed{3} + \boxed{5}$ *

 V ſemi-

V. $\left\{\begin{array}{l} \text{ſemiſum. } \frac{5a + a5}{2} = \boxed{0} + 12\tfrac{1}{2}\boxed{2} + 25\boxed{4} + 17\tfrac{1}{2}\boxed{6} + 5\boxed{8} + \tfrac{1}{2}\boxed{10} * \\ \text{ſemidiff. } \frac{5a - a5}{2} = 5\boxed{1} + 20\boxed{3} + 21\boxed{5} + 8\boxed{7} + \boxed{9} * \end{array}\right.$

VII. $\left\{\begin{array}{l} \text{ſemiſum. } \frac{7a + a7}{2} = \boxed{0} + 24\tfrac{1}{2}\boxed{2} + 98\boxed{4} + 147\boxed{6} + 105\boxed{8} + 38\tfrac{1}{2}\boxed{10} + \&c. \\ \text{ſemidiff. } \frac{7a - a7}{2} = 7\boxed{1} + 56\boxed{3} + 126\boxed{5} + 120\boxed{7} + 55\boxed{9} + \&c. \end{array}\right.$

IX. $\left\{\begin{array}{l} \text{ſemiſum. } \frac{9a + a9}{2} = \boxed{0} + 40\tfrac{1}{2}\boxed{2} + 270\boxed{4} + 693\boxed{6} + 891\boxed{8} + 643\tfrac{1}{2}\boxed{10} + \&c. \\ \text{ſemidiff. } \frac{9a - a9}{2} = 9\boxed{1} + 120\boxed{3} + 462\boxed{5} + 792\boxed{7} + 715\boxed{9} + \&c. \end{array}\right.$

&c.

Poſtquam Series completa fuerit, ſi capiantur denuo terminorum differentiæ, ponendo $1b = 1a - A$, $b1 = A - a1$, $c = 1b - b1$, &c. & ſcribantur b, d, f, h, &c. pro ſemiſummis ipſarum $1b$ & $b1$, $1d$ & $d1$, $1f$ & $f1$, $1h$ & $h1$, &c. habebitur differentiarum ordo Medialis a, b, c, d, e, f, g, h, &c. Hic vero prorſus idem erit atque ordo ille quadratus $\boxed{0}$, $\boxed{1}$, $\boxed{2}$, $\boxed{3}$, $\boxed{4}$, $\boxed{5}$, $\boxed{6}$, $\boxed{7}$, &c. per quem terminos interſerendos invenimus. Effectio itaque Problematis vocetur *Biſectio per quadratum ordinem Medialem*, vel etiam *Biſectio Medialis*.

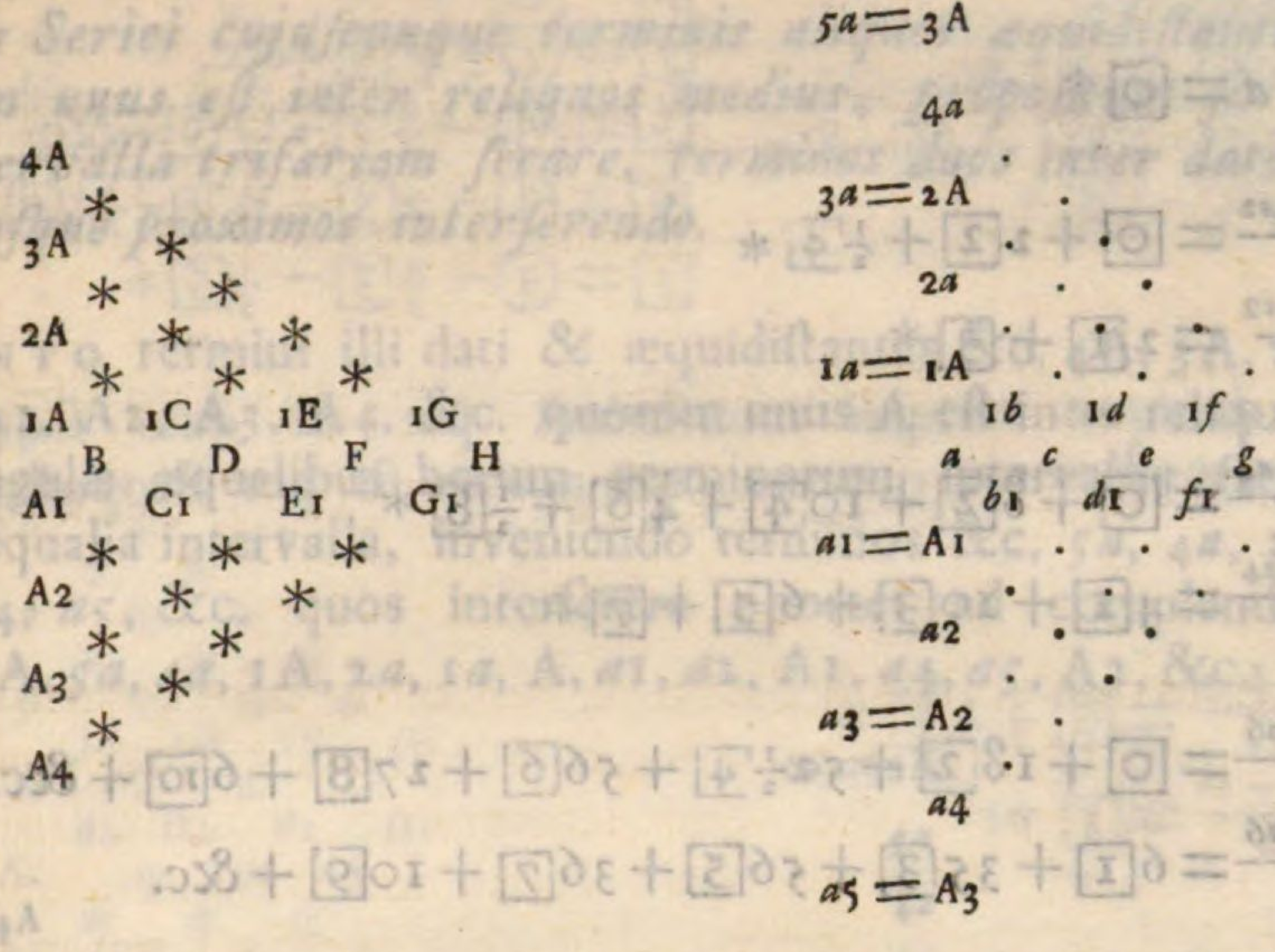
PROPOSITIO II.

Datis Seriei cujuſcunque terminis aliquot æquidiſtantibus, quorum bini ſunt inter reliquos medii; propoſitum ſit eorundem intervalla bifariam ſecare, terminum unum inter datorum binos quoſque proximos interſerendo.

SUNTO termini illi dati & æquidiſtantes &c, 4A, 3A, 2A, 1A, A1, A2, A3, A4, &c. quorum bini 1A & A1 ſunt inter reliquos medii: ex ſingulis æqualibus horum terminorum intervallis facienda ſunt duo æqualia intervalla, inveniendo terminos &c, $4a$, $2a$, a, $a2$, $a4$, &c. quos interſerere oportet ad complendam Seriem &c, 3A, $4a$, 2A, $2a$, 1A, a, A1, $a2$, A2, $a4$, A3, &c.

Terminorum datorum capiantur differentiæ primæ, ponendo $B = 1A - A1$, $1B = 2A - 1A$, $2B = 3A - 2A$, &c. $B1 = A1 - A2$, $B2 = A2 - A3$, &c. ſecundæ, ponendo $2C = 2B - 1B$, $1C = 1B - B$,

$C1 = B - B1$, $C2 = B1 - B2$, &c. tertiæ, ponendo $1D = 2C - 1C$, $D = 1C - C1$, $D1 = C1 - C2$, &c. & sic deinceps continuanda erit



operatio, signis + & − probe observatis, donec ad ultimas differentias perventum fuerit. Deinde excerpantur differentiæ mediæ

$$\begin{array}{cccc} 1A & 1C & 1E & 1G \\ B & D & F & H, \&c. \\ A1 & C1 & E1 & G1 \end{array}$$

& scribendo A, C, E, G, &c. pro semisummis ipsarum 1A & A1, 1C & C1, 1E & E1, 1G & G1, &c. habebitur differentiarum primarius ordo Bimedialis A, B, C, D, E, F, G, H, &c.

Differentiæ rotundæ inveniuntur ut in Propositione superiore, sumendo nempe $\textcircled{0} = A$, $\textcircled{1} = \frac{1}{2}B$, $\textcircled{2} = \frac{1}{4}C$, $\textcircled{3} = \frac{1}{8}D$, $\textcircled{4} = \frac{1}{16}E$, $\textcircled{5} = \frac{1}{32}F$, &c.

Differentiæ quadratæ ex rotundis colliguntur per hæcce Theoremata *priora*, quorum continuatio fit per Propositionem nonam.

&c.

$\boxed{10} = \textcircled{10} -$ &c.

$\boxed{8} = \textcircled{8} - \frac{1}{2}\boxed{10} -$ &c.

$\boxed{6} = \textcircled{6} - \frac{5}{4}\boxed{8} - \frac{9}{16}\boxed{10} -$ &c.

$\boxed{4} = \textcircled{4} - \boxed{6} - \frac{5}{16}\boxed{8} - \frac{1}{32}\boxed{10}$ *

$\boxed{2} = \textcircled{2} - \frac{3}{4}\boxed{4} - \frac{1}{8}\boxed{6}$ *

$\boxed{0} = \textcircled{0} - \frac{1}{2}\boxed{2}$ *

&c.

$\boxed{9} = \textcircled{9} -$ &c.

$\boxed{7} = \textcircled{7} - \frac{3}{4}\boxed{9} -$ &c.

$\boxed{5} = \textcircled{5} - \frac{1}{2}\boxed{7} - \frac{1}{16}\boxed{9}$ *

$\boxed{3} = \textcircled{3} - \frac{1}{4}\boxed{5}$ *

$\boxed{1} = \textcircled{1}$ *

Dantur

Dantur denique termini interſerendi per ea quæ ſequuntur Theoremata *poſteriora*, quorum continuatio fit per Propoſitionem decimam.

Terminus medius $a = \boxed{0}$ *

II. $\begin{cases} \text{ſemiſum. } \frac{2a + a2}{2} = \boxed{0} + 2\boxed{2} + \frac{1}{2}\boxed{4} \; * \\ \text{ſemidiff. } \frac{2a - a2}{2} = 2\boxed{1} + \boxed{3} \; * \end{cases}$

IV. $\begin{cases} \text{ſemiſum. } \frac{4a + a4}{2} = \boxed{0} + 8\boxed{2} + 10\boxed{4} + 4\boxed{6} + \frac{1}{2}\boxed{8} \; * \\ \text{ſemidiff. } \frac{4a - a4}{2} = 4\boxed{1} + 10\boxed{3} + 6\boxed{5} + \boxed{7} \; * \end{cases}$

VI. $\begin{cases} \text{ſemiſum. } \frac{6a + a6}{2} = \boxed{0} + 18\boxed{2} + 52\tfrac{1}{2}\boxed{4} + 56\boxed{6} + 27\boxed{8} + 6\boxed{10} + \&c. \\ \text{ſemidiff. } \frac{6a - a6}{2} = 6\boxed{1} + 35\boxed{3} + 56\boxed{5} + 36\boxed{7} + 10\boxed{9} + \&c. \end{cases}$

VIII. $\begin{cases} \text{ſemiſum. } \frac{8a + a8}{2} = \boxed{0} + 32\boxed{2} + 168\boxed{4} + 336\boxed{6} + 330\boxed{8} + 176\boxed{10} + \&c. \\ \text{ſemidiff. } \frac{8a - a8}{2} = 8\boxed{1} + 84\boxed{3} + 252\boxed{5} + 330\boxed{7} + 220\boxed{9} + \&c. \end{cases}$

X. $\begin{cases} \text{ſemiſum. } \frac{10a + a10}{2} = \boxed{0} + 50\boxed{2} + 412\tfrac{1}{2}\boxed{4} + 1320\boxed{6} + 2145\boxed{8} + 2002\boxed{10} + \&c. \\ \text{ſemidiff. } \frac{10a - a10}{2} = 10\boxed{1} + 165\boxed{3} + 792\boxed{5} + 1716\boxed{7} + 2002\boxed{9} + \&c. \end{cases}$

&c.

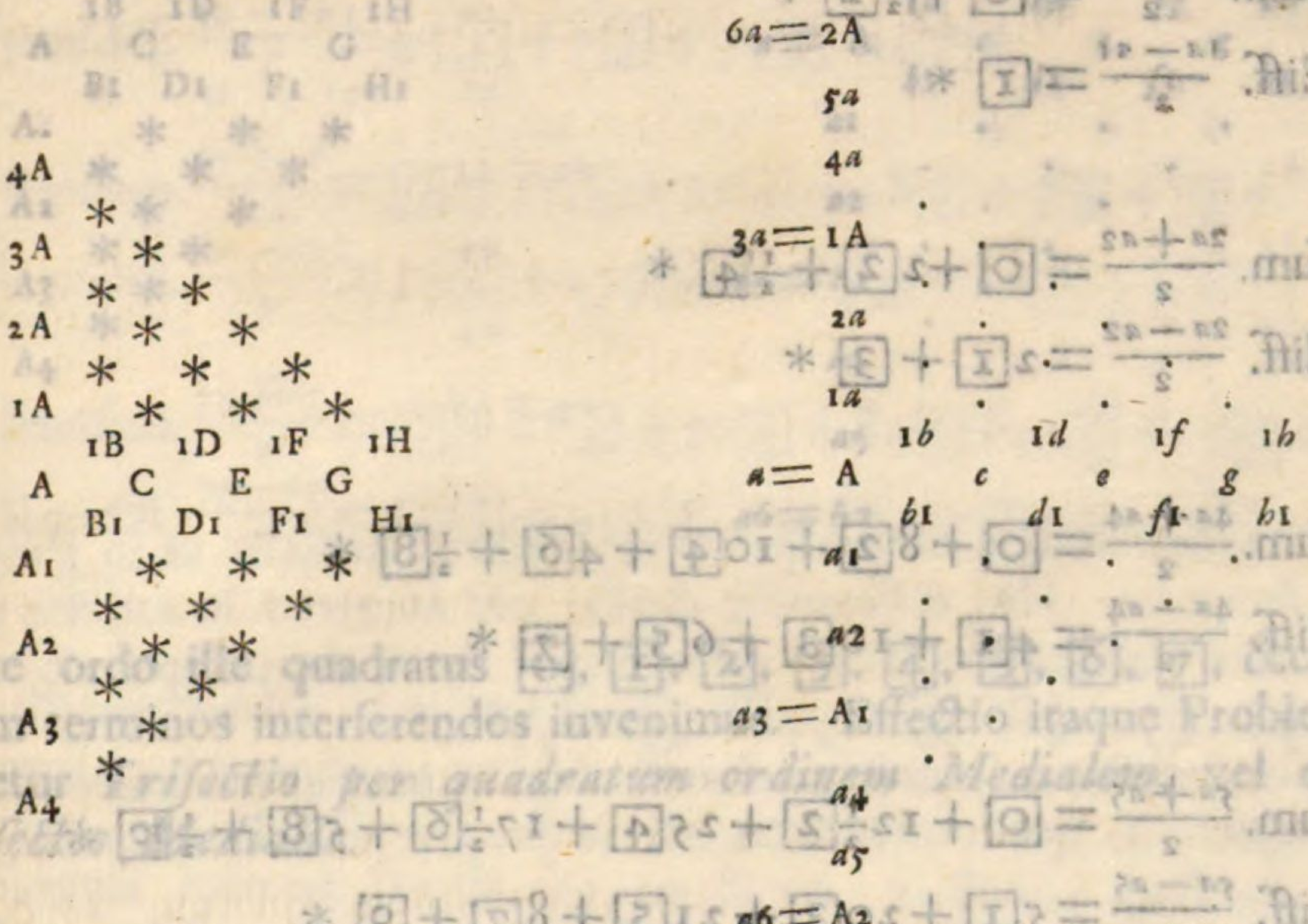
Poſtquam Series completa fuerit, ſi capiantur denuo terminorum differentiæ, ponendo $1b = 1A - a$, $b1 = a - A1$, $c = 1b - b1$, &c. & ſcribantur b, d, f, h, &c. pro ſemiſummis ipſarum $1b$ & $b1$, $1d$ & $d1$, $1f$ & $f1$, $1h$ & $h1$, &c. habebitur differentiarum ordo Medialis contrarius a, b, c, d, e, f, g, h, &c. nam primarius ordo erat Bimedialis. Hic vero Medialis ordo contrarius prorſus idem erit atque ordo ille quadratus $\boxed{0}$, $\boxed{1}$, $\boxed{2}$, $\boxed{3}$, $\boxed{4}$, $\boxed{5}$, $\boxed{6}$, $\boxed{7}$, &c. per quem terminos interſerendos invenimus. Effectio itaque Problematis vocetur *Biſectio per quadratum ordinem Medialem contrarium*, vel etiam, *Biſectio Medialis contraria*.

PRO-

PROPOSITIO III.

Datis Seriei cujuſcunque terminis aliquot æquidiſtantibus, quorum unus eſt inter reliquos medius; propoſitum ſit eorundem intervalla trifariam ſecare, terminos duos inter datorum binos quoſque proximos interſerendo.

SUNTO termini illi dati & æquidiſtantes &c, 4A, 3A, 2A, 1A, A, A1, A2, A3, A4, &c. quorum unus A eſt inter reliquos medius: ex ſingulis æqualibus horum terminorum intervallis facienda ſunt tria æqualia intervalla, inveniendo terminos &c, *5a*, *4a*, *2a*, *1a*, *a1*, *a2*, *a4*, *a5*, &c. quos interſerere oportet ad complendam Seriem &c, 2A, *5a*, *4a*, 1A, *2a*, *1a*, A, *a1*, *a2*, A1, *a4*, *a5*, A2, &c.



Inveniatur differentiarum primarius ordo Medialis ut in Propoſitione prima. Hæ differentiæ diviſæ per numeros in ratione tripli creſcentes dabunt ordinem rotundum, ſumendo nempe ⓪ = A, ① = $\frac{1}{3}$B, ② = $\frac{1}{9}$C, ③ = $\frac{1}{27}$D, ④ = $\frac{1}{81}$E, &c.

Ex ordine rotundo colligendus eſt ordo quadratus per hæcce Theoremata *priora*, quorum continuatio fit per Propoſitionem ſeptimam.

&c.

$\boxed{10} = \textcircled{10} - \text{\&c.}$

$\boxed{8} = \textcircled{8} - \frac{8}{3}\boxed{10} - \text{\&c.}$

$\boxed{6} = \textcircled{6} - 2\boxed{8} - \frac{5}{3}\boxed{10} - \text{\&c.}$

$\boxed{4} = \textcircled{4} - \frac{4}{3}\boxed{6} - \frac{2}{3}\boxed{8} - \frac{4}{27}\boxed{10} - \text{\&c.}$

$\boxed{2} = \textcircled{2} - \frac{2}{3}\boxed{4} - \frac{1}{9}\boxed{6}\ *$

$\boxed{0} = \textcircled{0}\ *$

&c.

$\boxed{9} = \textcircled{9} - \text{\&c.}$

$\boxed{7} = \textcircled{7} - \frac{10}{3}\boxed{9} - \text{\&c.}$

$\boxed{5} = \textcircled{5} - \frac{8}{3}\boxed{7} - \frac{25}{9}\boxed{9} - \text{\&c.}$

$\boxed{3} = \textcircled{3} - 2\boxed{5} - \frac{4}{3}\boxed{7} - \frac{10}{27}\boxed{9} - \text{\&c.}$

$\boxed{1} = \textcircled{1} - \frac{4}{3}\boxed{3} - \frac{1}{3}\boxed{5}\ *$

Dantur denique termini interſerendi per ea quæ ſequuntur Theoremata *poſteriora*, quorum continuatio fit per Propoſitionem decimam.

I. ſemiſum. $\frac{1a + a1}{2} = \boxed{0} + \frac{1}{2}\boxed{2}\ *$
ſemidiff. $\frac{1a - a1}{2} = \boxed{1}\ *$

II. ſemiſum. $\frac{2a + a2}{2} = \boxed{0} + 2\boxed{2} + \frac{1}{2}\boxed{4}\ *$
ſemidiff. $\frac{2a - a2}{2} = 2\boxed{1} + \boxed{3}\ *$

IV. ſemiſum. $\frac{4a + a4}{2} = \boxed{0} + 8\boxed{2} + 10\boxed{4} + 4\boxed{6} + \frac{1}{2}\boxed{8}\ *$
ſemidiff. $\frac{4a - a4}{2} = 4\boxed{1} + 10\boxed{3} + 6\boxed{5} + \boxed{7}\ *$

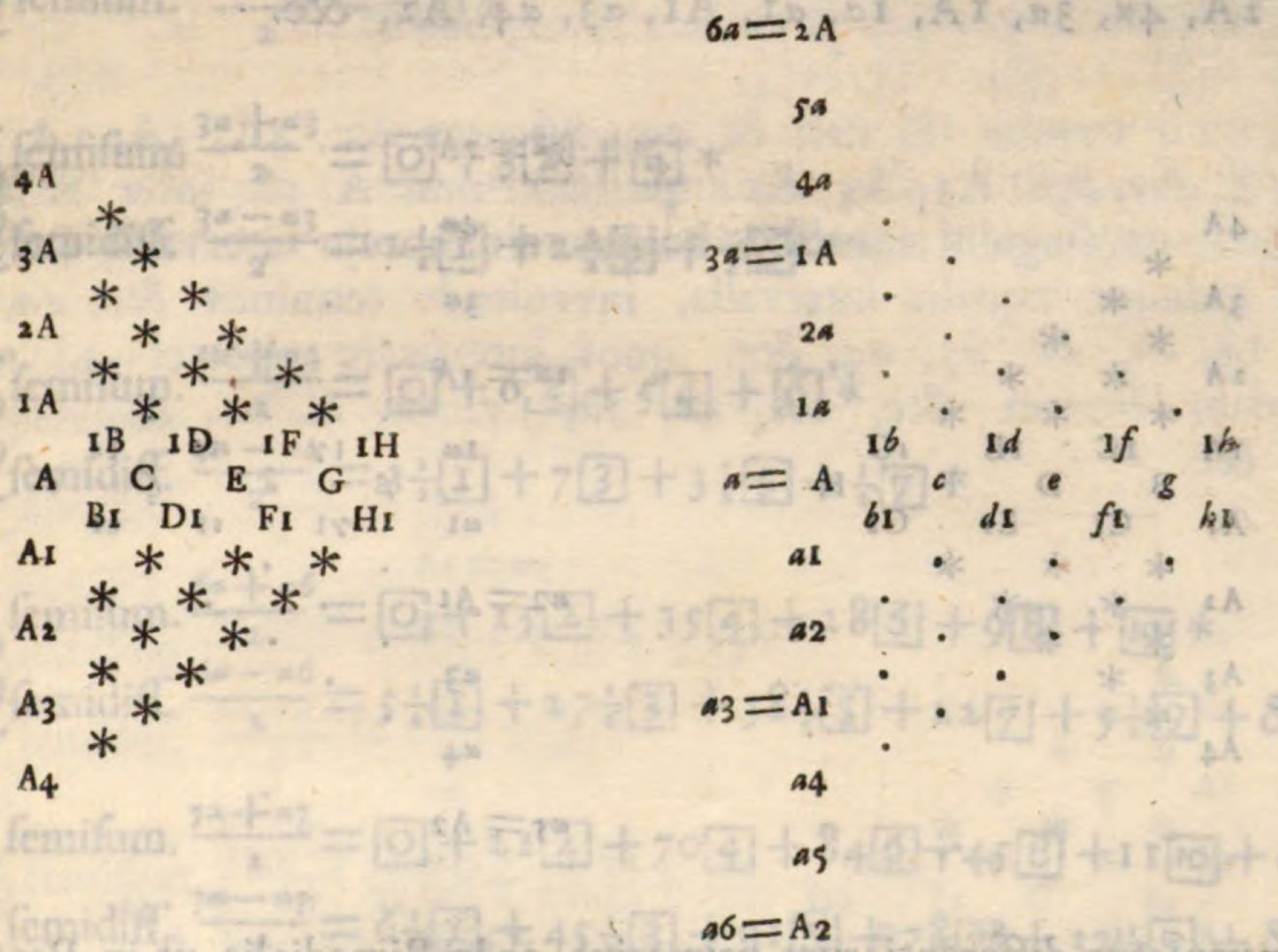
V. ſemiſum. $\frac{5a + a5}{2} = \boxed{0} + 12\tfrac{1}{2}\boxed{2} + 25\boxed{4} + 17\tfrac{1}{2}\boxed{6} + 5\boxed{8} + \frac{1}{2}\boxed{10}\ *$
ſemidiff. $\frac{5a - a5}{2} = 5\boxed{1} + 20\boxed{3} + 21\boxed{5} + 8\boxed{7} + \boxed{9}\ *$

VII. ſemiſum. $\frac{7a + a7}{2} = \boxed{0} + 24\tfrac{1}{2}\boxed{2} + 98\boxed{4} + 147\boxed{6} + 105\boxed{8} + 38\tfrac{1}{2}\boxed{10} + \text{\&c.}$
ſemidiff. $\frac{7a - a7}{2} = 7\boxed{1} + 56\boxed{3} + 126\boxed{5} + 120\boxed{7} + 55\boxed{9} + \text{\&c.}$

&c.

Poſt-

Postquam Series completa fuerit, si capiantur denuo terminorum differentiæ, ponendo $1b = 1a - A$, $b1 = A - a1$, $c = 1b - b1$, &c. & scribantur b, d, f, h, &c. pro semisummis ipsarum $1b$ & $b1$, $1d$ & $d1$, $1f$ & $f1$, $1h$ & $h1$, &c. habebitur differentiarum ordo Medialis a, b, c, d, e, f, g, h, &c. Hic vero prorsus idem erit



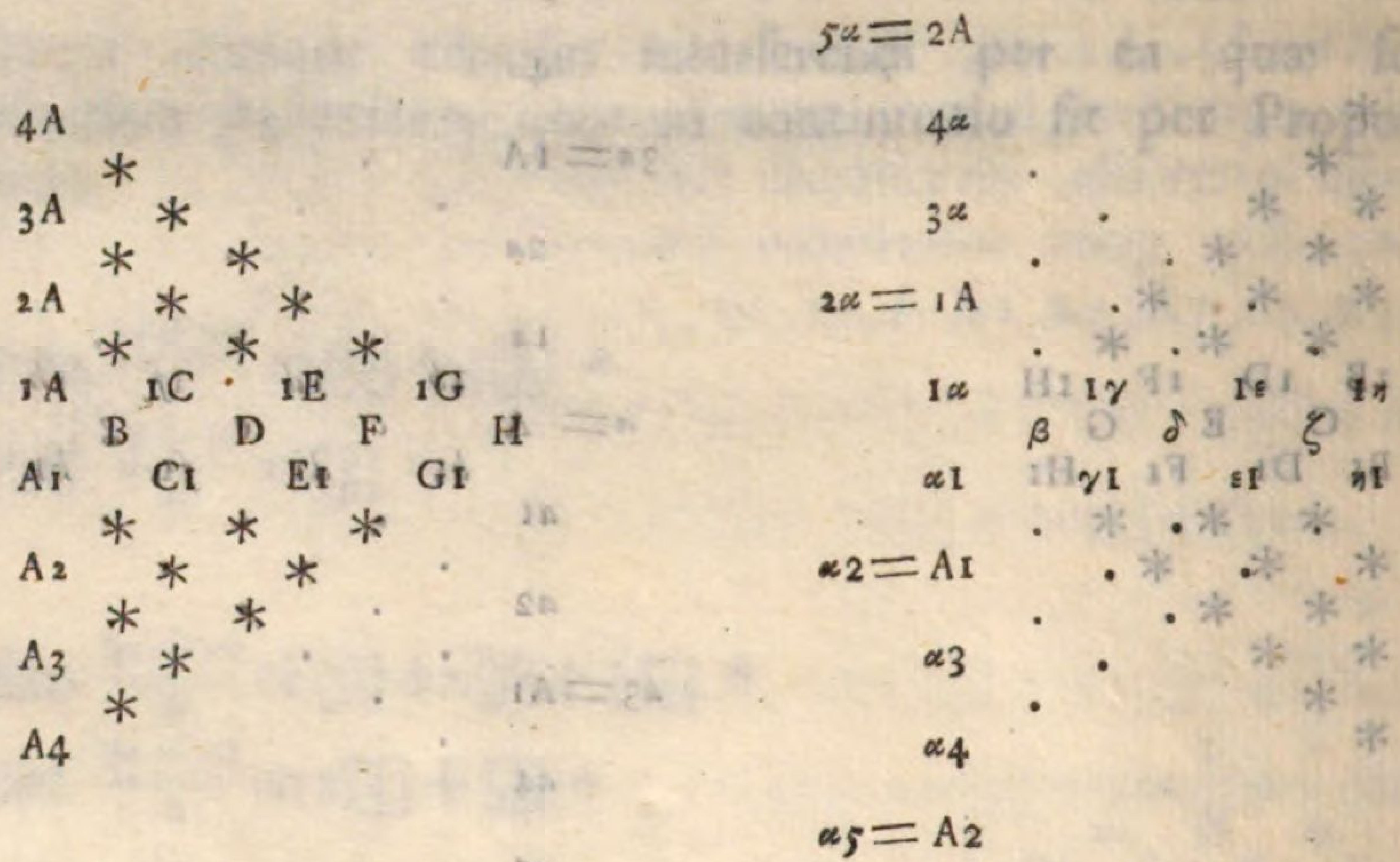
atque ordo ille quadratus $\boxed{0}$, $\boxed{1}$, $\boxed{2}$, $\boxed{3}$, $\boxed{4}$, $\boxed{5}$, $\boxed{6}$, $\boxed{7}$, &c. per quem terminos interserendos invenimus. Effectio itaque Problematis vocetur *Trisectio per quadratum ordinem Medialem*, vel etiam, *Trisectio Medialis*.

PROPOSITIO IV.

Datis Seriei cujuscunque terminis aliquot æquidistantibus, quorum bini sunt inter reliquos medii; propositum sit eorundem intervalla trifariam secare, terminos duos inter datorum binos quosque proximos interserendo.

SUNTO

Sunto termini illi dati & æquidiſtantes &c, 4A, 3A, 2A, 1A, A1, A2, A3, A4, &c. quorum bini 1A & A1 ſunt inter reliquos medii: ex ſingulis æqualibus horum terminorum intervallis facienda ſunt tria æqualia intervalla, inveniendo terminos &c, 4α, 3α, 1α, α1, α3, α4, &c. quos interſerere oportet ad complendam Seriem &c, 2A, 4α, 3α, 1A, 1α, α1, A1, α3, α4, A2, &c.



Inveniatur differentiarum primarius ordo Bimedialis ut in Propoſitione ſecunda. Hæ differentiæ diviſæ per numeros in ratione tripli creſcentes dabunt ordinem rotundum, ſumendo nempe $\textcircled{0} = A$, $\textcircled{1} = \frac{1}{3}B$, $\textcircled{2} = \frac{1}{9}C$, $\textcircled{3} = \frac{1}{27}D$, $\textcircled{4} = \frac{1}{81}E$, &c.

Ex ordine rotundo colligendus eſt ordo quadratus per hæcce Theoremata *priora*, quorum continuatio fit per Propoſitionem octavam.

&c.

$\boxed{10} = \textcircled{10} -$ &c.

$\boxed{8} = \textcircled{8} - \frac{11}{3}\boxed{10} -$ &c.

$\boxed{6} = \textcircled{6} - 3\boxed{8} - \frac{11}{3}\boxed{10} -$ &c.

$\boxed{4} = \textcircled{4} - \frac{7}{3}\boxed{6} - 2\boxed{8} - \frac{22}{27}\boxed{10} -$ &c.

$\boxed{2} = \textcircled{2} - \frac{5}{3}\boxed{4} - \frac{7}{9}\boxed{6} - \frac{1}{9}\boxed{8}$ *

$\boxed{0} = \textcircled{0} - \boxed{2}$ *

&c.

$\boxed{9} = \textcircled{9} -$ &c.

$\boxed{7} = \textcircled{7} - \frac{7}{3}\boxed{9} -$ &c.

$\boxed{5} = \textcircled{5} - \frac{5}{3}\boxed{7} - \frac{10}{9}\boxed{9} -$ &c.

$\boxed{3} = \textcircled{3} - \boxed{5} - \frac{1}{3}\boxed{7} - \frac{1}{27}\boxed{9}$ *

$\boxed{1} = \textcircled{1} - \frac{1}{3}\boxed{3}$ *

Dantur denique termini interſerendi per ea quæ ſequuntur Theoremata *poſteriora*, quorum continuatio fit per Propoſitionem decimam.

I. $\begin{cases} \text{ſemiſum.}\ \dfrac{1\alpha+\alpha 1}{2} = \boxed{0}\ * \\ \text{ſemidiff.}\ \dfrac{1\alpha-\alpha 1}{2} = \tfrac{1}{2}\boxed{1}\ * \end{cases}$

III. $\begin{cases} \text{ſemiſum.}\ \dfrac{3\alpha+\alpha 3}{2} = \boxed{0} + 3\boxed{2} + \boxed{4}\ * \\ \text{ſemidiff.}\ \dfrac{3\alpha-\alpha 3}{2} = 2\tfrac{1}{2}\boxed{1} + 2\tfrac{1}{2}\boxed{3} + \tfrac{1}{2}\boxed{5}\ * \end{cases}$

IV. $\begin{cases} \text{ſemiſum.}\ \dfrac{4\alpha+\alpha 4}{2} = \boxed{0} + 6\boxed{2} + 5\boxed{4} + \boxed{6}\ * \\ \text{ſemidiff.}\ \dfrac{4\alpha-\alpha 4}{2} = 3\tfrac{1}{2}\boxed{1} + 7\boxed{3} + 3\tfrac{1}{2}\boxed{5} + \tfrac{1}{2}\boxed{7}\ * \end{cases}$

VI. $\begin{cases} \text{ſemiſum.}\ \dfrac{6\alpha+\alpha 6}{2} = \boxed{0} + 15\boxed{2} + 35\boxed{4} + 28\boxed{6} + 9\boxed{8} + \boxed{10}\ * \\ \text{ſemidiff.}\ \dfrac{6\alpha-\alpha 6}{2} = 5\tfrac{1}{2}\boxed{1} + 27\tfrac{1}{2}\boxed{3} + 38\tfrac{1}{2}\boxed{5} + 22\boxed{7} + 5\tfrac{1}{2}\boxed{9} + \&c. \end{cases}$

VII. $\begin{cases} \text{ſemiſum.}\ \dfrac{7\alpha+\alpha 7}{2} = \boxed{0} + 21\boxed{2} + 70\boxed{4} + 84\boxed{6} + 45\boxed{8} + 11\boxed{10} + \&c. \\ \text{ſemidiff.}\ \dfrac{7\alpha-\alpha 7}{2} = 6\tfrac{1}{2}\boxed{1} + 45\tfrac{1}{2}\boxed{3} + 91\boxed{5} + 78\boxed{7} + 32\tfrac{1}{2}\boxed{9} + \&c. \end{cases}$

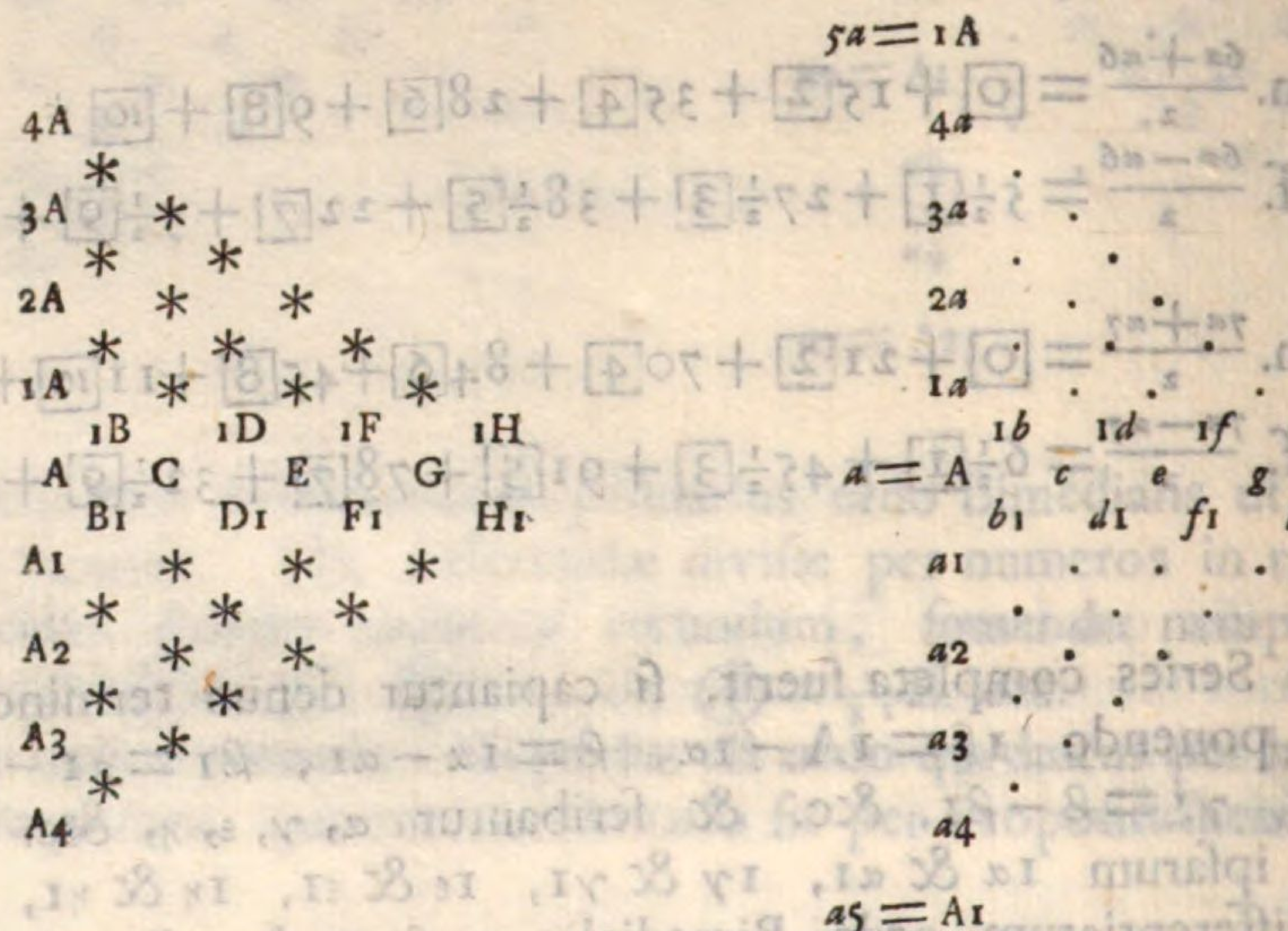
&c.

Poſtquam Series completa fuerit, ſi capiantur denuo terminorum differentiæ, ponendo $1\beta = 1\mathrm{A} - 1\alpha$, $\beta = 1\alpha - \alpha 1$, $\beta 1 = \alpha 1 - \mathrm{A}1$, $1\gamma = 1\beta - \beta$, $\gamma 1 = \beta - \beta 1$, &c. & ſcribantur $\alpha, \gamma, \epsilon, \eta$, &c. pro ſemiſummis ipſarum 1α & $\alpha 1$, 1γ & $\gamma 1$, 1ϵ & $\epsilon 1$, 1η & $\eta 1$, &c. habebitur differentiarum ordo Bimedialis $\alpha, \beta, \gamma, \delta, \epsilon, \zeta, \eta, \theta$, &c. Hic vero prorſus idem erit atque ordo ille quadratus $\boxed{0}$, $\boxed{1}$, $\boxed{2}$, $\boxed{3}$, $\boxed{4}$, $\boxed{5}$, $\boxed{6}$, $\boxed{7}$, &c. per quem terminos interſerendos invenimus. Effectio itaque Problematis vocetur *Triſectio per quadratum ordinem Bimedialem*, vel etiam, *Triſectio Bimedialis*.

PROPOSITIO V.

Datis Seriei cujuscunque terminis aliquot æquidistantibus, quorum unus est inter reliquos medius; propositum sit eorundem intervalla quinquifariam secare, terminos quatuor inter datorum binos quosque proximos interserendo.

SUNTO termini illi dati & æquidistantes &c, 4A, 3A, 2A, 1A, A, A1, A2, A3, A4, &c. quorum unus A est inter reliquos medius: ex singulis æqualibus horum terminorum intervallis facienda sunt quinque æqualia intervalla, inveniendo terminos &c, 4*a*, 3*a*, 2*a*, 1*a*, *a*1, *a*2, *a*3, *a*4, &c. quos interserere oportet ad complendam Seriem &c, 1A, 4*a*, 3*a*, 2*a*, 1*a*, A, *a*1, *a*2, *a*3, *a*4, A1, &c.



Inveniatur differentiarum primarius ordo Medialis ut in Propositione prima. Hæ differentiæ divisæ per numeros in ratione quintupli crescentes dabunt ordinem rotundum, sumendo nempe $\textcircled{0} = A$, $\textcircled{1} = \frac{1}{5}B$, $\textcircled{2} = \frac{1}{25}C$, $\textcircled{3} = \frac{1}{125}D$, $\textcircled{4} = \frac{1}{625}E$, &c.

Ex ordine rotundo colligendus est ordo quadratus per hæcce Theoremata *priora*, quorum continuatio fit per Propositionem septimam.

&c.

$\boxed{10} = \textcircled{10} -$ &c.

$\boxed{8} = \textcircled{8} - 8\boxed{10} -$ &c.

$\boxed{6} = \textcircled{6} - 6\boxed{8} - \frac{81}{5}\boxed{10} -$ &c.

$\boxed{4} = \textcircled{4} - 4\boxed{6} - \frac{34}{5}\boxed{8} - \frac{32}{5}\boxed{10} -$ &c.

$\boxed{2} = \textcircled{2} - 2\boxed{4} - \frac{7}{5}\boxed{6} - \frac{2}{5}\boxed{8} - \frac{1}{25}\boxed{10} *$

$\boxed{0} = \textcircled{0} *$

&c.

$\boxed{9} = \textcircled{9} -$ &c.

$\boxed{7} = \textcircled{7} - 10\boxed{9} -$ &c.

$\boxed{5} = \textcircled{5} - 8\boxed{7} - 27\boxed{9} -$ &c.

$\boxed{3} = \textcircled{3} - 6\boxed{5} - \frac{68}{5}\boxed{7} - 16\boxed{9} -$ &c.

$\boxed{1} = \textcircled{1} - 4\boxed{3} - \frac{21}{5}\boxed{5} - \frac{8}{5}\boxed{7} - \frac{1}{5}\boxed{9} *$

Dantur denique termini interſerendi per ea quæ ſequuntur Theoremata *poſteriora*, quorum continuatio fit per Propoſitionem decimam.

I. ſemiſum. $\frac{1a + a1}{2} = \boxed{0} + \frac{1}{2}\boxed{2} *$
ſemidiff. $\frac{1a - a1}{2} = \boxed{1} *$

II. ſemiſum. $\frac{2a + a2}{2} = \boxed{0} + 2\boxed{2} + \frac{1}{2}\boxed{4} *$
ſemidiff. $\frac{2a - a2}{2} = 2\boxed{1} + \boxed{3} *$

III. ſemiſum. $\frac{3a + a3}{2} = \boxed{0} + 4\tfrac{1}{2}\boxed{2} + 3\boxed{4} + \frac{1}{2}\boxed{6} *$
ſemidiff. $\frac{3a - a3}{2} = 3\boxed{1} + 4\boxed{3} + \boxed{5} *$

IV. ſemiſum. $\frac{4a + a4}{2} = \boxed{0} + 8\boxed{2} + 10\boxed{4} + 4\boxed{6} + \frac{1}{2}\boxed{8} *$
ſemidiff. $\frac{4a - a4}{2} = 4\boxed{1} + 10\boxed{3} + 6\boxed{5} + \boxed{7} *$

VI. ſemiſum. $\frac{6a + a6}{2} = \boxed{0} + 18\boxed{2} + 52\tfrac{1}{2}\boxed{4} + 56\boxed{6} + 27\boxed{8} + 6\boxed{10} +$ &c.
ſemidiff. $\frac{6a - a6}{2} = 6\boxed{1} + 35\boxed{3} + 56\boxed{5} + 36\boxed{7} + 10\boxed{9} +$ &c.

&c.

Poſtquam Series completa fuerit, ſi capiantur denuo terminorum differentiæ, ponendo $1b = 1a - A$, $b1 = A - a1$, $c = 1b - b1$, &c. & ſcribantur b, d, f, h, &c. pro ſemiſummis ipſarum $1b$ & $b1$, $1d$ & $d1$, $1f$ & $f1$, $1h$ & $h1$, &c. habebitur differentiarum ordo Medialis a, b, c, d, e, f, g, h, &c. Hic vero prorſus idem erit atque

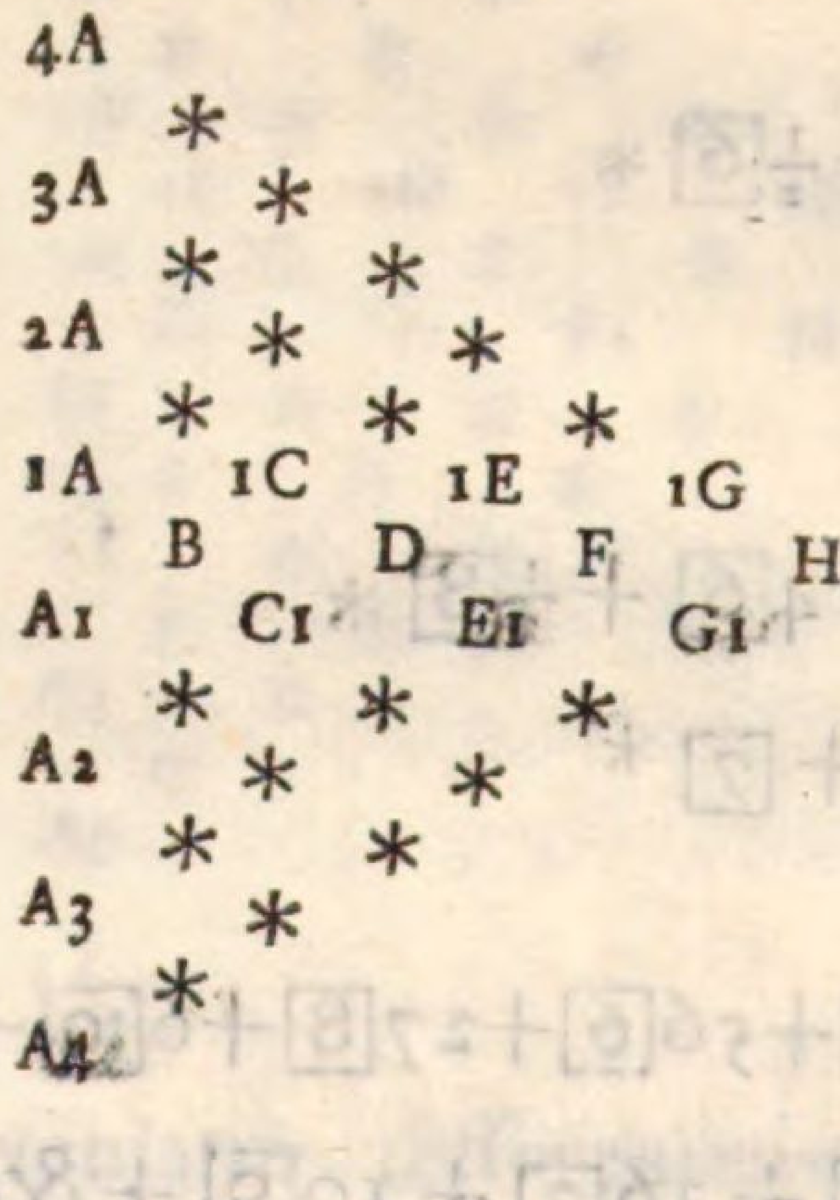
ordo

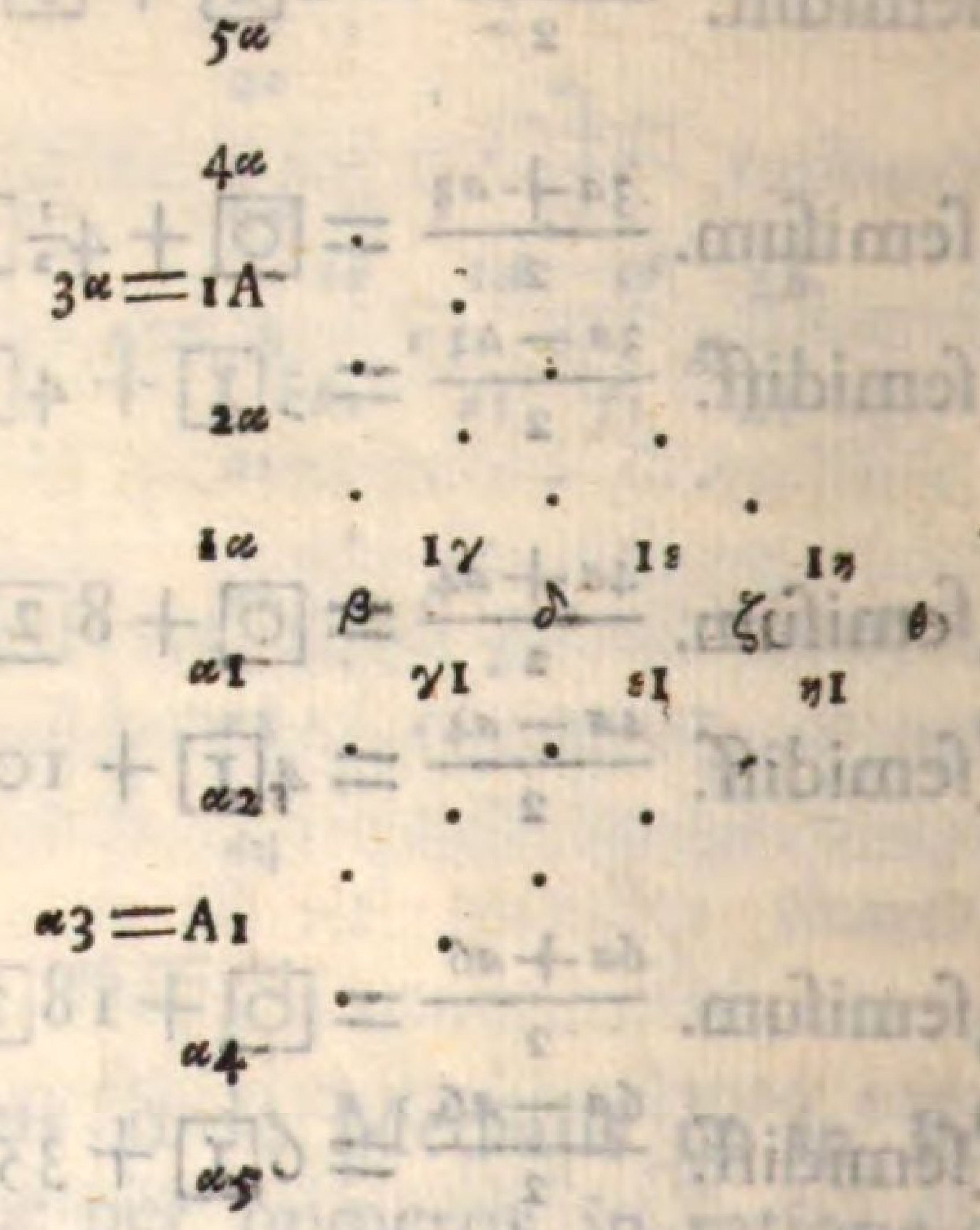
ordo ille quadratus [0], [1], [2], [3], [4], [5], [6], [7], &c. per quem terminos interferendos invenimus. Effectio itaque Problematis vocetur *Quinquisectio per quadratum ordinem Medialem*, vel etiam, *Quinquisectio Medialis*.

PROPOSITIO VI.

Datis Seriei cujuscunque terminis aliquot æquidistantibus, quorum bini sunt inter reliquos medii; propositum sit eorundem intervalla quinquifariam secare, terminos quatuor inter datorum binos quosque proximos interserendo.

SUNTO termini illi dati & æquidistantes &c, 4A, 3A, 2A, 1A, A1, A2, A3, A4, &c. quorum bini 1A & A1 sunt inter reliquos medii: ex singulis æqualibus horum terminorum intervallis facienda sunt quinque æqualia intervalla, inveniendo terminos &c, 5α, 4α, 2α, 1α, α1, α2, α4, α5 &c. quos interserere oportet ad complendam Seriem &c, 5α, 4α, 1A, 2α, 1α, α1, α2, A1, α4, α5, &c.





Inveniatur differentiarum primarius ordo Bimedialis ut in Propositione secunda. Hæ differentiæ divisæ per numeros in ratione quintupli crescentes dabunt ordinem rotundum, sumendo nempe (0) = A, (1) = $\frac{1}{5}$B, (2) = $\frac{1}{25}$C, (3) = $\frac{1}{125}$D, (4) = $\frac{1}{625}$E, &c.

Ex

Ex ordine rotundo colligendus eſt ordo quadratus per hæcce Theoremata *priora*, quorum continuatio fit per Propoſitionem octavam.

&c.

$\boxed{10} = \textcircled{10} - \&c.$

$\boxed{8} = \textcircled{8} - 11\boxed{10} - \&c.$

$\boxed{6} = \textcircled{6} - 9\boxed{8} - \tfrac{176}{5}\boxed{10} - \&c.$

$\boxed{4} = \textcircled{4} - 7\boxed{6} - \tfrac{29}{5}\boxed{8} - \tfrac{154}{5}\boxed{10} - \&c.$

$\boxed{2} = \textcircled{2} - 5\boxed{4} - \tfrac{42}{5}\boxed{6} - \tfrac{33}{5}\boxed{8} - \tfrac{66}{25}\boxed{10} - \&c.$

$\boxed{0} = \textcircled{0} - 3\boxed{2} - \boxed{4}\,*$

&c.

$\boxed{9} = \textcircled{9} - \&c.$

$\boxed{7} = \textcircled{7} - 7\boxed{9} - \&c.$

$\boxed{5} = \textcircled{5} - 5\boxed{7} - 11\boxed{9} - \&c.$

$\boxed{3} = \textcircled{3} - 3\boxed{5} - \tfrac{18}{5}\boxed{7} - \tfrac{11}{5}\boxed{9} - \&c.$

$\boxed{1} = \textcircled{1} - \boxed{3} - \tfrac{1}{5}\boxed{5}\,*$

Dantur denique termini interſerendi per ea quæ ſequuntur Theoremata *poſteriora*, quorum continuatio fit per Propoſitionem decimam.

I. ſemiſum. $\frac{1\alpha + \alpha 1}{2} = \boxed{0}\,*$
ſemidiff. $\frac{1\alpha - \alpha 1}{2} = \tfrac{1}{2}\boxed{1}\,*$

II. ſemiſum. $\frac{2\alpha + \alpha 2}{2} = \boxed{0} + \boxed{2}\,*$
ſemidiff. $\frac{2\alpha - \alpha 2}{2} = 1\tfrac{1}{2}\boxed{1} + \tfrac{1}{2}\boxed{3}\,*$

IV. ſemiſum. $\frac{4\alpha + \alpha 4}{2} = \boxed{0} + 6\boxed{2} + 5\boxed{4} + \boxed{6}\,*$
ſemidiff. $\frac{4\alpha - \alpha 4}{2} = 3\tfrac{1}{2}\boxed{1} + 7\boxed{3} + 3\tfrac{1}{2}\boxed{5} + \tfrac{1}{2}\boxed{7}\,*$

V. ſemiſum. $\frac{5\alpha + \alpha 5}{2} = \boxed{0} + 10\boxed{2} + 15\boxed{4} + 7\boxed{6} + \boxed{8}\,*$
ſemidiff. $\frac{5\alpha - \alpha 5}{2} = 4\tfrac{1}{2}\boxed{1} + 15\boxed{3} + 13\tfrac{1}{2}\boxed{5} + 4\tfrac{1}{2}\boxed{7} + \tfrac{1}{2}\boxed{9}\,*$

VI. ſemiſum. $\frac{6\alpha + \alpha 6}{2} = \boxed{0} + 15\boxed{2} + 35\boxed{4} + 28\boxed{6} + 9\boxed{8} + \boxed{10}\,*$
ſemidiff. $\frac{6\alpha - \alpha 6}{2} = 5\tfrac{1}{2}\boxed{1} + 27\tfrac{1}{2}\boxed{3} + 38\tfrac{1}{2}\boxed{5} + 22\boxed{7} + 5\tfrac{1}{2}\boxed{9} + \&c.$

&c.

Postquam Series completa fuerit, si capiantur denuo terminorum differentiæ, ponendo $1\beta = 2\alpha - 1\alpha$, $\beta = 1\alpha - \alpha 1$, $\beta 1 = \alpha 1 - \alpha 2$, $1\gamma = 1\beta - \beta$, $\gamma 1 = \beta - \beta 1$, &c. & scribantur α, γ, ϵ, η, &c. pro semisummis ipsarum 1α & $\alpha 1$, 1γ & $\gamma 1$, 1ϵ & $\epsilon 1$, 1η & $\eta 1$, &c. habebitur differentiarum ordo Bimedialis α, β, γ, δ, ϵ, ζ, η, θ, &c. Hic vero prorsus idem erit atque ordo ille quadratus $\boxed{0}$, $\boxed{1}$, $\boxed{2}$, $\boxed{3}$, $\boxed{4}$, $\boxed{5}$, $\boxed{6}$, $\boxed{7}$, &c. per quem terminos interserendos invenimus. Effectio itaque Problematis vocetur *Quinquisectio per quadratum ordinem Bimedialem*, vel etiam, *Quinquisectio Bimedialis*.

Quæ huc usque tradita sunt abunde mihi sufficere videntur ad computum Tabularum; nam quæ potissimum apud nos in usum receptæ sunt divisiones, illæ quidem satis commode meo judicio per Bisectionem, Trisectionem & Quinquisectionem absolvantur. Verum cum Methodus nostra generalior sit & ad omnes sese divisiones extendat, modum quo Sectiones reliquæ perficiuntur in sequentibus exponere non gravabor, præsertim cum usui esse possit ad continuationem Theorematum quæ sunt in superioribus allata.

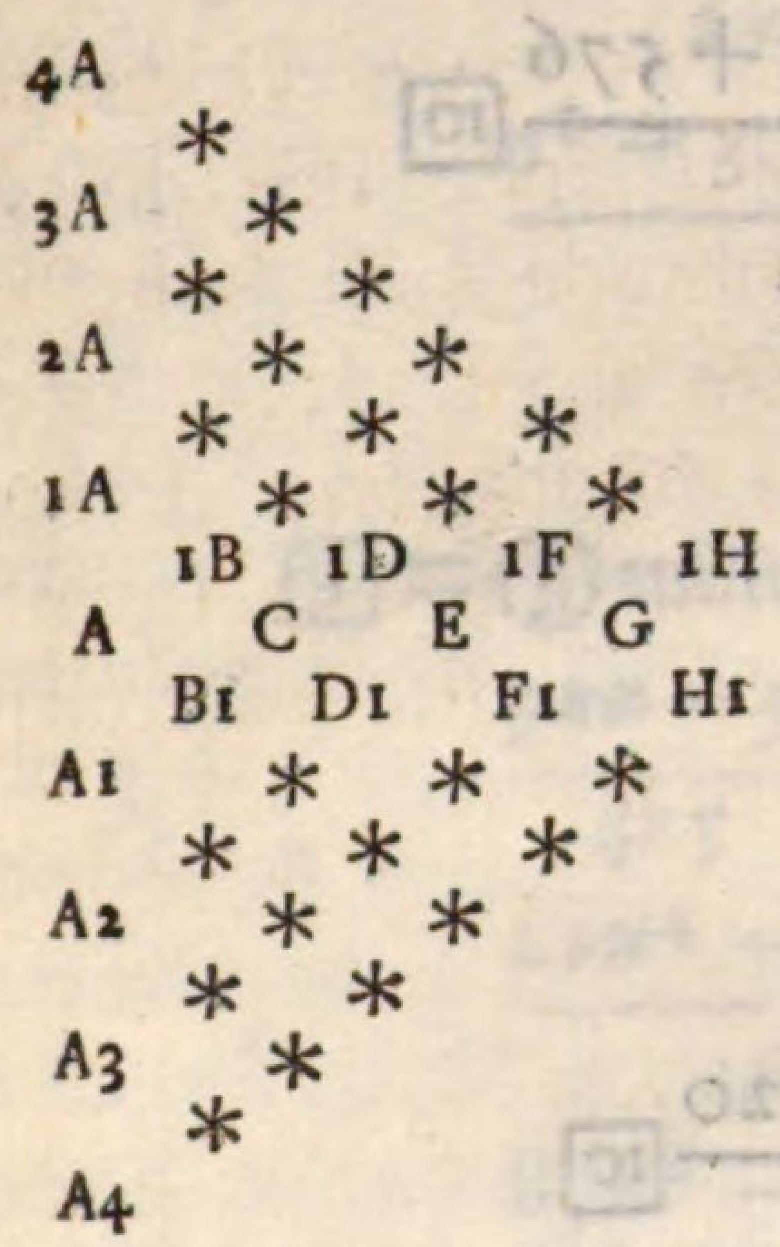
PROPOSITIO VII.

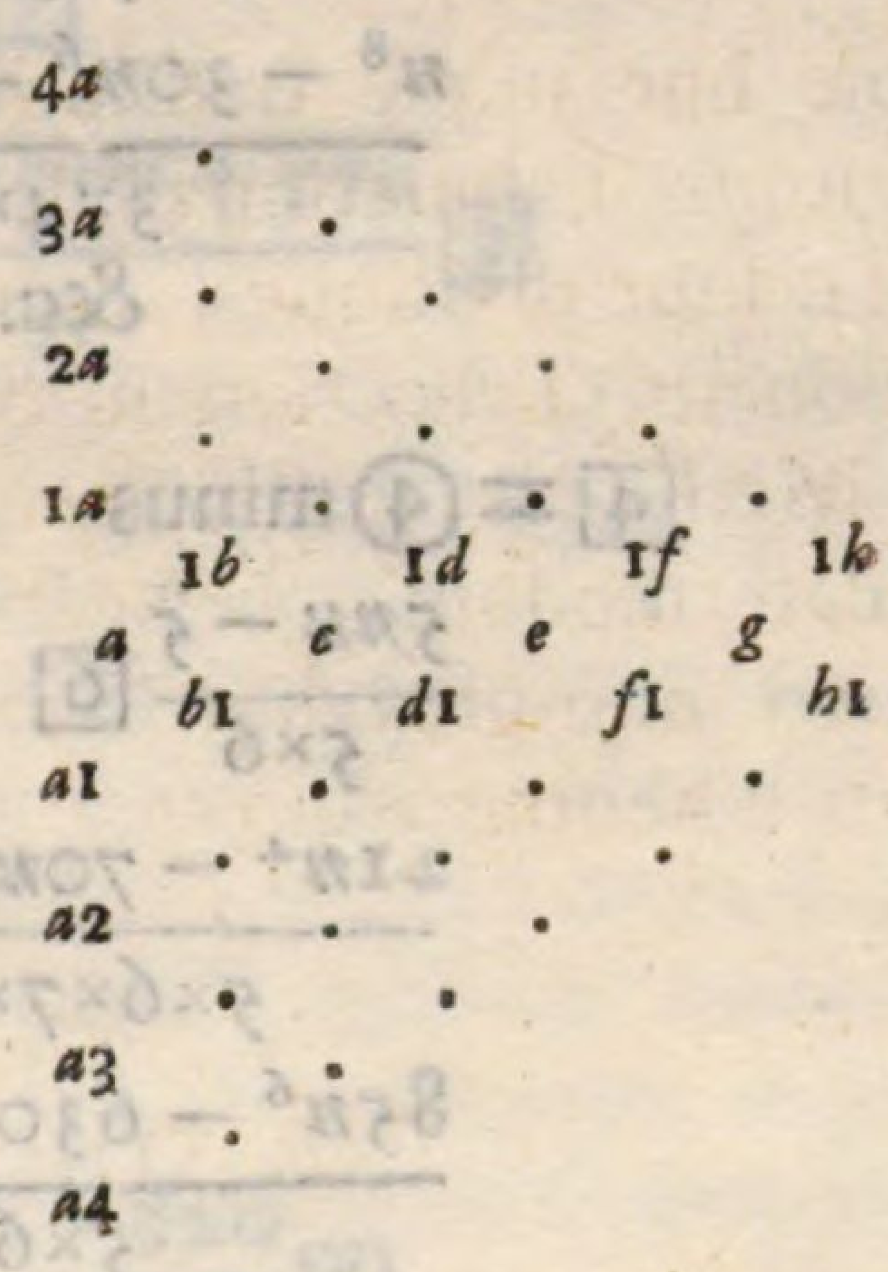
Dato differentiarum ordine primario Mediali, propositum sit Medialem ordinem quadratum invenire pro quibuscunque Sectionibus.

Esto ille datus differentiarum primarius ordo Medialis A, B, C, D, E, F, G, H, &c. qui deductus sit ex Serie Mediali terminorum æquidistantium &c, 4A, 3A, 2A, 1A, A, A1, A2, A3, A4, &c. quemadmodum in Proposit. I. III. V. Horum terminorum intervalla secanda sint in partes æquales quotlibetcunque quarum numerus sit n, interpositis terminis novis quorum numerus erit $n-1$, ad complendam Seriem Medialem &c, $4a$, $3a$, $2a$, $1a$, a, $a1$, $a2$, $a3$, $a4$, &c. cujus terminus medius a idem est atque terminus medius A Seriei prioris, & cujus intervalla minora sunt quam intervalla Seriei prioris in ratione 1 ad n. Ex hac Serie deductæ intelligantur differentiæ,

sumendo

ſumendo $1b = 1a - a$, $b1 = a - a1$, $c = 1b - b1$, &c. & ſi excerpantur $\begin{matrix} & 1b & & 1d & & 1f & & 1h \\ a & & c & & e & & g & \\ & b1 & & d1 & & f1 & & h1 \end{matrix}$ &c. quæ inter omnes medium habent locum, & ſcribantur b, d, f, h, &c. pro ſemiſummis ipſarum $1b$ & $b1$, $1d$ & $d1$, $1f$ & $f1$, $1h$ & $h1$; erit a, b, c, d, e, f, g, h, &c. ordo ille quadratus Medialis quem invenire oportet quemque ſic denotare ſoleo $\boxed{0}, \boxed{1}, \boxed{2}, \boxed{3}, \boxed{4}, \boxed{5}, \boxed{6}, \boxed{7}$, &c.





Dividatur primarius ordo A, B, C, D, E, F, G, H, &c. per numeros creſcentes in ratione 1 ad n; & habebitur ordo rotundus $\textcircled{0}, \textcircled{1}, \textcircled{2}, \textcircled{3}, \textcircled{4}, \textcircled{5}, \textcircled{6}, \textcircled{7}$, &c. ſumendo nempe $\textcircled{0} = A$, $\textcircled{1} = \frac{1}{n}B$, $\textcircled{2} = \frac{1}{nn}C$, $\textcircled{3} = \frac{1}{n^3}D$, $\textcircled{4} = \frac{1}{n^4}E$, &c.

Deinde ex ordine rotundo colligendus eſt ordo quadratus per ſequentia Theoremata; incipit vero calculus a differentiis remotiſſimis & regrediendo continuatur ad priores.

$\boxed{2} = ⓶$ minus $\frac{nn-1}{3\times4}\boxed{4}$ $\frac{n^4-5nn+4}{3\times4\times5\times6}\boxed{6}$ $\frac{n^6-14n^4+49nn-36}{3\times4\times5\times6\times7\times8}\boxed{8}$ $\frac{n^8-30n^6+273n^4-820nn+576}{3\times4\times5\times6\times7\times8\times9\times10}\boxed{10}$ &c.

$\boxed{4} = ⓸$ minus $\frac{5nn-5}{5\times6}\boxed{6}$ $\frac{21n^4-70nn+49}{5\times6\times7\times8}\boxed{8}$ $\frac{85n^6-630n^4+1365nn-820}{5\times6\times7\times8\times9\times10}\boxed{10}$ &c.

$\boxed{6} = ⓺$ minus $\frac{14nn-14}{7\times8}\boxed{8}$ $\frac{147n^4-420nn+273}{7\times8\times9\times10}\boxed{10}$ &c.

$\boxed{8} = ⓼$ minus $\frac{30nn-30}{9\times10}\boxed{10}$ &c.

$\boxed{10} = ⓾$ minus &c.

$\boxed{1} = \textcircled{1}$ minus

$$\frac{nn-1}{2\times3}\boxed{3}$$

$$\frac{n^4-5nn+4}{2\times3\times4\times5}\boxed{5}$$

$$\frac{n^6-14n^4+49nn-36}{2\times3\times4\times5\times6\times7}\boxed{7}$$

$$\frac{n^8-30n^6+273n^4-820nn+576}{2\times3\times4\times5\times6\times7\times8\times9}\boxed{9}$$

&c.

$\boxed{3} = \textcircled{3}$ minus

$$\frac{5nn-5}{4\times5}\boxed{5}$$

$$\frac{21n^4-70nn+49}{4\times5\times6\times7}\boxed{7}$$

$$\frac{85n^6-630n^4+1365nn-820}{4\times5\times6\times7\times8\times9}\boxed{9}$$

&c.

$\boxed{5} = \textcircled{5}$ minus

$$\frac{14nn-14}{6\times7}\boxed{7}$$

$$\frac{147n^4-420nn+273}{6\times7\times8\times9}\boxed{9}$$

&c.

$\boxed{7} = \textcircled{7}$ minus

$$\frac{30nn-30}{8\times9}\boxed{9}$$

&c.

$\boxed{9} = \textcircled{9}$ minus

&c.

Theoremata prima ſic producuntur $nn-1 = nn-1$; $\overline{nn-1}\times\overline{nn-4} = n^4 - 5nn + 4$; $\overline{nn-1}\times\overline{nn-4}\times\overline{nn-9} = n^6 - 14n^4 + 49nn - 36$; $\overline{nn-1}\times\overline{nn-4}\times\overline{nn-9}\times\overline{nn-16} = n^8 - 30n^6 + 273n^4 - 820nn + 576$, &c. Reliqua Theoremata e primis derivantur dividendo per nn, n^4, n^6, &c. & multiplicando per numeros quos exhibet adjecta Tabella. Hujus prima Series tranſverſa conſtat ex unitatibus. Secunda 0, 1, 5, 21, 85, &c. efficitur ducendo terminum quemque præcedentem in 4 & addendo numerum ſupra poſitum in ſerie prima: ſic $4\times 0 + 1 = 1$, $4\times 1 + 1 = 5$, $4\times 5 + 1 = 21$, $4\times 21 + 1 = 85$, $4\times 85 + 1 = 341$, &c. Tertia 0, 0, 1, 14, 147, 1408, &c. efficitur ducendo terminum quemque præcedentem in 9 & addendo numerum ſupra poſitum in Serie ſecunda: ſic $9\times 0 + 0 = 0$, $9\times 0 + 1 = 1$, $9\times 1 + 5 = 14$, $9\times 14 + 21 = 147$, $9\times 147 + 85 = 1408$, &c. Quarta & ſequentes ad eundem modum efficiuntur, ducendo terminos præcedentes in quadratos numeros 16, 25, 36, &c. Uſus harum Serierum ad continuationem Theorematum facile patebit ſi perpendantur Exempla ſequentia.

1	1	1	1	1	1	1	&c.
4	0	1	5	21	85	341	&c.
9	0	0	1	14	147	1408	&c.
16	0	0	0	1	30	627	&c.
25	0	0	0	0	1	55	&c.

$$n^4 - 5nn + 4$$
$$\frac{5}{nn},\ \frac{1}{nn},\ \frac{0}{nn}$$
$$5nn - 5$$

$$n^6 - 14n^4 + 49nn - 36$$
$$\frac{21}{nn},\ \frac{5}{nn},\ \frac{1}{nn},\ \frac{0}{nn}$$
$$21n^4 - 70nn + 49$$

$$n^6 - 14n^4 + 49nn - 36$$
$$\frac{14}{n^4},\ \frac{1}{n^4},\ \frac{0}{n^4},\ \frac{0}{n^4}$$
$$14nn - 14$$

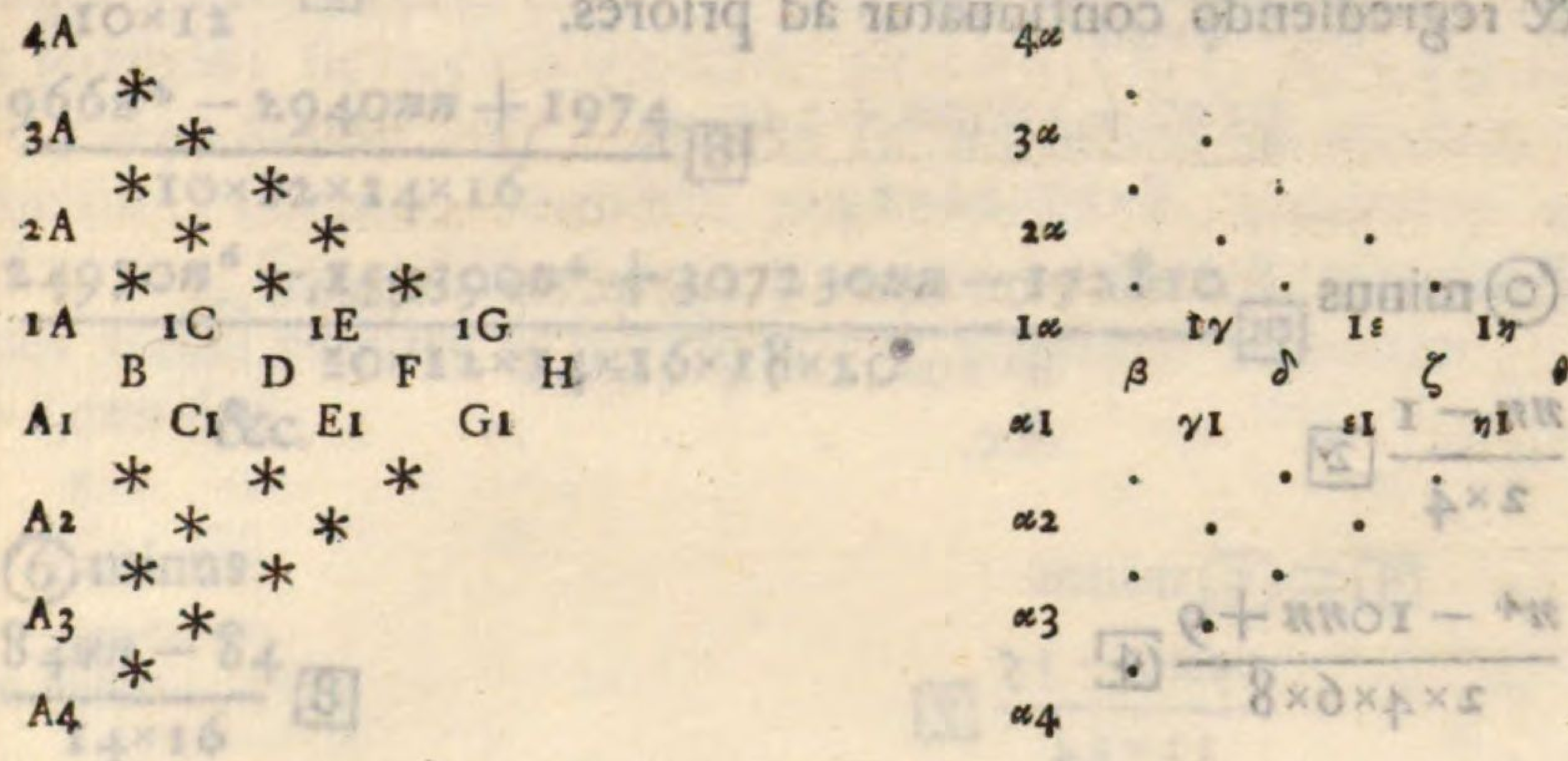
$$n^8 - 30n^6 + 273n^4 - 820nn + 576$$
$$\frac{85}{nn},\ \frac{21}{nn},\ \frac{5}{nn},\ \frac{1}{nn},\ \frac{0}{nn}$$
$$85n^6 - 630n^4 + 1365nn - 820$$
&c.

$$n^8 - 30n^6 + 273n^4 - 820nn + 576$$
$$\frac{147}{n^4},\ \frac{14}{n^4},\ \frac{1}{n^4},\ \frac{0}{n^4},\ \frac{0}{n^4}$$
$$147n^4 - 420nn + 273$$
&c.

PROPOSITIO VIII.

Dato differentiarum ordine primario Bimediali; propositum sit Bimedialem ordinem quadratum invenire pro quibuscunque Sectionibus ab impari numero denominatis.

ESTO ille datus differentiarum primarius ordo Bimedialis A, B, C, D, E, F, G, H, &c. qui deductus sit ex Serie Bimediali terminorum æquidistantium &c, 4A, 3A, 2A, 1A, A1, A2, A3, A4, &c. quemadmodum in Proposit. IV & VI. Horum terminorum intervalla secanda sint in partes æquales quotlibetcunque quarum numerus impar sit n, interpositis terminis novis quorum numerus erit



$n-1$, ad complendam Seriem Bimedialem &c, 4α, 3α, 2α, 1α, $\alpha 1$, $\alpha 2$, $\alpha 3$, $\alpha 4$, &c. cujus bini termini medii 1α & $\alpha 1$ medium habent locum inter 1A & A1 binos medios Seriei prioris, & cujus intervalla minora sunt quam intervalla Seriei prioris in ratione 1 ad n. Ex hac Serie deductæ intelligantur differentiæ, sumendo $\beta = 1\alpha - \alpha 1$, $1\gamma = 1\beta - \beta$, $\gamma 1 = \beta - \beta 1$, $\delta = 1\gamma - \gamma 1$, &c. & si excerpantur

$1\alpha \quad 1\gamma \quad 1\varepsilon \quad 1\eta$

$\beta \quad \delta \quad \zeta \quad \theta$ &c. quæ inter omnes medium habent locum,

$\alpha 1 \quad \gamma 1 \quad \varepsilon 1 \quad \eta 1$

& scribantur $\alpha, \gamma, \varepsilon, \eta$, &c. pro semisummis ipsarum 1α & $\alpha 1$, 1γ & $\gamma 1$

ιε & ει, ιη & ηι, &c. erit α, β, γ, δ, ε, ζ, η, θ, &c. ordo ille quadratus Bimedialis quem invenire oportet, quemque sic denotare soleo [0], [1], [2], [3], [4], [5], [6], [7], &c.

Dividatur primarius ordo A, B, C, D, E, F, G, H, &c. per numeros crescentes in ratione 1 ad n; & habebitur ordo rotundus ⓪, ①, ②, ③, ④, ⑤, ⑥, ⑦, &c. sumendo nempe ⓪ $=$ A, ① $=\frac{1}{n}$B, ② $=\frac{1}{nn}$C, ③ $=\frac{1}{n^3}$D, ④ $=\frac{1}{n^4}$E, ⑤ $=\frac{1}{n^5}$F, ⑥ $=\frac{1}{n^6}$G, ⑦ $=\frac{1}{n^7}$H, &c.

Deinde ex ordine rotundo colligendus est ordo quadratus per sequentia Theoremata; incipit vero calculus a differentiis remotissimis & regrediendo continuatur ad priores.

[0] $=$ ⓪ minus

$$\frac{nn-1}{2\times4}\,[2]$$

$$\frac{n^4-10nn+9}{2\times4\times6\times8}\,[4]$$

$$\frac{n^6-35n^4+259nn-225}{2\times4\times6\times8\times10\times12}\,[6]$$

$$\frac{n^8-84n^6+1974n^4-12916nn+11025}{2\times4\times6\times8\times10\times12\times14\times16}\,[8]$$

$$\frac{n^{10}-165n^8+8778n^6-172810n^4+1057221nn-893025}{2\times4\times6\times8\times10\times12\times14\times16\times18\times20}\,[10]$$

&c.

$\boxed{2} = \textcircled{2}$ minus

$$\frac{10nn - 10}{6\times 8}\ \boxed{4}$$

$$\frac{91n^4 - 350nn + 259}{6\times 8\times 10\times 12}\ \boxed{6}$$

$$\frac{820n^6 - 7644n^4 + 19740nn - 12916}{6\times 8\times 10\times 12\times 14\times 16}\ \boxed{8}$$

$$\frac{7381n^8 - 135300n^6 + 798798n^4 - 1728100nn + 1057221}{6\times 8\times 10\times 12\times 14\times 16\times 18\times 20}\ \boxed{10}$$

&c.

$\boxed{4} = \textcircled{4}$ minus

$$\frac{35nn - 35}{10\times 12}\ \boxed{6}$$

$$\frac{966n^4 - 2940nn + 1974}{10\times 12\times 14\times 16}\ \boxed{8}$$

$$\frac{24970n^6 - 159390n^4 + 307230nn - 172810}{10\times 12\times 14\times 16\times 18\times 20}\ \boxed{10}$$

&c.

$\boxed{6} = \textcircled{6}$ minus

$$\frac{84nn - 84}{14\times 16}\ \boxed{8}$$

$$\frac{5082n^4 - 13860nn + 8778}{14\times 16\times 18\times 20}\ \boxed{10}$$

&c.

$\boxed{8} = \textcircled{8}$ minus

$$\frac{165nn - 165}{18\times 20}\ \boxed{10}$$

&c.

$\boxed{10} = \textcircled{10}$ minus

&c.

$$\boxed{1} = \textcircled{1} \text{ minus}$$
$$\frac{nn-1}{4\times 6}\boxed{3}$$
$$\frac{n^4-10nn+9}{4\times 6\times 8\times 10}\boxed{5}$$
$$\frac{n^6-35n^4+259nn-225}{4\times 6\times 8\times 10\times 12\times 14}\boxed{7}$$
$$\frac{n^8-84n^6+1974n^4-12916nn+11025}{4\times 6\times 8\times 10\times 12\times 14\times 16\times 18}\boxed{9}$$
&c.

$$\boxed{3} = \textcircled{3} \text{ minus}$$
$$\frac{10nn-10}{8\times 10}\boxed{5}$$
$$\frac{91n^4-350nn+259}{8\times 10\times 12\times 14}\boxed{7}$$
$$\frac{820n^6-7644n^4+19740nn-12916}{8\times 10\times 12\times 14\times 16\times 18}\boxed{9}$$
&c.

$$\boxed{5} = \textcircled{5} \text{ minus}$$
$$\frac{35nn-35}{12\times 14}\boxed{7}$$
$$\frac{966n^4-2940nn+1974}{12\times 14\times 16\times 18}\boxed{9}$$
&c.

$$\boxed{7} = \textcircled{7} \text{ minus}$$
$$\frac{84nn-84}{16\times 18}\boxed{9}$$
&c.

$$\boxed{9} = \textcircled{9} \text{ minus}$$
&c.

Theoremata prima sic producuntur $\overline{nn-1} = nn-1$; $\overline{nn-1}\times\overline{nn-9} = n^4-10nn+9$; $\overline{nn-1}\times\overline{nn-9}\times\overline{nn-25} = n^6-35n^4+259nn-225$; $\overline{nn-1}\times\overline{nn-9}\times\overline{nn-25}\times\overline{nn-49} = n^8-84n^6+1974n^4-12916nn+11025$, &c. Reliqua Theoremata e primis derivantur dividendo per nn, n^4, n^6, &c. & multiplicando per numeros quos exhibet adjecta Tabella. Hujus prima Series transversa constat ex unitatibus. Secunda 0, 1, 10, 91, 820, &c. efficitur ducendo terminum quemque præcedentem in 9 & addendo numerum supra positum in Serie prima: sic $9\times0+1=1$, $9\times1+1=10$, $9\times10+1=91$, $9\times91+1=820$, $9\times820+1=7381$, &c. Tertia 0, 0, 1, 35, 966, &c. efficitur ducendo terminum quemque præcedentem in 25 & addendo numerum supra positum in Serie secunda: sic $25\times0+0=0$, $25\times0+1=1$, $25\times1+10=35$, $25\times35+91=966$, &c. Quarta & sequentes ad eundem modum efficiuntur, ducendo terminos præcedentes in quadratos numeros 49, 81, 121, &c. Usus harum Serierum ad continuationem Theorematum facile patebit si perpendantur Exempla sequentia.

1	1	1	1	1	1	1	&c.
9	0	1	10	91	820	7381	&c.
25	0	0	1	35	966	24970	&c.
49	0	0	0	1	84	5082	&c.
81	0	0	0	0	1	165	&c.
121	0	0	0	0	0	1	&c.

$n^4-10nn+9$

$\frac{10}{nn},\ \frac{1}{nn},\ \frac{0}{nn}$

$10nn-10$

$n^6-35n^4+259nn-225$

$\frac{91}{nn},\ \frac{10}{nn},\ \frac{1}{nn},\ \frac{0}{nn}$

$91n^4-350nn+259$

$n^6-35n^4+259nn-225$

$\frac{35}{n^4},\ \frac{1}{n^4},\ \frac{0}{n^4},\ \frac{0}{n^4}$

$35nn-35$

$n^8-84n^6+1974n^4-12916nn+11025$

$\frac{820}{nn},\ \frac{91}{nn},\ \frac{10}{nn},\ \frac{1}{nn},\ \frac{0}{nn}$

$820n^6-7644n^4+19740nn-12916$

&c.

$n^8-84n^6+1974n^4-12916nn+11025$

$\frac{966}{n^4},\ \frac{35}{n^4},\ \frac{1}{n^4},\ \frac{0}{n^4},\ \frac{0}{n^4}$

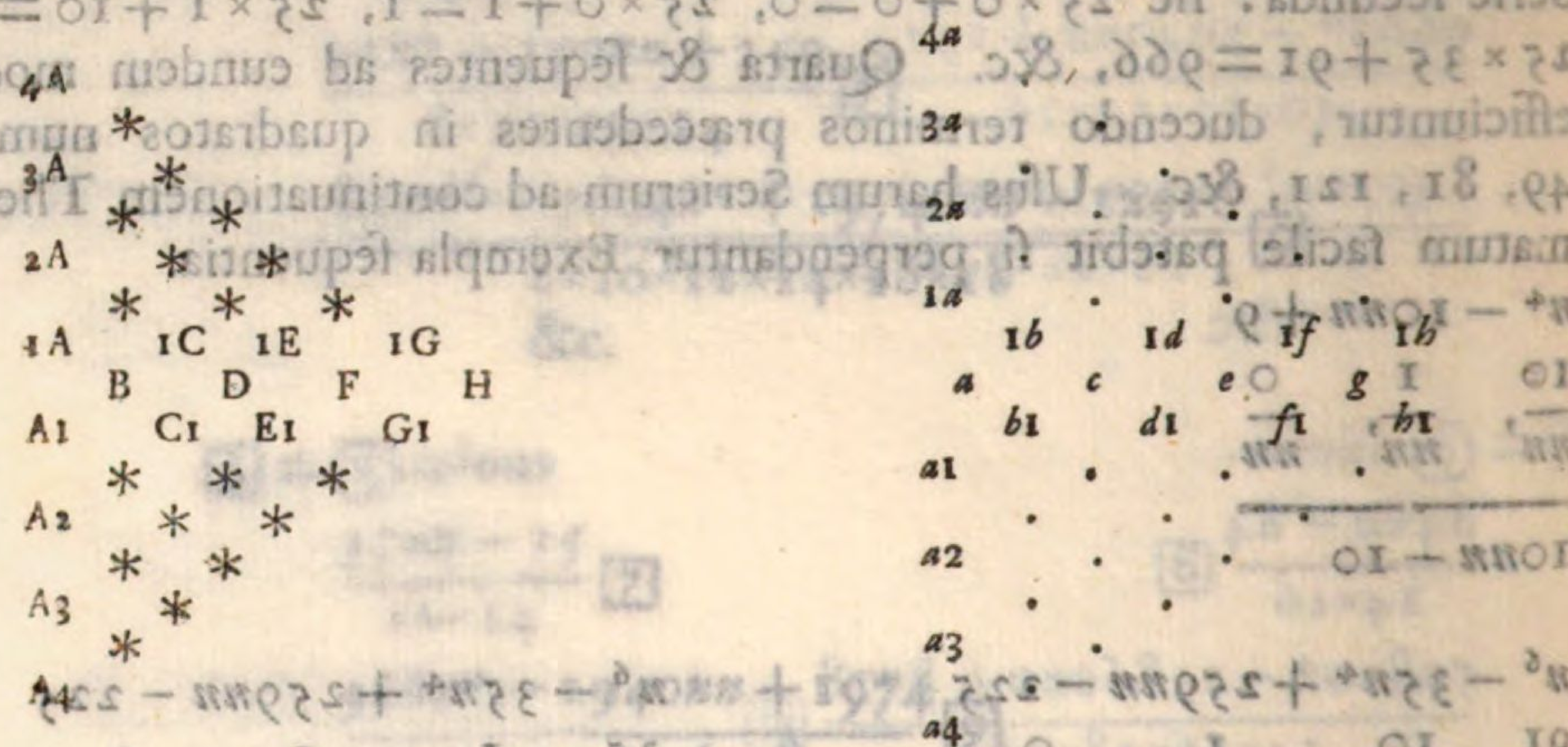
$966n^4-2940nn+1974$

&c.

PROPOSITIO IX.

Dato differentiarum ordine primario Bimediali, propositum sit Medialem contrarium ordinem quadratum invenire pro quibuscunque Sectionibus a pari numero denominatis.

Esto ille datus differentiarum primarius ordo Bimedialis A, B, C, D, E, F, G, H, &c. qui deductus sit ex Serie Bimediali terminorum æquidistantium &c. 4A, 3A, 2A, 1A, A1, A2, A3, A4, &c. quemadmodum in Propositione II. Horum terminorum intervalla secanda sint in partes æquales quotlibetcunque quarum numerus n par sit, interpositis terminis novis quorum numerus erit $n-1$, ad complendam Seriem Medialem &c. $4a$, $3a$, $2a$, $1a$, a, $a1$, $a2$, $a3$, $a4$, &c.



cujus terminus medius a medium habet locum inter 1A & A1 binos medios Seriei prioris, & cujus intervalla minora sunt quam intervalla Seriei prioris in ratione 1 ad n. Ex hac Serie deductæ intelligantur differentiæ, sumendo $1b = 1a - a$, $b1 = a - a1$, $c = 1b - b1$, &c. & si excerpantur $\begin{matrix} & 1b & & 1d & & 1f & & 1h \\ a & & c & & e & & g & \\ & b1 & & d1 & & f1 & & h1 \end{matrix}$ &c. quæ inter omnes medium habent locum, & scribantur b, d, f, h, &c. pro semisummis ipsarum $1b$ & $b1$, $1d$ & $d1$, $1f$ & $f1$, $1h$ & $h1$, &c. erit a, b, c, d, e, f,

g, *h*, &c. ordo ille quadratus Medialis contrarius quem invenire oportet, quemque ſic denotare ſoleo $\boxed{0}$, $\boxed{1}$, $\boxed{2}$, $\boxed{3}$, $\boxed{4}$, $\boxed{5}$, $\boxed{6}$, $\boxed{7}$, &c.

Dividatur primarius ordo A, B, C, D, E, F, G, H, &c. per numeros creſcentes in ratione 1 ad n; & habebitur ordo rotundus $\textcircled{0}$, $\textcircled{1}$, $\textcircled{2}$, $\textcircled{3}$, $\textcircled{4}$, $\textcircled{5}$, $\textcircled{6}$, $\textcircled{7}$, &c. ſumendo nempe $\textcircled{0}=\text{A}$, $\textcircled{1}=\frac{1}{n}\text{B}$, $\textcircled{2}=\frac{1}{nn}\text{C}$, $\textcircled{3}=\frac{1}{n^3}\text{D}$, $\textcircled{4}=\frac{1}{n^4}\text{E}$, $\textcircled{5}=\frac{1}{n^5}\text{F}$, $\textcircled{6}=\frac{1}{n^6}\text{G}$, $\textcircled{7}=\frac{1}{n^7}\text{H}$, &c.

Deinde ex ordine rotundo colligendus eſt ordo quadratus per ſequentia Theoremata in quibus m denotat ſemiſſem numeri paris n. Horum vero continuatio ſimillima eſt continuationi Theorematum in Propoſitionibus ſeptima & octava, nec indiget ulteriore deſcriptione.

$\boxed{0}=\textcircled{0}$ minus

$$\frac{mm}{1\times 2}\boxed{2}$$

$$\frac{m^4-mm}{1\times 2\times 3\times 4}\boxed{4}$$

$$\frac{m^6-5m^4+4mm}{1\times 2\times 3\times 4\times 5\times 6}\boxed{6}$$

$$\frac{m^8-14m^6+49m^4-36mm}{1\times 2\times 3\times 4\times 5\times 6\times 7\times 8}\boxed{8}$$

$$\frac{m^{10}-30m^8+273m^6-820m^4+576mm}{1\times 2\times 3\times 4\times 5\times 6\times 7\times 8\times 9\times 10}\boxed{10}$$

&c.

$\boxed{2} = \textcircled{2}$ minus

$$\frac{10mm - 1}{3 \times 4}\boxed{4}$$

$$\frac{91m^4 - 50mm + 4}{3 \times 4 \times 5 \times 6}\boxed{6}$$

$$\frac{820m^6 - 1274m^4 + 490mm - 36}{3 \times 4 \times 5 \times 6 \times 7 \times 8}\boxed{8}$$

$$\frac{7381m^8 - 24600m^6 + 24843m^4 - 8200mm + 576}{3 \times 4 \times 5 \times 6 \times 7 \times 8 \times 9 \times 10}\boxed{10}$$

&c.

$\boxed{4} = \textcircled{4}$ minus

$$\frac{35mm - 5}{5 \times 6}\boxed{6}$$

$$\frac{966m^4 - 490mm + 49}{5 \times 6 \times 7 \times 8}\boxed{8}$$

$$\frac{24970m^6 - 28980m^4 + 9555mm - 820}{5 \times 6 \times 7 \times 8 \times 9 \times 10}\boxed{10}$$

&c.

$\boxed{6} = \textcircled{6}$ minus

$$\frac{84mm - 14}{7 \times 8}\boxed{8}$$

$$\frac{5082m^4 - 2520mm + 273}{7 \times 8 \times 9 \times 10}\boxed{10}$$

&c.

$\boxed{8} = \textcircled{8}$ minus

$$\frac{165mm - 30}{9 \times 10}\boxed{10}$$

&c.

$\boxed{10} = \textcircled{10}$ minus

&c.

$$\boxed{1} = \bigcirc\!\!\!\!\!1 \text{ minus}$$

$$\frac{mm-1}{2\times3}\,\boxed{3}$$

$$\frac{m^4-5mm+4}{2\times3\times4\times5}\,\boxed{5}$$

$$\frac{m^6-14m^4+49mm-36}{2\times3\times4\times5\times6\times7}\,\boxed{7}$$

$$\frac{m^8-30m^6+273m^4-820mm+576}{2\times3\times4\times5\times6\times7\times8\times9}\,\boxed{9}$$

&c.

$$\boxed{3} = \bigcirc\!\!\!\!\!3 \text{ minus}$$

$$\frac{10mm-5}{4\times5}\,\boxed{5}$$

$$\frac{91m^4-140mm+49}{4\times5\times6\times7}\,\boxed{7}$$

$$\frac{820m^6-2730m^4+2730mm-820}{4\times5\times6\times7\times8\times9}\,\boxed{9}$$

&c.

$$\boxed{5} = \bigcirc\!\!\!\!\!5 \text{ minus}$$

$$\frac{35mm-14}{6\times7}\,\boxed{7}$$

$$\frac{966m^4-1050mm+273}{6\times7\times8\times9}\,\boxed{9}$$

&c.

$$\boxed{7} = \bigcirc\!\!\!\!\!7 \text{ minus}$$

$$\frac{84mm-30}{8\times9}\,\boxed{9}$$

&c.

$$\boxed{9} = \bigcirc\!\!\!\!\!9 \text{ minus}$$

&c.

PROPOSITIO X.

Datis Seriei cujuscunque terminis aliquot æquidistantibus, quorum intervalla secanda sunt in æquales quotlibetcunque partes; propositum sit invenire terminos interserendos.

Si data Series fuerit Medialis, dabitur quadratus differentiarum ordo Medialis per Propositionem septimam. Si data Series fuerit Bimedialis, & Sectio denominata fuerit a pari numero, dabitur quadratus differentiarum ordo Medialis contrarius per Propositionem nonam. In utroque casu sit &c, $4a, 3a, 2a, 1a, a, a1, a2, a3, a4$, &c. Series illa nova Medialis quam invenire oportet: & terminorum a medio loco æquidistantium semisummæ $\frac{1a+a1}{2}$, $\frac{2a+a2}{2}$, $\frac{3a+a3}{2}$, $\frac{4a+a4}{2}$, &c. dabuntur per sequentium Tabularum posteriorem; semidifferentiæ vero $\frac{1a-a1}{2}$, $\frac{2a-a2}{2}$, $\frac{3a-a3}{2}$, $\frac{4a-a4}{2}$, &c. dabuntur per Tabulam priorem.

Quod si Series data fuerit Bimedialis, Sectio autem denominata ab impari numero; dabitur quadratus differentiarum ordo Bimedialis per Propositionem octavam. In hoc ultimo casu sit &c, $4\alpha, 3\alpha, 2\alpha, 1\alpha, \alpha 1, \alpha 2, \alpha 3, \alpha 4$, &c. Series illa nova Bimedialis quam invenire oportet: & terminorum a medio loco æquidistantium semisummæ $\frac{1\alpha+\alpha 1}{2}$, $\frac{2\alpha+\alpha 2}{2}$, $\frac{3\alpha+\alpha 3}{2}$, $\frac{4\alpha+\alpha 4}{2}$, &c. dabuntur per Tabulam priorem; semidifferentiæ vero $\frac{1\alpha-\alpha 1}{2}$, $\frac{2\alpha-\alpha 2}{2}$, $\frac{3\alpha-\alpha 3}{2}$, $\frac{4\alpha-\alpha 4}{2}$, &c. dabuntur per Tabulam posteriorem.

Numeri in transversum locati e regione semisummæ & semidifferentiæ cujusque quæsitæ ducendi sunt respective in differentias quadratas supra positas quæ summum locum obtinent in utraque Tabula, & aggregatum factorum omnium signo + conjunctorum erit semisumma vel semidifferentia quæsita. Ceterum usus Tabularum per Exempla quæ habentur in Propositionibus sex prioribus plenius elucebit. Continuatio numerorum facillima est per additionem solam & primo intuitu patet.

TABULA PRIOR.			0	1	2	3	4	5	6	7	8	9	10
I	Semisum Bimed.	$\frac{1\alpha + \alpha 1}{2}$	1										
	Semi diff. Medial	$\frac{1a - a1}{2}$		1									
II	Semisum Bimed.	$\frac{2\alpha + \alpha 2}{2}$	1		1								
	Semidiff. Medial	$\frac{2a - a2}{2}$		2		1							
III	Semisum Bimed.	$\frac{3\alpha + \alpha 3}{2}$	1		3		1						
	Semidiff. Medial	$\frac{3a - a3}{2}$		3		4		1					
IV	Semisum Bimed.	$\frac{4\alpha + \alpha 4}{2}$	1		6		5		1				
	Semidiff. Medial	$\frac{4a - a4}{2}$		4		10		6		1			
V	Semisum Bimed.	$\frac{5\alpha + \alpha 5}{2}$	1		10		15		7		1		
	Semidiff. Medial	$\frac{5a - a5}{2}$		5		20		21		8		1	
VI	Semisum Bimed.	$\frac{6\alpha + \alpha 6}{2}$	1		15		35		28		9		1
	Semidiff. Medial	$\frac{6a - a6}{2}$		6		35		56		36		10	
VII	Semisum Bimed.	$\frac{7\alpha + \alpha 7}{2}$	1		21		70		84		45		11
	Semidiff. Medial	$\frac{7a - a7}{2}$		7		56		126		120		55	
VIII	Semisum Bimed.	$\frac{8\alpha + \alpha 8}{2}$	1		28		126		210		165		66
	Semidiff. Medial	$\frac{8a - a8}{2}$		8		64		252		330		220	
IX	Semisum Bimed.	$\frac{9\alpha + \alpha 9}{2}$	1		36		210		462		495		286
	Semidiff. Medial	$\frac{9a - a9}{2}$		9		120		462		792		715	
X	Semisum Bimed.	$\frac{10\alpha + \alpha 10}{2}$	1		45		330		924		1287		1001
	Semidiff. Medial	$\frac{10a - a10}{2}$		10		165		792		1716		2002	

TABULA POSTERIOR			0	1	2	3	4	5	6	7	8	9	10
I	Semidiff. Bimed.	$\frac{1\alpha - \alpha 1}{2}$		$\frac{1}{2}$									
	Semisum Medial	$\frac{1a + a1}{2}$	1		$\frac{1}{2}$								
II	Semidiff Bimed	$\frac{2\alpha - \alpha 2}{2}$		$1\frac{1}{2}$		$\frac{1}{2}$							
	Semisum Medial	$\frac{2a + a2}{2}$	1		2		$\frac{1}{2}$						
III	Semidiff. Bimed	$\frac{3\alpha - \alpha 3}{2}$		$2\frac{1}{2}$		$2\frac{1}{2}$		$\frac{1}{2}$					
	Semisum Medial	$\frac{3a + a3}{2}$	1		$4\frac{1}{2}$		3		$\frac{1}{2}$				
IV	Semidiff. Bimed.	$\frac{4\alpha - \alpha 4}{2}$		$3\frac{1}{2}$		7		$3\frac{1}{2}$		$\frac{1}{2}$			
	Semisum Medial	$\frac{4a + a4}{2}$	1		8		10		4		$\frac{1}{2}$		
V	Semidiff Bimed	$\frac{5\alpha - \alpha 5}{2}$		$4\frac{1}{2}$		15		$13\frac{1}{2}$		$4\frac{1}{2}$		$\frac{1}{2}$	
	Semisum Medial	$\frac{5a + a5}{2}$	1		$12\frac{1}{2}$		25		$17\frac{1}{2}$		5		$\frac{1}{2}$
VI	Semidiff. Bimed	$\frac{6\alpha - \alpha 6}{2}$		$5\frac{1}{2}$		$27\frac{1}{2}$		$38\frac{1}{2}$		22		$5\frac{1}{2}$	
	Semisum Medial	$\frac{6a + a6}{2}$	1		18		$52\frac{1}{2}$		56		27		6
VII	Semidiff. Bimed.	$\frac{7\alpha - \alpha 7}{2}$		$6\frac{1}{2}$		$45\frac{1}{2}$		91		78		$32\frac{1}{2}$	
	Semisum Medial	$\frac{7a + a7}{2}$	1		$24\frac{1}{2}$		98		147		105		$38\frac{1}{2}$
VIII	Semidiff. Bimed	$\frac{8\alpha - \alpha 8}{2}$		$7\frac{1}{2}$		70		189		225		$137\frac{1}{2}$	
	Semisum Medial	$\frac{8a + a8}{2}$	1		32		168		336		330		176
IX	Semidiff Bimed.	$\frac{9\alpha - \alpha 9}{2}$		$8\frac{1}{2}$		102		357		561		$467\frac{1}{2}$	
	Semisum Medial.	$\frac{9a + a9}{2}$	1		$40\frac{1}{2}$		270		693		891		$643\frac{1}{2}$
X	Semidiff. Bimed.	$\frac{10\alpha - \alpha 10}{2}$		$9\frac{1}{2}$		$142\frac{1}{2}$		627		1254		$1358\frac{1}{2}$	
	Semisum Medial.	$\frac{10a + a10}{2}$	1		50		$412\frac{1}{2}$		1320		2145		2002

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In construendis Tabulis debet intercalatio terminorum eo usque continuari, si commode fieri poterit, donec eorum differentiæ primæ vel æquales evadant vel proxime saltem æquales, ut ita per partem proportionalem vulgari methodo, inveniri possit vel terminus quilibet intermedius ex dato intervallo quo distat a proxime minore termino Tabulari vel etiam vicissim intervallum ex dato termino.

Si terminus intermedius quæratur, intervallum quo distat a proxime minore termino Tabulari ducendum est in utrinque proximorum Tabularium differentiam primam; & facta erit pars proportionalis, quæ adjecta Tabulari proxime minori dabit intermedium quæsitum. Correctio termini sic inventi per differentiam secundam ad octavam hujus differentiæ partem assurgere quidem potest, ulterius procedere non potest. Efficitur autem ducendo continue semissem differentiæ secundæ in bina intervalla quibus distat quæsitus terminus ab utrinque proximis Tabularibus. Et subducenda est a termino prius invento si termini Tabulares & eorum differentiæ primæ simul crescant vel simul decrescant, aliter addenda est.

Si vicissim quæratur intervallum quo distat terminus intermedius datus a Tabulari proxime minore, dividenda est horum differentia per differentiam primam Tabularium dato utrinque proximorum & habebitur intervallum quæsitum. Intervalli sic inventi correctio per differentiam secundam veræ proxima efficitur ducendo continue semissem differentiæ secundæ in excessum termini intermedii dati supra Tabularem proxime minorem inque defectum ejusdem a Tabulari proxime majore, & dividendo factum per cubum differentiæ primæ. Addenda est autem intervallo prius invento si termini Tabulares & eorum differentiæ primæ simul crescant vel simul descrescant, aliter subducenda est. Notetur in operationibus hic descriptis terminorum Tabularium intervalla supponi esse unitates, & per differentiam secundam intelligi semper semisummam binarum proxime adstantium si forte inæquales provenerint.

Sunt & correctiones ulteriores per differentias tertias, quartas, quintas, &c. sed omitti poterunt cum perraro futuræ sint utiles præsertim si commodissimæ Tabularum formæ probe consulatur.

Invenio utique correctionem intermedii termini per differentiam tertiam nunquam superare differentiæ hujus integræ partes $\frac{\sqrt{3}}{216}$ seu octo millesimas circiter. Libet tamen harum omnium correctionum pro termino intermedio synopsin subjungere non inelegantem & ad infinitum euntem. Sint &c, L, M, N, P, Q, R, &c. termini in Tabula dispositi æqualibus intervallis & sint ista intervalla unitates, quæratur autem terminus Z qui medium aliquem locum habeat inter proxime minorem N & proxime majorem P, distet vero a terminis N, M, L, &c; P, Q, R, &c. intervallis n, m, l, &c; p, q, r, &c; ita ut sit $n+p=1$; atque $m=1+n$, $l=2+n$, &c; $q=1+p$,

&c. L. M. N. P. Q. R. &c.
* * B * *
* C1 1C *
* D *
E1 1E
F

$r=2+p$, &c. sit insuper $o=n-\frac{1}{2}=\frac{1}{2}-m$. Deinde capiantur differentiæ $B=P-N$, $1B=Q-P$, $2B=R-Q$, &c. $B1=N-M$, $B2=M-L$, &c. $2C=2B-1B$, $1C=1B-B$, $C1=B-B1$, $C2=B1-B2$, &c. $1D=2C-1C$, $D=1C-C1$, $D1=C1-C2$, &c. $1E=1D-D$, $E1=D-D1$, &c. $F=1E-E1$, &c. signisque + & − probe observatis & positis C, E, G, &c. pro semisummis ipsarum 1C & C1, 1E & E1, 1G & G1, &c. erit $Z=N+nB-\frac{np}{2}C-\frac{nop}{6}D+\frac{mnpq}{24}E+\frac{mnopq}{120}F-\frac{lmnpqr}{720}G-\frac{lmnopqr}{5040}H++\&c.$

Postscriptum.

Sequuntur Theoremata quædam ad Solutionem sex priorum Propositionum per differentiarum ordines rotundos, ne forte desiderari videantur. Differentiarum rotundarum inventio supra docetur in Propositionibus ad quas ipsa spectant Theoremata. Theoremata vero inveniri possunt & quousque commodum videbitur continuari per tertiam Propositionem Tractatus a Cel. *Newtono* conscripti & nuper editi de Methodo Differentiali, si modo Propositio illa prius debite concinnetur.

Theore-

Theoremata ad Solutionem Propositionis Primæ.

I. semisum. $\frac{1a + a1}{2} = \textcircled{0} + \frac{1}{2}\textcircled{2} - \frac{1}{8}\textcircled{4} + \frac{1}{16}\textcircled{6} - \frac{5}{128}\textcircled{8} + \frac{7}{256}\textcircled{10}$ − &c.
semidiff. $\frac{1a - a1}{2} = \textcircled{1} - \frac{1}{2}\textcircled{3} + \frac{3}{8}\textcircled{5} - \frac{5}{16}\textcircled{7} + \frac{35}{128}\textcircled{9}$ − &c.

III. semisum. $\frac{3a + a3}{2} = \textcircled{0} + \frac{9}{2}\textcircled{2} + \frac{15}{8}\textcircled{4} - \frac{7}{16}\textcircled{6} + \frac{27}{128}\textcircled{8} - \frac{33}{256}\textcircled{10}$ + &c.
semidiff. $\frac{3a - a3}{2} = 3\textcircled{1} + \frac{5}{2}\textcircled{3} - \frac{7}{8}\textcircled{5} + \frac{9}{16}\textcircled{7} - \frac{55}{128}\textcircled{9}$ + &c.

V. semisum. $\frac{5a + a5}{2} = \textcircled{0} + \frac{25}{2}\textcircled{2} + \frac{175}{8}\textcircled{4} + \frac{105}{16}\textcircled{6} - \frac{165}{128}\textcircled{8} + \frac{143}{256}\textcircled{10}$ − &c.
semidiff. $\frac{5a - a5}{2} = 5\textcircled{1} + \frac{35}{2}\textcircled{3} + \frac{63}{8}\textcircled{5} - \frac{33}{16}\textcircled{7} + \frac{143}{128}\textcircled{9}$ − &c.

VII. semisum. $\frac{7a + a7}{2} = \textcircled{0} + \frac{49}{2}\textcircled{2} + \frac{735}{8}\textcircled{4} + \frac{1617}{16}\textcircled{6} + \frac{3003}{128}\textcircled{8} - \frac{1001}{256}\textcircled{10}$ + &c.
semidiff. $\frac{7a - a7}{2} = 7\textcircled{1} + \frac{105}{2}\textcircled{3} + \frac{693}{8}\textcircled{5} + \frac{429}{16}\textcircled{7} - \frac{715}{128}\textcircled{9}$ + &c.

IX. semisum. $\frac{9a + a9}{2} = \textcircled{0} + \frac{81}{2}\textcircled{4} + \frac{2072}{8}\textcircled{4} + \frac{9009}{16}\textcircled{6} + \frac{57915}{128}\textcircled{8} + \frac{21879}{256}\textcircled{10}$ − &c.
semidiff. $\frac{9a - a9}{2} = 9\textcircled{1} + \frac{231}{2}\textcircled{3} + \frac{3003}{8}\textcircled{5} + \frac{6435}{16}\textcircled{7} + \frac{12155}{128}\textcircled{9}$ − &c.
&c.

Theoremata ad Solutionem Propositionis Secundæ.

Terminus medius $a = \textcircled{0} - \frac{1}{2}\textcircled{2} + \frac{3}{8}\textcircled{4} - \frac{5}{16}\textcircled{6} + \frac{35}{128}\textcircled{8} - \frac{63}{256}\textcircled{10}$ + &c.

II. semisum. $\frac{2a + a2}{2} = \textcircled{0} + \frac{3}{2}\textcircled{2} - \frac{5}{8}\textcircled{4} + \frac{7}{16}\textcircled{6} - \frac{45}{128}\textcircled{8} + \frac{77}{256}\textcircled{10}$ − &c.
semidiff. $\frac{2a - a2}{2} = 2\textcircled{1} + \textcircled{3} - \frac{1}{4}\textcircled{5} + \frac{1}{8}\textcircled{7} - \frac{5}{64}\textcircled{9}$ + &c.

IV. semisum. $\frac{4a + a4}{2} = \textcircled{0} + \frac{15}{2}\textcircled{2} + \frac{35}{8}\textcircled{4} - \frac{21}{16}\textcircled{6} + \frac{99}{128}\textcircled{8} - \frac{143}{256}\textcircled{10}$ + &c.
semidiff. $\frac{4a - a4}{2} = 4\textcircled{1} + 10\textcircled{3} + \frac{7}{2}\textcircled{5} - \frac{3}{4}\textcircled{7} + \frac{11}{32}\textcircled{9}$ − &c.

VI. semisum. $\frac{6a + a6}{2} = \textcircled{0} + \frac{35}{2}\textcircled{2} + \frac{315}{8}\textcircled{4} + \frac{231}{16}\textcircled{6} - \frac{429}{128}\textcircled{8} + \frac{429}{256}\textcircled{10}$ − &c.
semidiff. $\frac{6a - a6}{2} = 6\textcircled{1} + 35\textcircled{3} + \frac{189}{4}\textcircled{5} + \frac{99}{8}\textcircled{7} - \frac{143}{64}\textcircled{9}$ + &c.
&c.

VIII. semisum. $\frac{8a+a8}{2} = \textcircled{0} + \frac{63}{2}\textcircled{2} + \frac{1155}{8}\textcircled{4} + \frac{3003}{16}\textcircled{6} + \frac{6435}{128}\textcircled{8} - \frac{2431}{256}\textcircled{10} +$ &c.

semidiff. $\frac{8a-a8}{2} = 8\textcircled{1} + 84\textcircled{3} + 231\textcircled{5} + \frac{429}{2}\textcircled{7} + \frac{715}{16}\textcircled{9} -$ &c.

X. semisum, $\frac{10a+a10}{2} = \textcircled{0} + \frac{99}{2}\textcircled{2} + \frac{3003}{8}\textcircled{4} + \frac{15015}{16}\textcircled{6} + \frac{109395}{128}\textcircled{8} + \frac{46189}{256}\textcircled{10} -$ &c.

semidiff. $\frac{10a-a10}{2} = 10\textcircled{1} + 165\textcircled{3} + \frac{3003}{4}\textcircled{5} + \frac{10725}{8}\textcircled{7} + \frac{60775}{64}\textcircled{9} +$ &c.

&c.

Theoremata ad Solutionem Propositionis Tertiæ.

I. semisum. $\frac{1a+a1}{2} = \textcircled{0} + \frac{1}{2}\textcircled{2} - \frac{1}{3}\textcircled{4} + \frac{7}{18}\textcircled{6} - \frac{5}{9}\textcircled{8} + \frac{143}{162}\textcircled{10} -$ &c.

semidiff. $\frac{1a-a1}{2} = \textcircled{1} - \frac{4}{3}\textcircled{3} + \frac{7}{3}\textcircled{5} - \frac{40}{9}\textcircled{7} + \frac{715}{81}\textcircled{9} -$ &c.

II. semisum, $\frac{2a+a2}{2} = \textcircled{0} + 2\textcircled{2} - \frac{5}{6}\textcircled{4} + \frac{8}{9}\textcircled{6} - \frac{11}{9}\textcircled{8} + \frac{154}{81}\textcircled{10} -$ &c.

semidiff. $\frac{2a-a2}{2} = 2\textcircled{1} - \frac{5}{3}\textcircled{3} + \frac{8}{3}\textcircled{5} - \frac{44}{9}\textcircled{7} + \frac{770}{81}\textcircled{9} -$ &c.

IV. semisum. $\frac{4a+a4}{2} = \textcircled{0} + 8\textcircled{2} + \frac{14}{3}\textcircled{4} - \frac{28}{9}\textcircled{6} + \frac{65}{18}\textcircled{8} - \frac{416}{81}\textcircled{10} +$ &c.

semidiff. $\frac{4a-a4}{2} = 4\textcircled{1} + \frac{14}{3}\textcircled{3} - \frac{14}{3}\textcircled{5} + \frac{65}{9}\textcircled{7} - \frac{1040}{81}\textcircled{9} +$ &c.

V. semisum. $\frac{5a+a5}{2} = \textcircled{0} + \frac{25}{2}\textcircled{2} + \frac{50}{3}\textcircled{4} - \frac{55}{9}\textcircled{6} + \frac{55}{9}\textcircled{8} - \frac{1309}{162}\textcircled{10} +$ &c.

semidiff. $\frac{5a-a5}{2} = 5\textcircled{1} + \frac{40}{3}\textcircled{3} - \frac{22}{3}\textcircled{5} + \frac{88}{9}\textcircled{7} - \frac{1309}{81}\textcircled{9} +$ &c.

VII. semisum. $\frac{7a+a7}{2} = \textcircled{0} + \frac{49}{2}\textcircled{2} + \frac{245}{3}\textcircled{4} + \frac{637}{18}\textcircled{6} - \frac{182}{9}\textcircled{8} + \frac{1729}{81}\textcircled{10} -$ &c.

semidiff. $\frac{7a-a7}{2} = 7\textcircled{1} + \frac{140}{3}\textcircled{3} + \frac{91}{3}\textcircled{5} - \frac{208}{9}\textcircled{7} + \frac{2470}{81}\textcircled{9} -$ &c.

VIII. semisum. $\frac{8a+a8}{2} = \textcircled{0} + 32\textcircled{2} + \frac{440}{3}\textcircled{4} + \frac{1232}{9}\textcircled{6} - \frac{374}{9}\textcircled{8} + \frac{2992}{81}\textcircled{10} -$ &c.

semidiff. $\frac{8a-a8}{2} = 8\textcircled{1} + \frac{220}{3}\textcircled{3} + \frac{308}{3}\textcircled{5} - \frac{374}{9}\textcircled{7} + \frac{3740}{81}\textcircled{9} -$ &c.

&c.

Theore-

Theoremata ad Solutionem Propoſitionis Quartæ.

I. ſemiſum. $\frac{1\alpha + \alpha 1}{2} = \textcircled{0} - \textcircled{2} + \frac{5}{3}\textcircled{4} - \frac{28}{9}\textcircled{6} + \frac{55}{9}\textcircled{8} - \frac{1001}{81}\textcircled{10} + \&c.$
ſemidiff. $\frac{1\alpha - \alpha 1}{2} = \frac{1}{2}\textcircled{1} - \frac{1}{6}\textcircled{3} + \frac{1}{6}\textcircled{5} - \frac{2}{9}\textcircled{7} + \frac{55}{162}\textcircled{9} - \&c.$

III. ſemiſum. $\frac{3\alpha + \alpha 3}{2} = \textcircled{0} + 2\textcircled{2} - \frac{7}{3}\textcircled{4} + \frac{35}{9}\textcircled{6} - \frac{65}{9}\textcircled{8} + \frac{1144}{81}\textcircled{10} - \&c.$
ſemidiff. $\frac{3\alpha - \alpha 3}{2} = \frac{5}{2}\textcircled{1} + \frac{5}{3}\textcircled{3} - \frac{7}{6}\textcircled{5} + \frac{25}{18}\textcircled{7} - \frac{325}{162}\textcircled{9} + \&c.$

IV. ſemiſum. $\frac{4\alpha + \alpha 4}{2} = \textcircled{0} + 5\textcircled{2} - \frac{10}{3}\textcircled{4} + \frac{44}{9}\textcircled{6} - \frac{77}{9}\textcircled{8} + \frac{1309}{81}\textcircled{10} - \&c.$
ſemidiff. $\frac{4\alpha - \alpha 4}{2} = \frac{7}{2}\textcircled{1} + \frac{35}{6}\textcircled{3} - \frac{7}{3}\textcircled{5} + \frac{22}{9}\textcircled{7} - \frac{539}{162}\textcircled{9} + \&c.$

VI. ſemiſum. $\frac{6\alpha + \alpha 6}{2} = \textcircled{0} + 14\textcircled{2} + \frac{35}{3}\textcircled{4} - \frac{91}{9}\textcircled{6} + \frac{130}{9}\textcircled{8} - \frac{1976}{81}\textcircled{10} + \&c.$
ſemidiff. $\frac{6\alpha - \alpha 6}{2} = \frac{11}{2}\textcircled{1} + \frac{77}{9}\textcircled{3} + \frac{77}{6}\textcircled{5} - \frac{143}{18}\textcircled{7} + \frac{715}{81}\textcircled{9} - \&c.$

VII. ſemiſum. $\frac{7\alpha + \alpha 7}{2} = \textcircled{0} + 20\textcircled{2} + \frac{110}{3}\textcircled{4} - \frac{154}{9}\textcircled{6} + \frac{187}{9}\textcircled{8} - \frac{2618}{81}\textcircled{10} + \&c.$
ſemidiff. $\frac{7\alpha - \alpha 7}{2} = \frac{13}{2}\textcircled{1} + \frac{130}{3}\textcircled{3} + \frac{143}{3}\textcircled{5} - \frac{143}{9}\textcircled{7} + \frac{2431}{162}\textcircled{9} - \&c.$

IX. ſemiſum. $\frac{9\alpha + \alpha 9}{2} = \textcircled{0} + 35\textcircled{2} + \frac{455}{3}\textcircled{4} + \frac{728}{9}\textcircled{6} - \frac{494}{9}\textcircled{8} + \frac{5434}{81}\textcircled{10} - \&c.$
ſemidiff. $\frac{9\alpha - \alpha 9}{2} = \frac{17}{2}\textcircled{1} + \frac{595}{6}\textcircled{3} + \frac{1547}{6}\textcircled{5} + \frac{884}{9}\textcircled{7} - \frac{4199}{81}\textcircled{9} + \&c.$

&c.

Theoremata ad Solutionem Propoſitionis Quintæ.

I. ſemiſum. $\frac{1a + a1}{2} = \textcircled{0} + \frac{1}{2}\textcircled{2} - \textcircled{4} + \frac{33}{10}\textcircled{6} - \frac{66}{5}\textcircled{8} + \frac{1463}{25}\textcircled{10} - \&c.$
ſemidiff. $\frac{1a - a1}{2} = \textcircled{1} - 4\textcircled{3} + \frac{99}{5}\textcircled{5} - \frac{528}{5}\textcircled{7} + \frac{2926}{5}\textcircled{9} - \&c.$

II. ſemiſum. $\frac{2a + a2}{2} = \textcircled{0} + 2\textcircled{2} - \frac{7}{2}\textcircled{4} + \frac{56}{5}\textcircled{6} - \frac{221}{5}\textcircled{8} + \frac{4862}{25}\textcircled{10} - \&c.$
ſemidiff. $\frac{2a - a2}{2} = 2\textcircled{1} - 7\textcircled{3} + \frac{168}{5}\textcircled{5} + \frac{884}{5}\textcircled{7} - \frac{4862}{5}\textcircled{9} - \&c.$

III. $\begin{cases} \text{semisum. } \frac{3a+a3}{2} = \textcircled{0} + \frac{9}{2}\textcircled{2} - 6\textcircled{4} + \frac{93}{5}\textcircled{6} - \frac{351}{5}\textcircled{8} + \frac{15249}{50}\textcircled{10} - \&c. \\ \text{semidiff. } \frac{3a-a3}{2} = 3\textcircled{1} - 8\textcircled{3} + \frac{182}{5}\textcircled{5} - \frac{936}{5}\textcircled{7} + \frac{5083}{5}\textcircled{9} - \&c. \end{cases}$

IV. $\begin{cases} \text{semisum. } \frac{4a+a4}{2} = \textcircled{0} + 8\textcircled{2} - 6\textcircled{4} + \frac{84}{5}\textcircled{6} - \frac{627}{10}\textcircled{8} + \frac{6688}{25}\textcircled{10} - \&c. \\ \text{semidiff. } \frac{4a-a4}{2} = 4\textcircled{1} - 6\textcircled{3} + \frac{126}{3}\textcircled{5} - \frac{627}{5}\textcircled{7} + \frac{3344}{5}\textcircled{9} - \&c. \end{cases}$

VI. $\begin{cases} \text{semisum. } \frac{6a+a6}{2} = \textcircled{0} + 18\textcircled{2} + \frac{33}{2}\textcircled{4} - \frac{176}{5}\textcircled{6} + \frac{594}{5}\textcircled{8} - \frac{12012}{25}\textcircled{10} + \&c. \\ \text{semidiff. } \frac{6a-a6}{2} = 6\textcircled{1} + 11\textcircled{3} - \frac{176}{5}\textcircled{5} + \frac{792}{5}\textcircled{7} - \frac{4004}{5}\textcircled{9} + \&c. \end{cases}$

VII. $\begin{cases} \text{semisum. } \frac{7a+a7}{2} = \textcircled{0} + \frac{49}{2}\textcircled{2} + 49\textcircled{4} - \frac{833}{10}\textcircled{6} + \frac{1309}{5}\textcircled{8} - \frac{51051}{50}\textcircled{10} + \&c. \\ \text{semidiff. } \frac{7a-a7}{2} = 7\textcircled{1} + 28\textcircled{3} - \frac{357}{5}\textcircled{5} + \frac{1496}{5}\textcircled{7} - \frac{7293}{5}\textcircled{9} + \&c. \end{cases}$

VIII. $\begin{cases} \text{semisum. } \frac{8a+a8}{2} = \textcircled{0} + 32\textcircled{2} + 104\textcircled{4} - \frac{624}{5}\textcircled{6} + \frac{1794}{5}\textcircled{8} - \frac{33488}{25}\textcircled{10} + \&c. \\ \text{semidiff. } \frac{8a-a8}{2} = 8\textcircled{1} + 52\textcircled{3} - \frac{468}{5}\textcircled{5} + \frac{1794}{5}\textcircled{7} - \frac{8372}{5}\textcircled{9} + \&c. \end{cases}$

IX. $\begin{cases} \text{semisum. } \frac{9a+a9}{2} = \textcircled{0} + \frac{81}{2}\textcircled{2} + 189\textcircled{4} - \frac{11907}{10}\textcircled{6} + \frac{15399}{5}\textcircled{8} - \frac{54549}{50}\textcircled{10} + \&c. \\ \text{semidiff. } \frac{9a-a9}{2} = 9\textcircled{1} + 84\textcircled{3} - \frac{399}{5}\textcircled{5} + \frac{1368}{5}\textcircled{7} - \frac{6061}{5}\textcircled{9} + \&c. \end{cases}$

&c.

Theoremata ad Solutionem Propositionis Sextæ.

I. $\begin{cases} \text{semisum. } \frac{1\alpha+\alpha 1}{2} = \textcircled{0} - 3\textcircled{2} + 14\textcircled{4} - \frac{364}{5}\textcircled{6} + \frac{1989}{5}\textcircled{8} - \frac{55913}{25}\textcircled{10} + \&c. \\ \text{semidiff. } \frac{1\alpha-\alpha 1}{2} = \frac{1}{2}\textcircled{1} - \frac{1}{2}\textcircled{3} + \frac{7}{5}\textcircled{5} - \frac{26}{5}\textcircled{7} + \frac{221}{10}\textcircled{9} - \&c. \end{cases}$

II. $\begin{cases} \text{semisum. } \frac{2\alpha+\alpha 2}{2} = \textcircled{0} - 2\textcircled{2} + 9\textcircled{4} - \frac{231}{5}\textcircled{6} + \frac{1254}{5}\textcircled{8} - \frac{35112}{25}\textcircled{10} + \&c. \\ \text{semidiff. } \frac{2\alpha-\alpha 2}{2} = \frac{3}{2}\textcircled{1} - \textcircled{3} + \frac{27}{10}\textcircled{5} - \frac{99}{10}\textcircled{7} + \frac{209}{5}\textcircled{9} - \&c. \end{cases}$

IV. ſemiſum. $\frac{4\alpha + \alpha 4}{2} = \textcircled{0} + 3\textcircled{2} - 11\textcircled{4} + \frac{264}{5}\textcircled{6} - \frac{1386}{5}\textcircled{8} + \frac{38038}{25}\textcircled{10} - \&c.$

ſemidiff. $\frac{4\alpha - \alpha 4}{2} = \frac{7}{2}\textcircled{1} + \frac{7}{2}\textcircled{3} - \frac{77}{10}\textcircled{5} + \frac{132}{5}\textcircled{7} - \frac{539}{5}\textcircled{9} + \&c.$

V. ſemiſum. $\frac{5\alpha + \alpha 5}{2} = \textcircled{0} + 7\textcircled{2} - 21\textcircled{4} + \frac{476}{5}\textcircled{6} - \frac{2431}{5}\textcircled{8} + \frac{65637}{25}\textcircled{10} - \&c.$

ſemidiff. $\frac{5\alpha - \alpha 5}{2} = \frac{9}{2}\textcircled{1} + \frac{21}{2}\textcircled{3} - \frac{189}{10}\textcircled{5} + \frac{306}{5}\textcircled{7} - \frac{2431}{10}\textcircled{9} + \&c.$

VI. ſemiſum. $\frac{6\alpha + \alpha 6}{2} = \textcircled{0} + 12\textcircled{2} - 26\textcircled{4} + \frac{546}{5}\textcircled{6} - \frac{2691}{5}\textcircled{8} + \frac{71162}{25}\textcircled{10} - \&c.$

ſemidiff. $\frac{6\alpha - \alpha 6}{2} = \frac{11}{2}\textcircled{1} + 22\textcircled{3} - \frac{143}{5}\textcircled{5} + \frac{429}{5}\textcircled{7} - \frac{3289}{10}\textcircled{9} + \&c.$

VII. ſemiſum. $\frac{7\alpha + \alpha 7}{2} = \textcircled{0} + 18\textcircled{2} - 21\textcircled{4} + \frac{399}{5}\textcircled{6} - \frac{1881}{5}\textcircled{8} + \frac{48488}{25}\textcircled{10} - \&c.$

ſemidiff. $\frac{7\alpha - \alpha 7}{2} = \frac{13}{2}\textcircled{1} + 39\textcircled{3} - \frac{273}{10}\textcircled{5} + \frac{741}{10}\textcircled{7} - \frac{2717}{10}\textcircled{9} + \&c.$

IX. ſemiſum. $\frac{9\alpha + \alpha 9}{2} = \textcircled{0} + 33\textcircled{2} + 44\textcircled{4} - \frac{616}{5}\textcircled{6} + \frac{2574}{5}\textcircled{8} - \frac{62062}{25}\textcircled{10} + \&c.$

ſemidiff. $\frac{9\alpha - \alpha 9}{2} = \frac{17}{2}\textcircled{1} + \frac{187}{2}\textcircled{3} + \frac{374}{5}\textcircled{5} - \frac{748}{5}\textcircled{7} + \frac{2431}{5}\textcircled{9} - \&c.$

X. ſemiſum. $\frac{10\alpha + \alpha 10}{2} = \textcircled{0} + 42\textcircled{2} + 119\textcircled{4} - \frac{1309}{5}\textcircled{6} + \frac{5049}{5}\textcircled{8} - \frac{116688}{25}\textcircled{10} + \&c.$

ſemidiff. $\frac{10\alpha - \alpha 10}{2} = \frac{19}{2}\textcircled{1} + 133\textcircled{3} + \frac{2261}{10}\textcircled{5} - \frac{3553}{10}\textcircled{7} + \frac{10659}{10}\textcircled{9} - \&c.$

&c.

DE

DESCENSU GRAVIUM.

DE

MOTU PENDULORUM

IN

CYCLOIDE.

DE

MOTU PROJECTILIUM.

AUCTORE

ROGERO COTES.

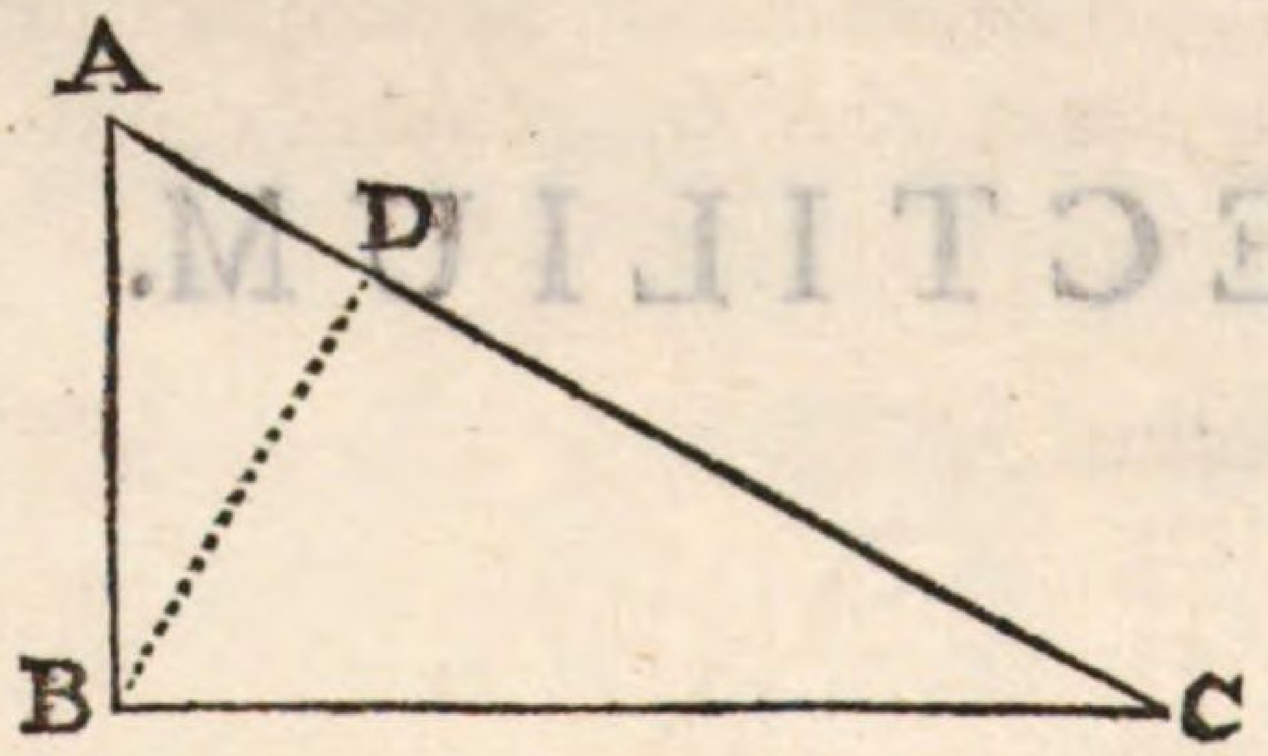
t

DE
DESCENSU GRAVIUM.

LEMMA.

Vis acceleratrix, qua grave corpus per planum quodvis inclinatum oblique descendens urgetur, est ut Elevatio plani illius directe & Longitudo ejusdem inverse.

SIT enim *AC* Longitudo plani utcunque inclinati ad horizontale planum *BC*, sitque *AB* ejusdem plani inclinati Elevatio seu perpendicularis altitudo supra *BC*. Jam si uniformis & data vis Gravitatis qua corpus recta deorsum tendit, exponatur per *AB*, & hæc resolvatur in binas vires *AD* & *DB*, ducendo *BD* perpendicularem ad *AC*: patebit harum solam *AD* eam esse qua corpus oblique descendens acceleratur, alteram vero *DB* per contrariam plani renitentiam omnino tolli. Est autem *AD* ad *AB* ut *AB* ad *AC*: adeoque vis acceleratrix ad vim gravitatis absolutam erit ut *AB* ad *AC*. Igitur cum data sit vis gravitatis; erit vis acceleratrix directe ut *AB* Elevatio plani inclinati & inverse ut *AC* Longitudo ejusdem. *Q.E.D.*



PROBLEMA.

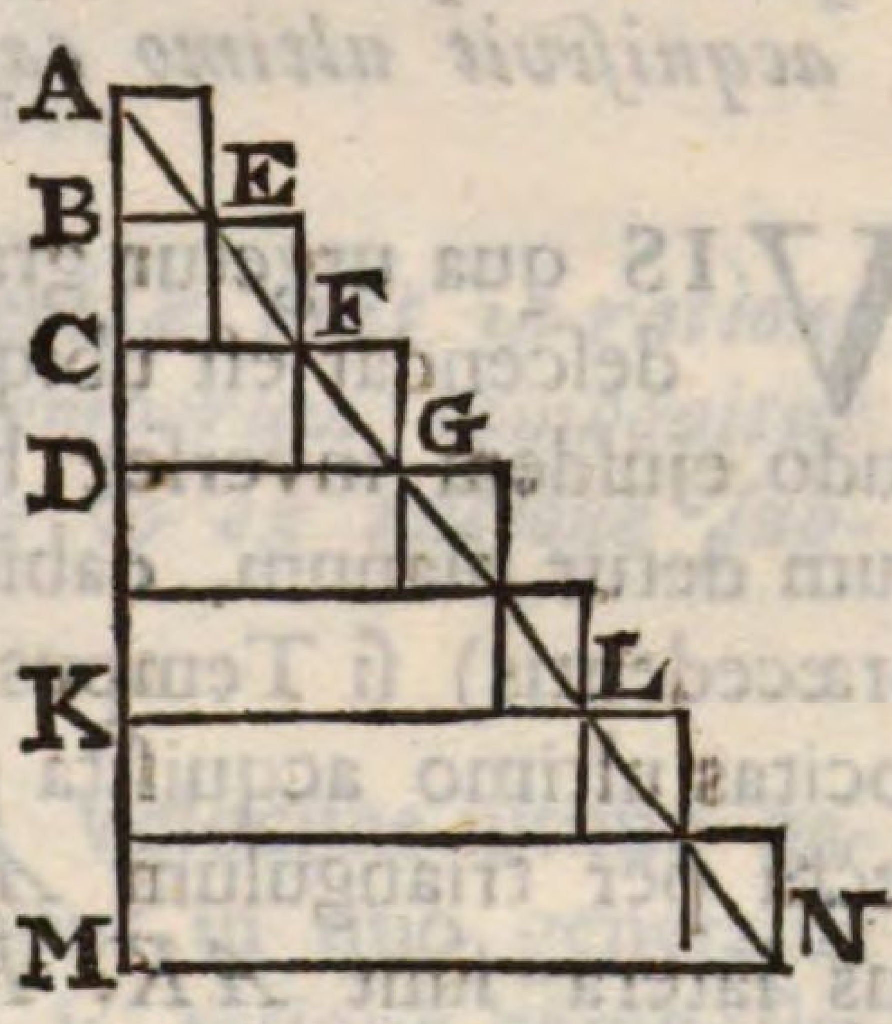
Motus rectilinei per æquales impulsus continuo propagati, in Medio non resistente, affectiones explicare.

IN Motu æquabili si detur Tempus, erit Longitudo spatii peracti ut Velocitas; si detur Velocitas, erit Longitudo ut Tempus: adeoque neutro dato, ut Velocitas & Tempus conjunctim. Commode ergo exponitur per Parallelogrammum rectangulum, cujus

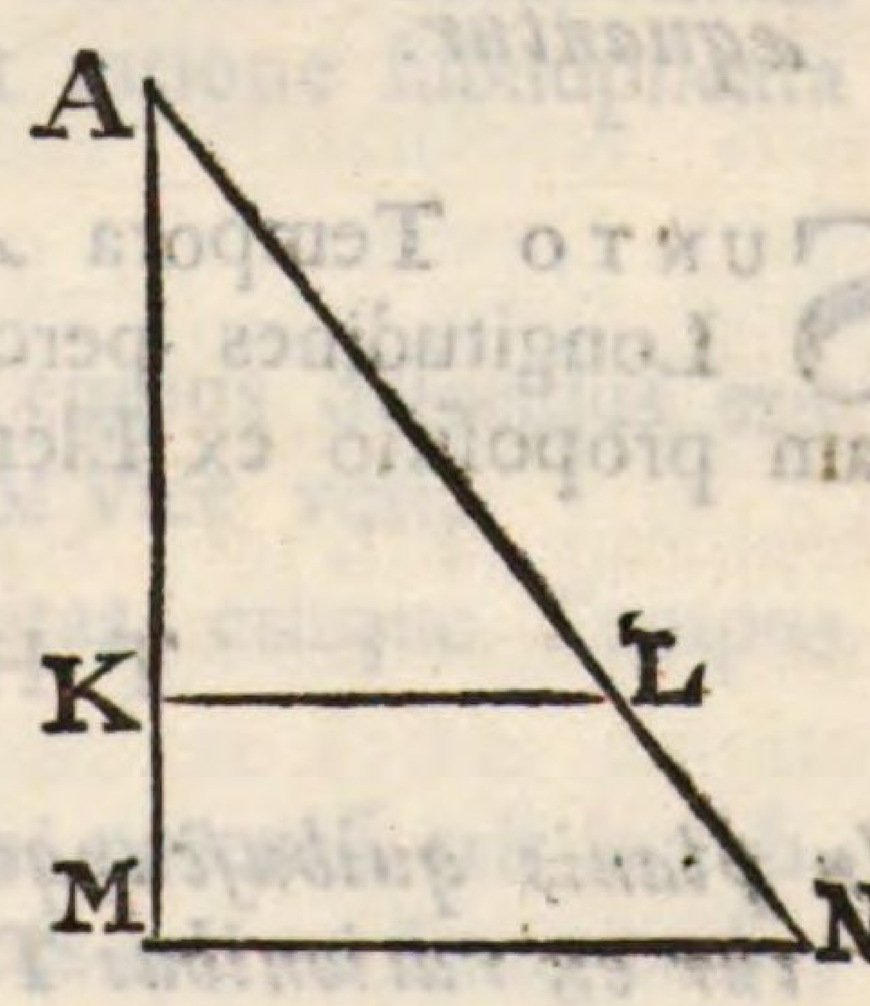
latera

latera Velocitatem & Tempus rite retulerint: Nam & hujus quoque ratio è rationibus laterum ſuorum componitur.

Dividatur jam recta *AM* in partes qualeſcunque æquales *AB*, *BC*, &c. Temporis partes æquales referentes, in quarum initiis agat vis eadem quælibetcunque ſucceſſive unico ſuo impulſu; faciatque ut mobile in primo tempore percurrat Spatium quale eſt *AE* rectangulum, huic æquale *CE* ſecundo tempore confecturum ſimulque (propter iteratum impulſum) huic aliud æquale *EF*; ita ut totum ſecundi temporis Spatium *BF* duplum ſit primi *AE*, totum tertii temporis Spatium *CG* triplum primi *AE*, atque ita deinceps. Velocitates etiam in ſingulis temporibus eodem modo accreſcent. Secunda *CF* dupla erit primæ *BE*, tertia *DG* tripla ejuſdem, &c. Quod ſi Spatiorum in temporibus integris *AK*, *AM* decurſorum exquiratur ratio, erunt hæc inter ſe ut areæ anguloſæ *AKL*, *AMN*; ultimæque Velocitates ut rectæ *KL*, *MN*.



Augeatur jam numerus & minuatur latitudo parallelogrammorum ad infinitum, ut ita actione continua vis impellentis progrediatur Mobile; & abibunt Figuræ anguloſæ è parallelogrammulis ſuis conflatæ in Triangula *AKL*, *AMN*. Itaque Spatia deſcripta in Temporibus *AK*, *AM* à Mobile æquabiliter accelerato, ſunt ut Triangula *AKL*, *AMN*, & Velocitates in iſtis Temporibus ultimo acquiſitæ, ſunt ut rectæ *KL*, *MN*. *Q.E.E.*



Corol. Patet ſi Vires aliæ atque aliæ idem Mobile eodem modo impellant, Velocitates in Temporibus quibuſvis genitas, eſſe ut Vires generantes & Tempora in quibus generantur conjunctim.

THEO-

THEOREMA I.

Longitudo peracta in dato Tempore à gravi è quiete casum inchoante, per quodcunque planum datum, dimidia est ejus quam pari tempore transiret motu æquabili cum Velocitate quam acquisivit ultimo casus momento.

VIS qua urgetur grave continuo impulsu ut per planum quodvis descendat est ubique ut Elevatio plani illius directe & Longitudo ejusdem inverse: hoc per Lemma præmissum constat. Itaque cum detur planum, dabitur Vis. Unde (per Solutionem Problematis præcedentis) si Tempus descensus exponatur per rectam *AK* & Velocitas ultimo acquisita per rectam *KL*, exponetur Longitudo confecta per triangulum *AKL*, dimidium scilicet parallelogrammi cujus latera sunt *AK*, *KL*, per quod utique exponitur Longitudo quæ tempore eodem *AK* describitur à mobili æquabiliter lato cum velocitate ultima *KL*. *Q.E.D.*

THEOREMA II.

In dato plano, ratio integra tum Temporis à quiete, tum Velocitatis genitæ & subduplicata ratio Longitudinis descriptæ æquantur.

SUNTO Tempora *AK*, *AM*; Velocitates genitæ *KL*, *MN*; Longitudines percursæ *AKL*, *AMN*, uti supra: & patet jam propositio ex Elementis. *Q.E.D.*

THEOREMA III.

In planis quibuscunque ratio Longitudinum emensarum componitur ex rationibus Temporum & Velocitatum ultimarum; ratio Temporum ex rationibus Longitudinum directe & Velocitatum inverse; ratio Velocitatum ex rationibus Longitudinum directe & Temporum inverse.

EXPONUNTUR, uti supra, Longitudines, Tempora & Velocitates per triangula eorumque latera bina circa datum angulum posita: constat itaque propositio ex Elementis. *Q.E.D.*

Corol. I.

Corol. 1. Si detur Longitudo, reciprocat ratio Temporis & Velocitatis; & vice versa.

Corol. 2. Si detur Tempus, erit Longitudo ut Velocitas; & vice versa.

Corol. 3. Si detur Velocitas, erit Longitudo ut Tempus; & vice versa.

THEOREMA IV.

In planis quibuscunque Velocitates ultimo genitæ sunt in ratione subduplicata Elevationum: Ratioque Temporum componitur ex ratione simplici Longitudinum directe & ex ratione subduplicata Elevationum inverse.

PER Corollarium Problematis præcedentis, Velocitas ultimo genita est ut Vis eam generans & ut Tempus in quo generatur conjunctim, hoc est (per Lemma præmissum) in ratione composita Elevationis plani directe, Longitudinis ejusdem inverse, & Temporis directe. Erit itaque Velocitas (per Theorema III.) ut Elevatio plani directe utque ipsa Velocitas inverse; atque adeo in ratione subduplicata Elevationis. *Q. E. D.*

Tempus autem, cum sit per Theorema III, ut Longitudo percursa directe & Velocitas inverse; rationem habebit compositam ex ratione simplici Longitudinis directe & ex ratione subduplicata Elevationis inverse. *Q. E. D.*

Corol. 1. Si detur Longitudo percursa; Tempus descensus erit reciproce in ratione subduplicata Elevationis; & vice versa.

Corol. 2. Si detur Elevatio; dabitur Velocitas, eritque Tempus directe ut Longitudo plani; & vice versa.

Corol. 3. Si detur Tempus; Longitudo plani percursi erit in subduplicata ratione Elevationis ejusdem, & Velocitas erit ut Longitudo; & vice versa.

Inde in Circulo, si diameter quævis ad horizontem statuatur normalis erunt Tempora descensus per Chordas quaslibet ab extremitate hujus ductas æqualia, & Velocitates ultimæ erunt ut ipsæ Chordæ: sunt enim Chordarum longitudines ad invicem in subduplicata ratione suarum elevationum.

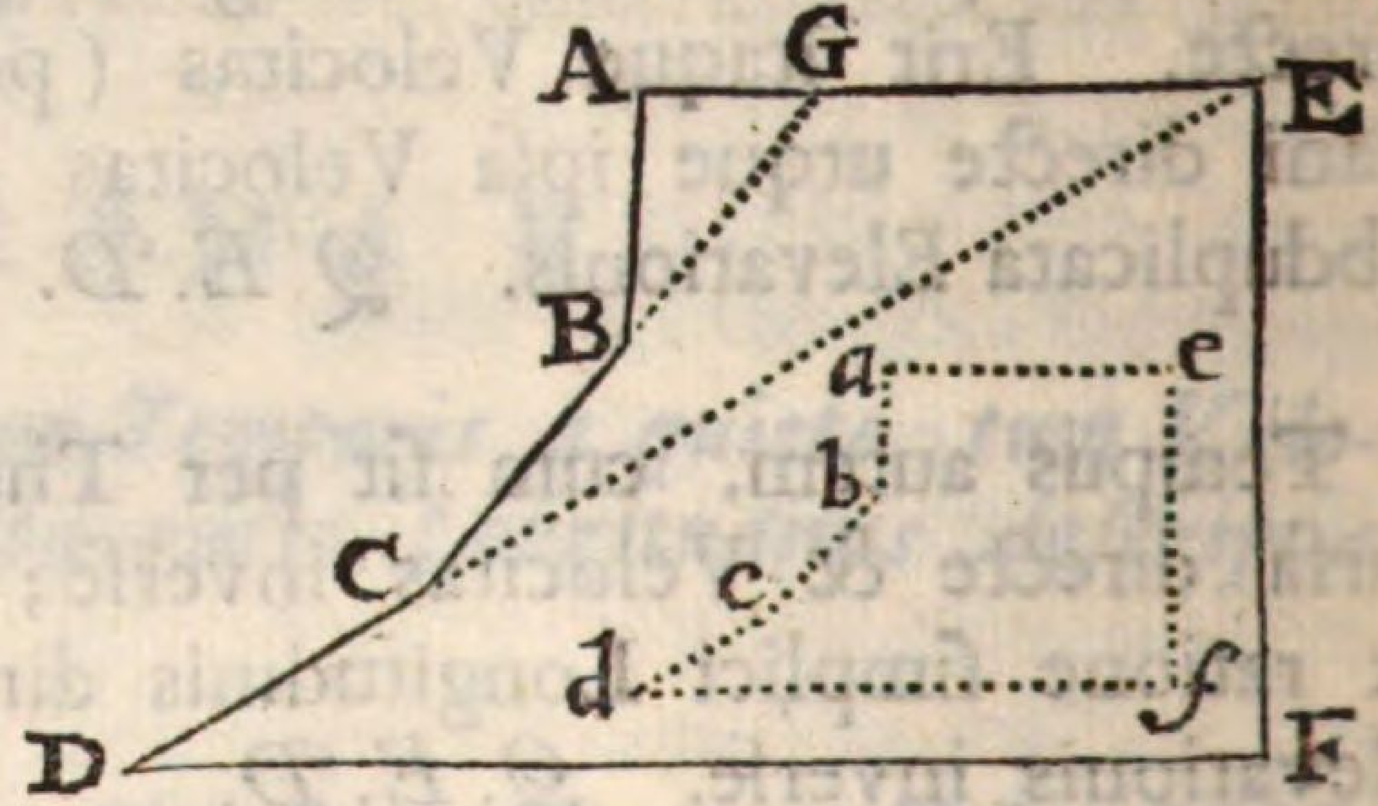
THEOREMA V.

Si ex altitudine eadem descendat mobile continuato motu per quotlibet ac quælibet plana contigua, utcunque inclinata; semper eandem in fine velocitatem acquiret, quæ nimirum æqualis erit ei quam acquireret cadendo perpendiculariter ex pari altitudine.

Hugenii Demonstratio.

SINT plana contigua *AB*, *BC*, *CD*; quorum terminus *A*, supra horizontalem lineam *DF* per infimum terminum *D* ductam, altitudinem habeat quanta est perpendicularis *EF*: descendatque mobile per plana illa ab *A* usque in *D*. Dico in *D* eam velocitatem habiturum quam, ex *E* cadens, haberet in *F*.

Producta enim *CB* occurrat rectæ *AE* in G. Itemque *DC* producta occurrat eidem *AE* in *E*. Quoniam itaque per *AB* descendens eandem acquirit velocitatem in termino *B*, atque descendens per *GB* (per Cor. 2. IV); manifestum est, cum flexus ad *B* nihil obstare motui ponatur, tantam velocitatem habiturum ubi in *C* pervenerit, quantam si per *GC* planum descendisset; hoc est, quantam haberet ex descensu per *EC*. Quare & reliquum planum *CD* eodem modo transibit ac si per *EC* advenisset, ac proinde in *D* denique parem velocitatem habebit, ac si descendisset per planum *ED*, hoc est, eandem quam ex casu perpendiculari per *EF*. *Q.E.D.*



Corol. Hinc liquet etiam per Circuli circumferentiam vel per Curvam quamlibet lineam descendente mobili (nam curvas tanquam ex infinitis rectis compositæ essent hic considerare licet) semper eandem illi velocitatem acquiri si ab æquali altitudine descenderit: tantamque eam esse velocitatem, quantam casu perpendiculari ex eadem altitudine adipisceretur.

THEO-

THEOREMA VI.

Si detur planorum quotlibet contiguorum Inclinatio atque ratio Longitudinum eorundem ad invicem; erit Tempus quo totum ſystema planorum continuato motu percurritur in ſubduplicata ratione Longitudinis ſyſtematis totius.

MANENTE conſtructione priori, conſtat ex Theoremate II. Tempus deſcenſus per *AB* eſſe in ſubduplicata ratione Longitudinis *AB*. Tempus item deſcenſus tum per *GB* tum per *GC* eſt in eadem ſubduplicata ratione Longitudinis *AB*. Tempus enim deſcenſus per *GB* eſt in ſubduplicata ratione Longitudinis *GB*, adeoque propter datam rationem *GB* ad *AB*, in ſubduplicata ratione Longitudinis *AB*: ſimiliterque Tempus deſcenſus per *GC* eſt etiam in eadem ratione: & diviſim Tempus deſcenſus per *BC* poſt *GB* vel *AB* eſt in eadem ratione. Igitur componendo, Tempus deſcenſus per *AB* & *BC* eſt adhuc in eadem ratione; & ſimiliter Tempus deſcenſus per quotlibet *AB*, *BC*, *CD* eſt in eadem ratione, hoc eſt, in ſubduplicata ratione Longitudinis *AB* vel Longitudinum $AB+BC+CD$: ſunt enim omnes ad invicem in datis rationibus per Hypotheſin. *Q.E.D.*

Corol. Hinc liquet ſimiles partes Curvarum ſimilium ſimiliterque poſitarum ad Horizontem deſcribi à mobili deſcendente in Temporibus quæ ſunt in ſubduplicata ratione partium deſcriptarum.

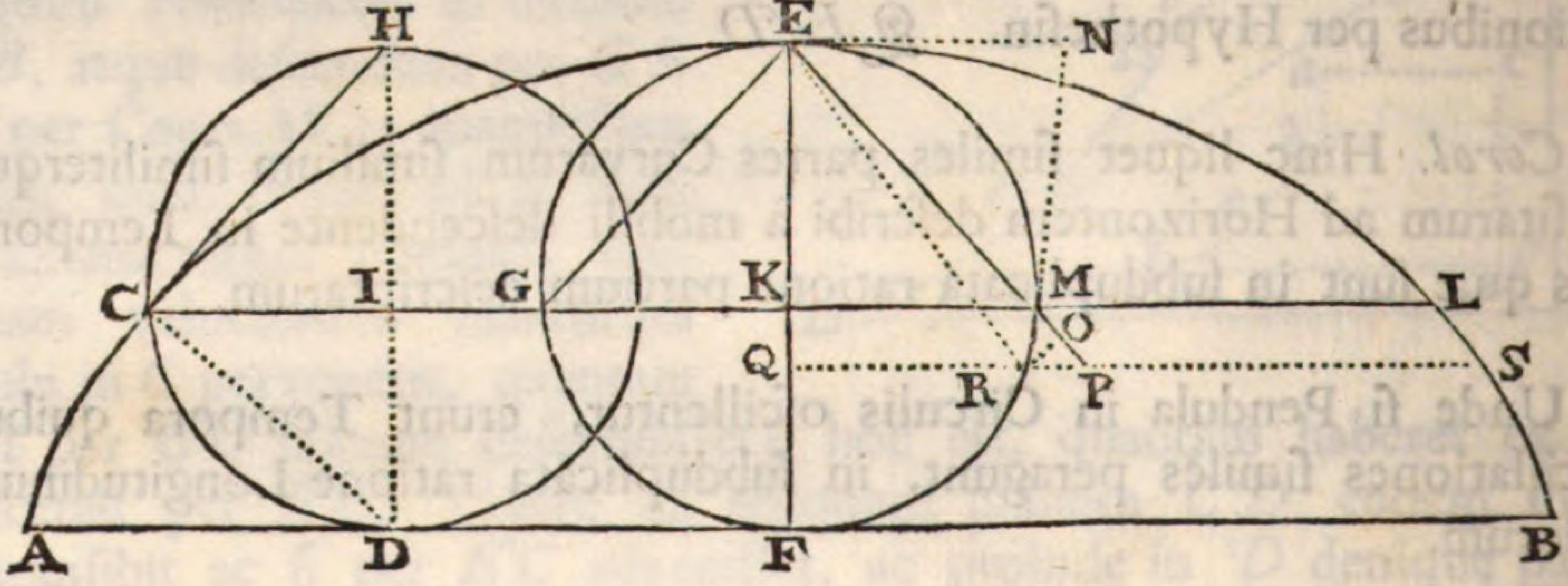
Unde ſi Pendula in Circulis oſcillentur, erunt Tempora quibus oſcillationes ſimiles peragunt, in ſubduplicata ratione Longitudinum Filorum.

DE

DE
MOTU PENDULORUM
IN
CYCLOIDE.

DEFINITIO.

SI Circulus *CDH* qui tangit *AB* rectam in puncto *D*, more rotarum provolvatur super eadem, & circuitum integrum progrediendo conficiat: punctum *C*, quod situm in ejus circumferentia sub initio tangebat rectam *AB* in puncto *A*, motu suo ex rectilineo & circulari composito describet curvam illam lineam *ACEB* quæ



Cyclois appellatur. Recta vero *AB* dicitur basis, & huic ad medium ejus punctum *F* perpendicularis *EF*, axis, punctum vero *E* vertex Cycloidis; Circulus autem *CDH* vel huic æqualis in alio situ *GFE* vocatur Circulus genitor.

LEMMA

LEMMA I.

Si circa Cycloidis axem EKF *describatur Circulus genitor* EGF *& à curvæ puncto quovis* C *ducatur* CIK *parallela basi* AB, *& secans Circulum in* G: *erit arcus Circularis* EG *æqualis rectæ* CG.

PATET ex Definitione totam Circuli circumferentiam æquari rectæ *AB* atque adeo dimidiam circumferentiam *EGF* æquari dimidiæ rectæ *AB*, nempe ipsi *AF*; Patet etiam Circuli æqualis *CDH*, diametro *DIH* descripti, arcum *CD* æquari rectæ *AD*; adeoque arcus *EG* æqualis erit rectæ *DF* vel *IK* vel *CG*. *Q.E.D.*

LEMMA II.

Iisdem positis, dico chordam EG *parallelam esse rectæ tangenti Cycloidem in puncto* C.

PATET ex generatione Cycloidis, chordam *CD* perpendicularem esse ad Curvam in puncto *C*; itaque chorda *HC* tanget eandem: est autem *EG* chorda parallela chordæ *HC*. *Q.E.D.*

LEMMA III.

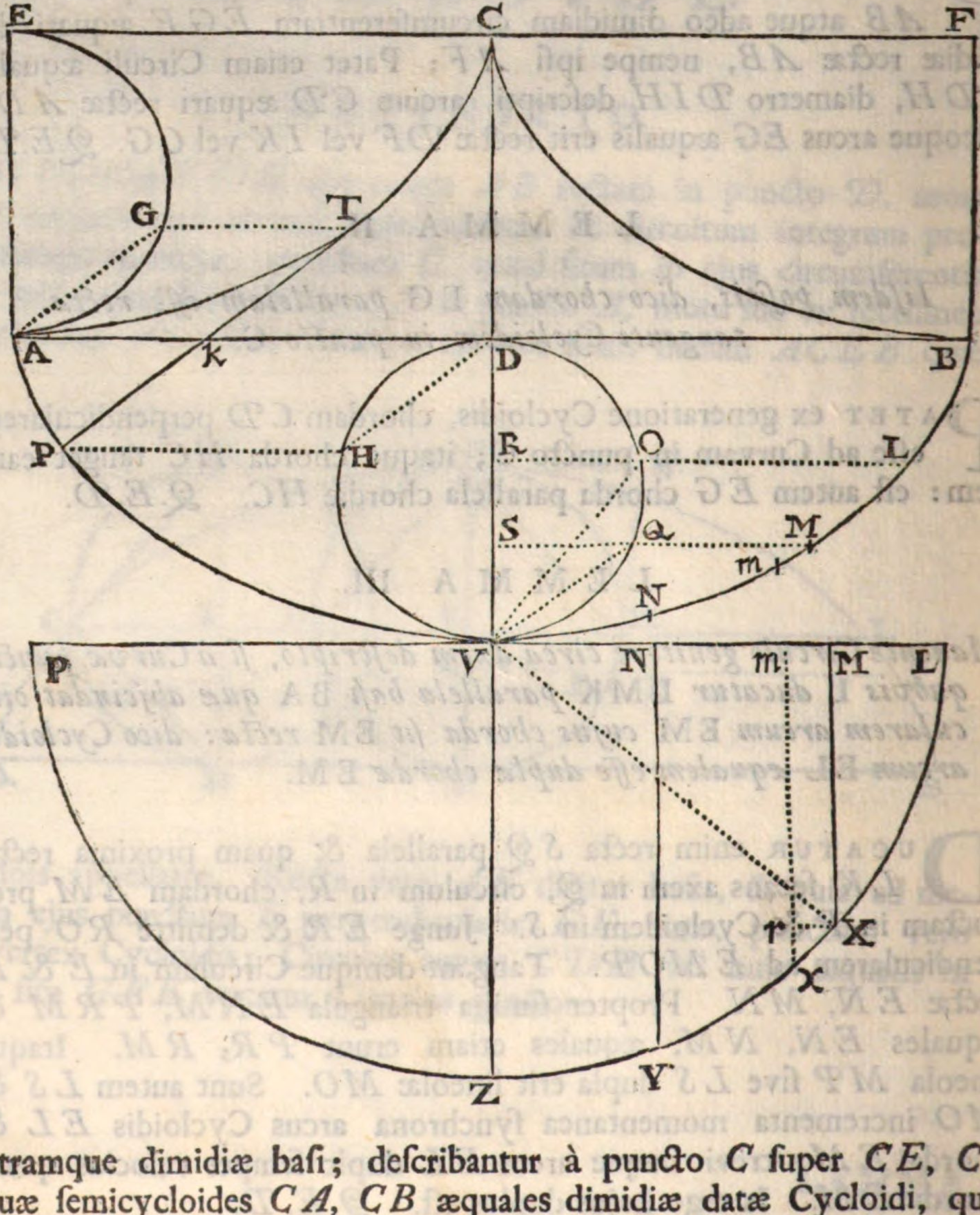
Manente Circulo genitore circa axem descripto, si à Curvæ puncto quovis L *ducatur* LMK *parallela basi* BA *quæ abscindat circularem arcum* EM *cujus chorda sit* EM *recta: dico Cycloidis arcum* EL *æqualem esse duplæ chordæ* EM.

DUCATUR enim recta *SQ* parallela & quam proxima rectæ *LK* secans axem in *Q*, circulum in *R*, chordam *EM* productam in *P* & Cycloidem in *S*. Junge *ER* & demitte *RO* perpendicularem ad *EMOP*. Tangant denique Circulum in *E* & *M* rectæ *EN*, *MN*. Propter similia triangula *ENM*, *PRM* & æquales *EN*, *NM*, æquales etiam erunt *PR*, *RM*. Itaque lineola *MP* sive *LS* dupla erit lineolæ *MO*. Sunt autem *LS* & *MO* incrementa momentanea synchrona arcus Cycloidis *EL* & chordæ *EM*: crevit itaque arcus *EL* duplo semper velocius quam chorda *EM*. Is ergo hujus duplus est. *Q.E.D.*

PROBLEMA.

Facere ut Corpus pendulum oſcilletur in Cycloide data.

DETUR Cyclois *AVB* ſuper baſi *AB* deſcripta; producatur axis ejus *VD* ad *C* ut æquales evadant *DC*, *DV*; per *C* age *ECF* parallelam baſi *AB*; cape *CE*, *CF* æqualem



utramque dimidiæ baſi; deſcribantur à puncto *C* ſuper *CE*, *CF* duæ ſemicycloides *CA*, *CB* æquales dimidiæ datæ Cycloidi, quarum vertices tangant baſim *AB* in punctis *A*, *B*. *A* puncto illo *C* filo

filo *CTP*, longitudinem *CV* æquante, pendeat corpus *P* & ita intra ſemicycloides oſcilletur, ut quoties pendulum digreditur à perpendiculo *CV* filum, parte ſui ſuperiore *CT*, applicetur ad ſemicycloidem illam *CTA* verſus quam peragitur motus & circum eam ceu obſtaculum flectatur, parteque reliqua *TP* cui ſemicyclois nondum objicitur, protendatur in lineam rectam; & pondus *P* oſcillabitur in Cycloide data *AVB*. *Q.E.F.*

Deſcribantur enim ſemicirculi genitores *AGE*, *DHV* ſuper axibus *AE*, *DV*. Age *TG*, *PH* parallelas baſi *AB*, & junge *AG*, *DH*.

Semicyclois *AC* æqualis eſt duplæ rectæ *AE*, per Lem. III, atque adeo æqualis rectæ *CV* vel filo integro *CTP*: itaque arcus Cycloidis *TA* æqualis erit parti liberæ fili *TP* quæ Cycloidem in *T* tangit. Unde cum *GA* parallela ſit tangenti *TK* (per Lem. II.) eique adeo æqualis, & dupla *GA* vel dupla *TK* æqualis ſit arcui *TA* vel rectæ *TP*, per Lem. III; erunt æquales *TK* & *KP*, & parallelæ *TG*, *PH* æquali intervallo diſtabunt à baſi *AB*. Abſcindunt ergo ſemicirculorum generantium æquales arcus *GA* & *HD*; unde parallelæ ſunt chordæ *AG*, *DH*, ac proinde *KP*, *DH*; & quadrilaterum *DHPK* eſt parallelogrammum & æquales ſunt *DK* & *HP*. Itaque cum *AK* recta ſit æqualis *GT* rectæ vel (per Lem. I.) arcui *AG* vel arcui *DH*; erit *HP* vel *DK* æqualis arcui reliquo *VH*, adeoque pondus *P* erit in Cycloide data *AVB*, per Lemma I. *Q.E.D.*

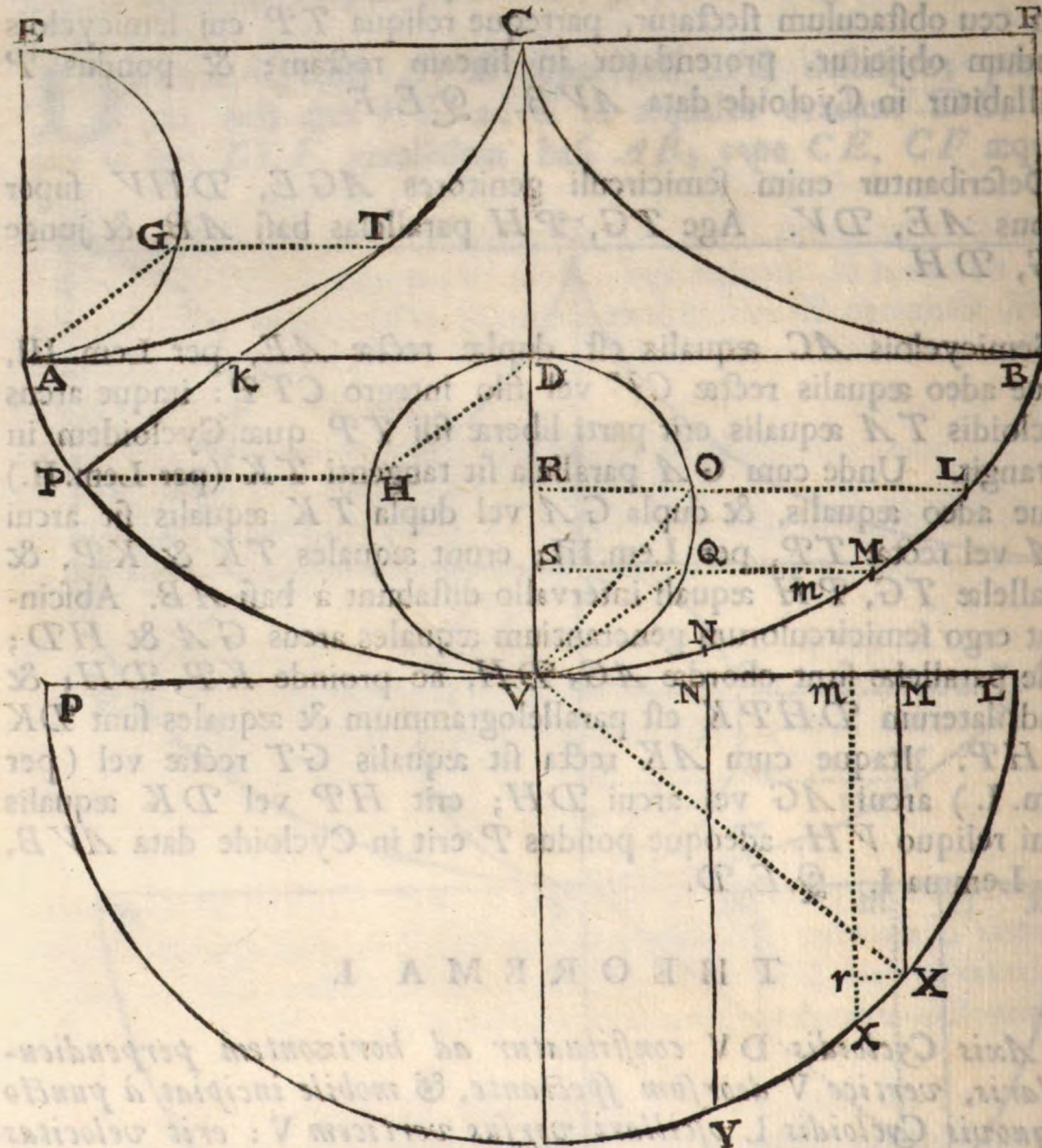
THEOREMA I.

Si Axis Cycloidis DV *conſtituatur ad horizontem perpendicularis, vertice* V *deorſum ſpectante, & mobile incipiat à puncto quovis Cycloidis* L *oſcillari verſus verticem* V: *erit velocitas ejus in loco quovis* M *ut* $\sqrt{VLq - VMq}$; *vel ſi in rectam protendatur curva* VNML *fiatque radius Circuli* LZP, *erit velocitas in loco* M *ut ſinus* MX *qui huic radio ad* M *punctum perpendiculariter inſiſtit.*

AD Axem *DRSV* demittantur perpendiculares *LR*, *MS* ſemicirculo genitori *DOQV* occurrentes in *O*, *Q*. & jungantur *OV*, *QV*.

Oſcillantis

Oscillantis Velocitas in loco M æqualis est velocitati corporis cujusvis cadentis ab R ad S, per Theor. V. de Desc. Grav. Est autem hæc velocitas (per Theor. II. de Desc. Grav.) in subduplicata



ratione Longitudinis RS vel $RV - SV$ vel quantitatis $RV \times VD - SV \times VD$ vel $VOq - VQq$ vel $VLq - VMq$, per Lem. III, vel in ratione integra sinus MX, per Constructionem. *Q.E.D.*

THEO-

THEOREMA II.

Iisdem positis, dico Circuli arcum quemlibet XxY *sinubus* MX, NY *interjectum, eodem tempore describi cum velocitate maxima* VZ *ad verticem Cycloidis acquisita, quo a Pendulo dimisso ab* L *percurritur arcus* MN *respondens rectæ* MmN, *quæ interjicitur iisdem sinubus. Eritque adeo tempus quo percurritur arcus ille* MN *ut Circuli arcus* XY.

NAM ductus intelligatur sinus *mrx* sinui *MX* vicinissimus & parallela sit lineola *Xr* lineolæ *Mm*.

Constat ex Theoremate præcedente lineolam *Mm* describi cum velocitate quæ est ut *MX* & lineolam *Xx* cum velocitate quæ est ut *VZ*. Sed propter similia triangula *MXV*, *rXx*, est *Xx* ad *Xr* vel *Mm*, ut *VX* vel *VZ* ad *MX*. Sunt igitur *Xx* & *Mm* ad invicem ut velocitates quibuscum describuntur. Æquali itaque tempore describuntur; ac pari ratione cæteræ partes correspondentes. Unde totus arcus Cycloidalis *MN* & Circularis *XY* æquali tempore describuntur. Est autem tempus quo describitur *MN*, vel quo describitur *XY* cum data velocitate *VZ* ut ipse arcus *XY*. *Q. E. D.*

THEOREMA III.

Tempus quo peragitur Oscillatio quævis in data Cycloide, est ad tempus descensus Gravis per axem Cycloidis, ut Circuli peripheria ad ejusdem diametrum; atque adeo Oscillationes omnes sunt Isochronæ.

TEMPUS quo describitur semiperipheria *LZP* cum velocitate maxima *VZ* est ad tempus quo describitur semidiameter *LV* cum eadem velocitate *VZ* ut Circuli peripheria ad ejusdem diametrum. Est autem tempus prius idem atque tempus oscillationis integræ *LVP*, per Theorema II. Tempus vero posterius idem est atque tempus descensus Gravis per Cycloidis axem *DV*: nam quo tempore semidiameter *LV* vel arcus *LV* vel dupla chorda *OV* percurritur cum velocitate maxima acquisita ad verticem *V* sive a pendulo oscillante per arcum *LV* sive a Gravi decidente per chordam *OV*; eodem cadit Grave ab *O* ad *V* (per Th. I. de Desc. Grav.) vel a *D* ad *V*, per Cor. 3. Th. IV. de Desc. Gravium. Itaque tempus oscillationis est ad tempus descensus Gravis per axem Cycloidis ut Circuli peripheria ad ejusdem diametrum. *Q. E. D.*

Cor. 1. Si corpora duo filis æqualibus suspensa, temporibus æqualibus oscillationes suas peragant; in descensu rectilineo æquabiliter

accelerabuntur: & propterea Pondera eorum erunt ut Quantitates Materiæ.

Cor. 2. Spatium quod a gravi e quiete dimiſſo, tempore quovis percurritur, eſt ad ſemiſſem longitudinis fili, quo idem grave ſuſpenſum eodem tempore oſcillationes ſingulas peragit; in duplicata ratione circuli peripheriæ ad diametrum. Cadat enim grave per fili longitudinem dimidiatam; & tempus deſcenſus erit ad tempus unius oſcillationis, ut circuli diameter ad peripheriam: augeatur jam tempus in hac ratione ut æquale fiat tempori oſcillationis; & ſpatium deſcriptum augebitur in duplicata illa ratione.

THEOREMA IV.

Tempus quo peragitur Oſcillatio quævis in Cycloide, ubi tum fili longitudo tum vis gravitatis variatur, eſt in ſubduplicata ratione longitudinis iſtius directe & vis gravitatis inverſe.

Tempus Oſcillationis eſt ut tempus deſcenſus per Axem per Theo. III; atque hoc tempus eſt in ſubduplicata ratione axis vel dupli axis vel fili longitudinis directe, & in ſubduplicata ratione vis gravitatis inverſe. Nam ſi detur vis, longitudo deſcenſus erit ut quadratum temporis; & ſi detur tempus, longitudo erit ut velocitas ac proinde ut vis: adeoque neutro dato, ut quadratum temporis & ut vis conjunctim. Unde tempus erit in ſubduplicata ratione longitudinis directe & vis inverſe. *Q. E. D.*

Cor. Longitudo fili, quæ per hanc demonſtrationem erat ut vis gravitatis & quadratum temporis ſimul; dato tempore erit ut vis illa ſimpliciter.

Scholium.

Patet vires acceleratrices eſſe ut velocitatum mutationes dato tempore quam minimo genitas. Erit itaque vis acceleratrix in loco M ut lineola rx, dato arcu minimo Xx, hoc eſt ut longitudo VM. Et viciſſim, ſi vis accleratrix ſit ut VM & motus continuetur ab L verſus V; erit velocitas in loco quovis M ut ſinus MX, tempuſque tranſeundi ab M ad N erit ut arcus XY.

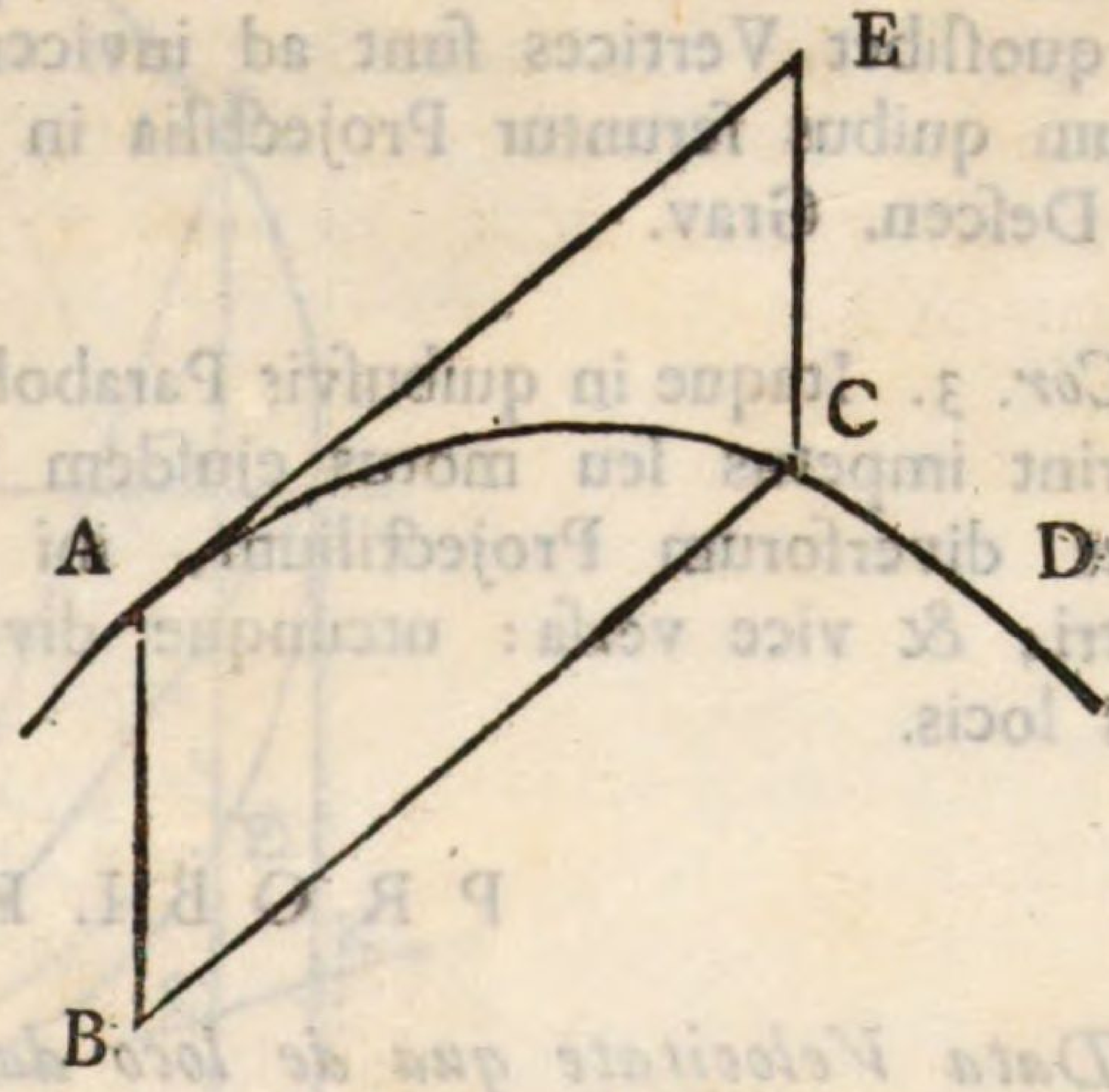
Quæcunque in præcedentibus de oſcillationibus in Cycloide dicta ſunt, etiam de oſcillationibus minimis in Circulo pariter obtinebunt: nam iſtiuſmodi oſcillationes utrobique eædem erunt. Quin & hoc fortaſſe non eſt prætereundum; nempe corporis funipenduli in dato circulo utcunque oſcillantis velocitatem ad punctum infimum eſſe ut chorda arcus toto deſcenſu deſcripti. Nam per Cor. Th. V. de Deſc. Gr. velocitas illa eadem erit ſive per arcum ſive per chordam deſcenderetur, & proinde erit ut chorda per Cor. 3. Theo. IV. de Deſc. Gravium.

DE MOTU PROJECTILIUM.

THEOREMA.

Curva descripta motu corporis oblique projecti est Parabola: & Velocitas Projectilis in quolibet puncto parabolæ ea est quam grave cadendo acquirere potest & casu suo describendo quartam Parametri partem pertinentis ad punctum illud datum pro Vertice sumptum.

EXEAT Projectile de loco A secundum rectam AE & sit ACD curva descripta. Ponatur AE longitudo quam dato quovis tempore Projectile transiret motu æquabili orto ex vi impressa si nulla esset gravitas: atque AB alia longitudo quam grave eodem tempore transiret descendendo ab A versus centrum Telluris. Completo parallelogrammo $EABC$, facile patebit projectile motu composito latum in fine ejusdem temporis perventurum esse ad punctum C. Est autem AE ut tempus quo describeretur, adeoque AEq vel BCq ut temporis quadratum. Est etiam AB ut quadratum ejusdem temporis per Theor. II. de Desc. Grav. Igitur AEq vel BCq est ut AB. Proinde curvæ descriptæ puncta C sita sunt in Parabola cujus Diameter est AB, Vertex A & Parameter ad hunc verticem pertinens est $\frac{BCq}{AB}$ vel $\frac{AEq}{EC}$.



Projectilis in puncto quovis A ea est Velocitas qua longitudo AE eodem tempore describi posset quo grave descenderet ab E ad C. Velocitas autem hoc descensu acquisita, est ad velocitatem illam qua longitudo AE eodem tempore describi posset, ut EC ad

ad $\frac{1}{2} AE$ per Theor. I. de Descen. Grav. Eademque velocitas acquisita in C cadendo ab E, est ad velocitatem acquirendam cadendo per quartam partem Parametri nempe $\frac{\frac{1}{4} AEq}{EC}$, in subduplicata ratione longitudinis EC ad longitudinem $\frac{\frac{1}{4} AEq}{EC}$ (per Theor. II. de Desc. Grav.) vel in ratione integra ipsius EC ad $\frac{1}{2} AE$. Itaque Projectilis velocitas in puncto A ea est quam grave acquirere potest descendendo per quartem Parametri partem pertinentis ad Verticem A. *Q. E. D.*

Cor. 1. Si projectile dirigatur secundum AE & parabolæ describendæ parameter pertinens ad Verticem A sit æqualis quantitati $\frac{AEq}{EC}$; isthæc parabola transibit per punctum C.

Cor. 2. In eadem vel in diversis parabolis Parametri pertinentes ad quoslibet Vertices sunt ad invicem in duplicata ratione Velocitatum quibus feruntur Projectilia in istis verticibus, per Theor. II. de Descen. Grav.

Cor. 3. Itaque in quibusvis Parabolis ubicunque locorum æquales fuerint impetus seu motus ejusdem Projectilis vel æquales velocitates diversorum Projectilium, ibi semper æquales erunt Parametri; & vice versa: utcunque diversa fuerit directio motus in istis locis.

PROBLEMA.

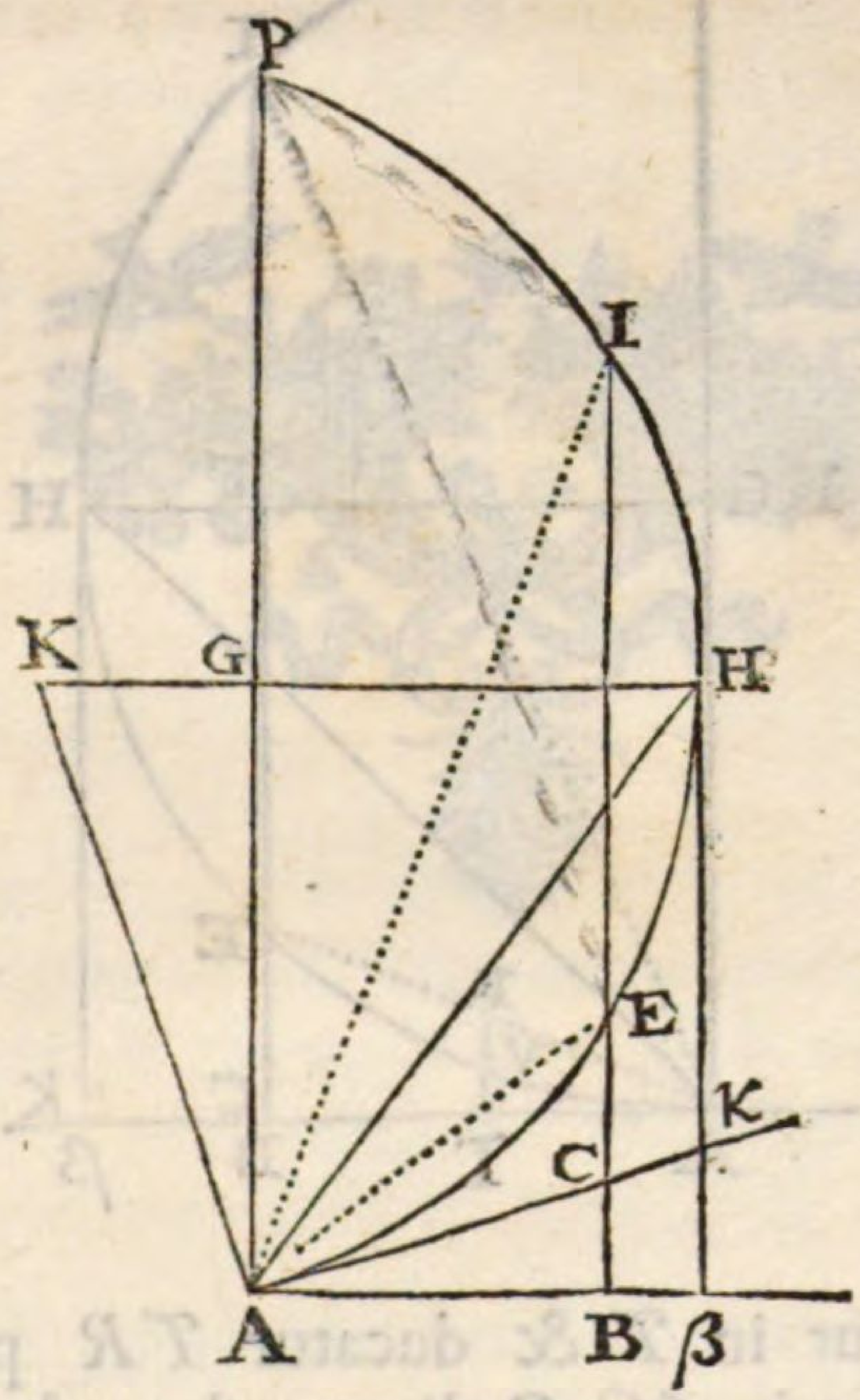
Data Velocitate qua de loco dato Projectile dirigendum est dataque positione Scopi feriendi: invenire Directiones vis imprimendæ.

DATA Velocitate dabitur Parameter Parabolæ describendæ. Constat enim per Theorema, si grave eousque descendat dum data illa velocitas cadendo fuerit acquisita: Altitudinem isto descensu percursam fore æqualem quartæ parti parametri.

Esto jam C scopus, positus in data recta AC. In loco A, de quo projectile dirigendum est, erige AP normalem ad Horizontem & æqualem Parametro inventæ. Dividat hanc perpendiculariter & bifariam

bifariam in *G* recta *KH*, cui in *K* occurat recta *AK* ad ipsam *AC* perpendicularis, centroque *K* & intervallo *KA* describatur circulus *AHP*. Denique per scopum *C* ad horizontalem *AB* normalis ducatur *BCEI* circulum secans (si fieri potest) in *E* & *I*: erunt *AE* & *AI* directiones quæsitæ. *Q.E.I.*

Nam, si jungantur *PE*, *PI*, propter similia triangula *PAE*, *AEC* erit *PA* æqualis $\frac{AEq}{EC}$. Itemque propter similia triangula *PAI*, *AIC* erit eadem *PA* æqualis $\frac{AIq}{IC}$. Igitur, cum sit *PA* parabolæ describendæ parameter pertinens ad punctum *A*, constat per Cor. 1. Theorematis, parabolam istam transituram esse per scopum *C*. *Q.E.D.*



Cor. 1. Patet rectam *AH* bisecare Angulum *CAP* mensuram visibilis distantiæ inter Scopum & Zenith.

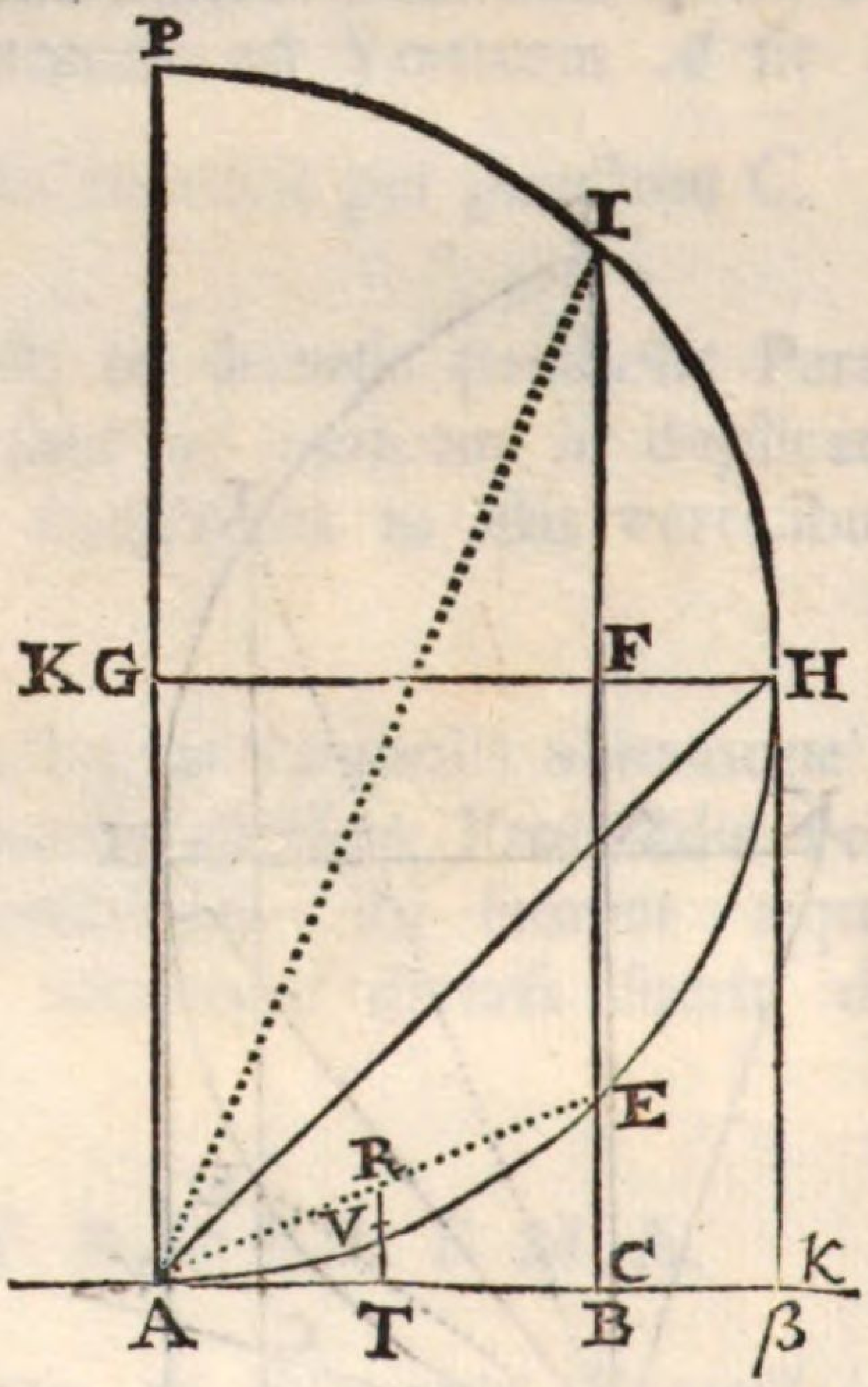
Cor. 2. Ubi puncta *E*, *I* concurrunt in *H* sive directiones binæ *AE*, *AI* concurrunt in unam *AH*, Scopi distantia *AC* evadet maxima $A\kappa$ quæ velocitate illa data transiri potest.

Cor. 3. Binæ quælibet directiones *AE*, *AI*, quarum utravis idem Scopus feriri potest, æqualiter distant à directione illa *AH*; hoc est, anguli *EAH*, *IAH* sunt æquales.

Cor. 4. Scopi diſtantia *AC* vel *AB* poſita in linea Horizontali, eſt ut ſinus dupli anguli Elevationis *EAC* vel *IAC*. Nam ſi recta *CEI* occurrat rectæ *GH* in *F*, erit *AC* vel *GF* æqualis ſinui arcus *AE* vel *AI*, qui metitur duplum angulum *EAC* vel *IAC*.

Cor. 5. Maxima autem Scopi diſtantia *Aκ* poſita in linea Horizontali adæquat *AG* ſemiſſem Parametri, adeoque datur: Et Scopus iſte *κ* attingi poteſt exiſtente angulo Elevationis *HAκ* ſemirecto.

Cor. 6. Si Projectile emittatur ſecundum *AE*, puncti altiſſimi parabolæ deſcribendæ Altitudo ſupra Horizontem adæquabit $\frac{1}{4}EC$.



Nam ſi *AC* biſecetur in *T* & ducatur *TR* parallela ad *CE* ſecanſque *AE* in *R*: erit *AC* Ordinata ad parabolæ axem *TR* quem vertex ejus principalis bifariam dividet in *V*. Itaque Altitudo *TV* eſt æqualis $\frac{1}{2}TR$ vel $\frac{1}{4}CE$. Cum vero *CE* ſit ſinus verſus arcus *AE* ſive dupli anguli *CAE*, patet projectilis Altitudinem *TV* eſſe ut Sinus verſus dupli anguli Elevationis.

Cor. 7. Altitudo autem Maxima projectilis emiſſi ſecundum *AP* normalem ad Horizontem adæquabit $\frac{1}{4}$ parametri *AP*; adeoque datur. Nam in hoc caſu coincidentibus *AE*, *EC* cum directione *AP*, altitudo illa *TV* vel $\frac{1}{4}CE$ jam evadet $\frac{1}{4}AP$.

Cor. 8.

Cor. 8. Tempus quo projectile emiſſum ſecundum AE feretur ab A ad C eſt ut Chorda AE vel ut $\frac{1}{2}AE$, hoc eſt, ut ſinus anguli elevationis EAC.

Cor. 9. Tempus autem Maximum projectili emiſſo ſecundum AP, normalem ad Horizontem, idem eſt quo grave deſcendet à P ad A per totam parametrum, adeoque datur. Nam in hoc caſu Projectile recta aſcendit ad altitudinem $\frac{1}{4}AP$, deinde vero ab eadem altitudine deſcendit. Tempora vero aſcenſus & deſcenſus ſimul ſumpta conficiunt duplum tempus ſolius deſcenſus vel tempus deſcenſus ab altitudine quadrupla, per Theor. II. de Deſcen. Grav.

FINIS.

EDITORIS NOTÆ
AD
HARMONIAM MENSURARUM.

NOTÆ AD LOGOMETRIAM.

Nota I. ad Propositionem I.

Capiatur *particula dupla vel tripla* &c. & *evadet ratio duplicata vel triplicata* &c. Nam si termini quotlibet *AP*, *AQ*, *AR*, *AS* &c fuerint continue proportionales, terminorum differentiæ *PQ*, *QR*, *RS* &c erunt in eadem ratione ac ipsi termini: adeoque si termini ipsi ad æqualitatem ita accedant ut quam minimæ sint eorum differentiæ, hæ differentiæ accedent itidem ad æqualitatem. Patet itaque, manente



AP, magnitudinem rationis inter *AQ* & *AP* esse ut *PQ*. Quo posito hæc est vis totius argumentationis. Si detur ratio inter *AQ* & *AP*, erit *PQ* ut *AP*; & si detur *AP*, erit *PQ* ut magnitudo rationis inter *AQ* & *AP*: quare neutro dato, erit *PQ* ut *AP* & magnitudo rationis inter *AQ* & *AP* conjunctim; adeoque $\frac{PQ}{AP}$ erit ut magnitudo rationis inter *AQ* & *AP*.

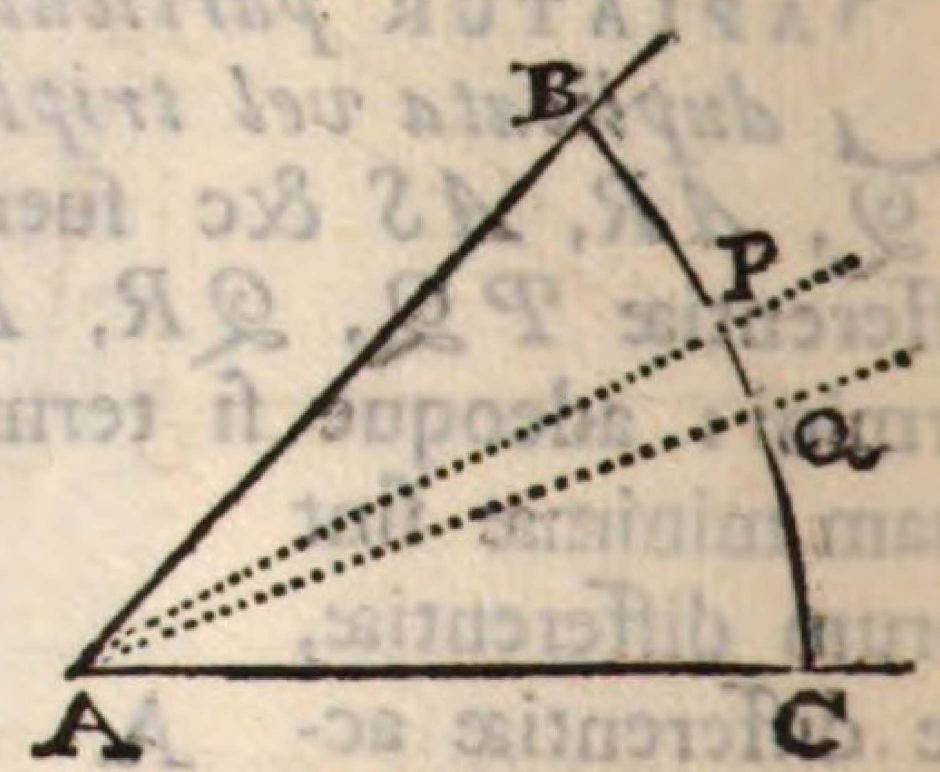
Quo melius intelligantur diversa Mensurarum Systemata eorumque Moduli, consule Prop. IV. V & VI. Hactenus de mensuris rationum, ne Lector forsan in limine hæreat. Libet jam paulisper dispicere de Mensuris Angulorum. Nam in tot rebus tanta est harmonia inter Angulorum & Rationum mensuras, ut contemplationem præbere possit nec injucundam nec inutilem ad sequentia.

PROPOSITIO.

Invenire mensuram anguli cujuscunque propositi.

DIVISUS concipiatur angulus propositus BAC in angulos innumeros PAQ, centroque A & intervallo quovis AP describatur arcus PQ: &, si Auctoris argumentationem a rationibus ad angulos huc transferamus, perducemur tandem ad hanc quantitatem generalem $M \times \frac{PQ}{AP}$, quæ commodissime designabit mensuram anguli PAQ ad Modulum M; atque ita pertrahemur ad sequentia, ut in Propositione prima.

Corol. 1. Modulus M est ad mensuram anguli PAQ, ut circuli Radius AP ad arcum PQ, qui angulum illum subtendit.



Corol. 2. Dato angulo BAC datur summa omnium $\frac{PQ}{AP}$, & summa omnium $M \times \frac{PQ}{AP}$ est ut M. Itaque mensura dati cujuscunque anguli est ut Modulus Systematis e quo desumitur.

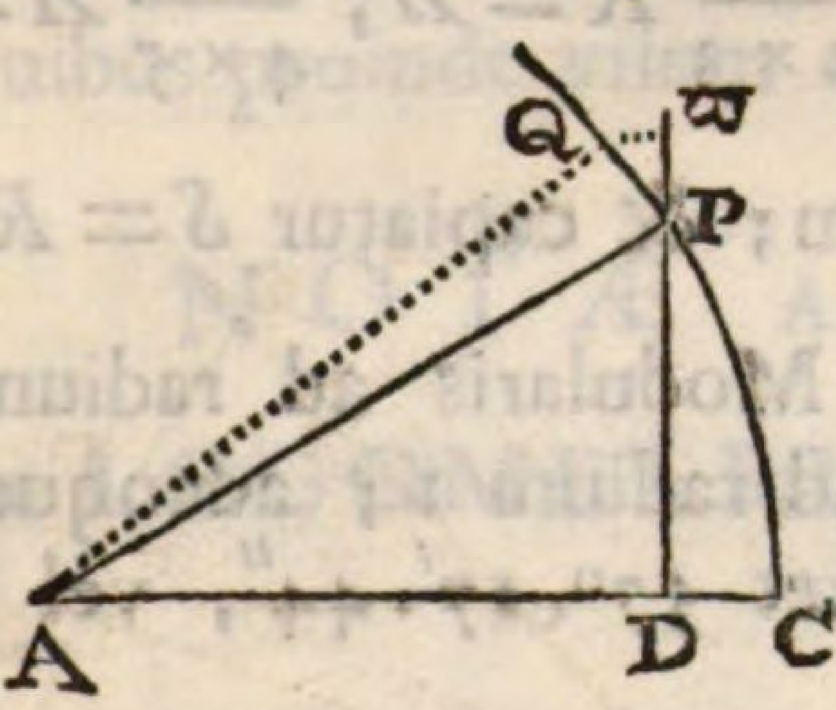
Corol. 3. Modulus ergo, in omni mensurarum systemate, semper æqualis fit mensuræ anguli cujusdam determinati & immutabilis, quem proinde *Angulum Modularem* vocabo.

Corol. 4. Angulus modularis is est, quem circuli arcus Radio æqualis subtendit. Patet per Corol. 1. Nam augeri intelligatur mensura anguli PAQ donec modulo M fuerit æqualis: & fiet arcus PQ æqualis radio AP.

In Canone Logarithmico exhibetur Systema quoddam mensurarum numeralium, quæ Logarithmi dicuntur: atque hujus systematis Modulus is est Logarithmus, qui metitur Rationem Modularem in Corol. 6. definitam. Similiter in Canone Trigonometrico sinuum & tangentium, exhibetur Systema quoddam mensurarum numeralium, quæ Gradus appellantur: atque hujus systematis Modulus is est Numerus

merus Graduum, qui metitur Angulum Modularem modo definitum, hoc est, qui continetur in arcu Radio æquali. Est autem hic Numerus ad Gradus 180 ut Circuli Radius ad Semicircumferentiam, hoc est ut 1 ad 3. 14159265389 &c. Unde Modulus Canonis Trigonometrici prodibit 57. 2957795130 &c. Cujus Reciprocus est 0. 0174532925 &c. Hujus moduli subsidio (quem in chartula quadam Auctoris manu descriptum inveni) commodissime computabis mensuras angulares, quemadmodum ostendam in Nota III.

Si rerum analogia delectari soleas, proponam tibi aliam methodum inveniendi Angulum Modularem: quæ ad amussim ei respondet, qua Auctor invenit Rationem Modularem in Scholio 2. Esto PD sinus cujusvis anguli PAD ad radium AP, & computetur hujus anguli mensura ad Modulum M. Centro A describatur arcus PQ & a puncto Q puncto P vicinissimo demittatur $Q\varpi$ normalis ad sinum DP productum. Ducatur AQ & vocentur AP, r; PD, x;



ejusque fluxio $P\varpi$, $\dot{x}$. Et fluxio mensuræ quæsitæ, quæ in propositione præcedente erat $M \times \frac{PQ}{AP}$, evadet $M \times \frac{P\varpi}{AD}$ vel $M \times \frac{\dot{x}}{\sqrt{rr - xx}}$ vel M in $\frac{\dot{x}}{r} + \frac{\dot{x}x^2}{2r^3} + \frac{3\dot{x}x^4}{8r^5} + \frac{5\dot{x}x^6}{16r^7} + \&c.$ Adeoque mensura illa fiet M in $\frac{x}{r} + \frac{x^3}{6r^3} + \frac{3x^5}{40r^5} + \frac{5x^7}{112r^7} + \&c.$ Unde si hæc mensura vocetur m: erit $\frac{m}{M} = \frac{x}{r} + \frac{x^3}{6r^3} + \frac{3x^5}{40r^5} + \frac{5x^7}{112r^7} + \&c.$ ac proinde $\frac{m^3}{M^3} = \frac{x^3}{r^3} + \frac{x^5}{2r^5} + \frac{37x^7}{120r^7} + \&c.$ Similiterque $\frac{m^5}{M^5} = \frac{x^5}{r^5} + \frac{5x^7}{6r^7} + \&c.$ ac denique $\frac{m^7}{M^7} = \frac{x^7}{r^7} + \&c.$ Ut igitur vicissim ex data mensura m inveniatur angulus quem metitur: addendo æqualia æqualibus habebitur $\frac{m}{M} - \frac{m^3}{6M^3} = \frac{x}{r} * - \frac{x^5}{120r^5} - \frac{34x^7}{5040r^7}$, &c.

atque

atque iterum $\frac{m}{M} - \frac{m^3}{6M^3} + \frac{m^5}{120M^5} = \frac{x}{r} * * + \frac{x^7}{5040r^7}$, &c. atque tandem $\frac{m}{M} - \frac{m^3}{6M^3} + \frac{m^5}{120M^5} - \frac{m^7}{5040M^7} + \&c = \frac{x}{r}$. Itaque anguli quæsiti sinus x erit ad radium r ut $\frac{m}{M} - \frac{m^3}{6M^3} + \frac{m^5}{120M^5} - \frac{m^7}{5040M^7} + \&c$ ad 1. Ponatur $m = M$ sive $\frac{m}{M} = 1$, & exinde sinus Anguli Modularis erit ad radium ut $1 - \frac{1}{6} + \frac{1}{120} - \frac{1}{5040} + \&c$ ad 1. Exhinc vero patet Corollarium sequens.

Corol. 5. Exposito radio R, si sumantur $\frac{1}{2 \times 3} R = A$; $\frac{1}{4 \times 5} A = B$; $\frac{1}{6 \times 7} B = C$; $\frac{1}{8 \times 9} C = D$; &c ad infinitum; & capiatur $S = R - A + B - C + D - \&c$: erit S sinus Anguli Modularis ad radium R. Hinc vero sinus iste est 0. 84147098 &c ad radium 1; adeoque per Canonem sinuum Angulus Modularis continet $57^{\circ}. 17'. 44''$, vel Gradus 57. 295 &c prout supra invenimus.

Nota II. ad Propositionem II.

Demonstratio. Quoniam ponitur $p = \log : \frac{126}{125}$ vel $\frac{2 \times 3^2 \times 7}{5^3}$; $q = \log : \frac{225}{224}$ vel $\frac{3^2 \times 5^2}{2^5 \times 7}$; $r = \log : \frac{2401}{2400}$ vel $\frac{7^4}{2^5 \times 3 \times 5^2}$; $s = \log : \frac{4375}{4374}$ vel $\frac{5^4 \times 7}{2 \times 3^7}$: si pro logarithmis Primorum 2, 3, 5, 7 scribantur respective a, b, c, d, erunt ex natura logarithmorum $p = a + 2b - 3c + d$; $q = -5a + 2b + 2c - d$; $r = -5a - b - 2c + 4d$; $s = -a - 7b + 4c + d$. Unde per notas analyseos regulas dabuntur logarithmi a, b, c, d. Reliqua satis patent.

Nota III. ad Propositionem III.

Per hanc propositionem Problematum Constructiones Logometricæ Effectionem practicam recipiunt ope Canonis Logarithmici. Huic similis est Effectio Constructionum Trigonometricarum ope Canonis Trigonometrici. Nam ex data ratione Radii ad Tangentem anguli dimetiendi,

dimetiendi, dabitur numerus graduum qui in angulo illo continetur. Vocetur hic numerus propofiti anguli *Menfura Trigonometrica*; &, per Corol. 2. Prop. in Nota I, Menfura cujufvis oblati anguli erit ad Modulum fuum, ut oblati anguli Menfura Trigonometrica ad Canonis Trigonometrici Modulum.

In propofitionibus præcedentibus Methodi fuæ principia Auctor tradidit, & in tribus fubfequentibus Methodum ipfam exemplis explanavit. His opibus inftructus tranfibis ad Operis partem tertiam, de Problematum Analyfi & Conftructione. Nam ad Scholium Generale gratior erit reditus, poftquam hujus Analyfeos adeo peritus fueris, ut iftiufmodi conftructiones proprio marte invenire poffis. Pendet autem hæc Analyfis e Theorematis in Operis parte fecunda, de quibus proinde vifum eft hæc fequentia annotare.

NOTÆ AD THEOREMATA TUM LOGOMETRICA TUM TRIGONOMETRICA.

NOTA IV.

De Menfurarum Harmonia in tranfitu a Ratione ad Angulum.

HORUM Theorematum inventionem Analyticam non eft inftituti mei hic tradere. Oftendam potius breviorem methodum fyntheticam, qua Theorema quodlibet ad examen revocare poffimus. Primo itaque demonftrabo mirabilem illam Menfurarum utriufque generis Harmoniam, quæ in Auctoris præfatione fic defcribitur.

In his Theorematis quantitates R, T, S (quarum valor femper apponitur) defignant vel Rationem vel Angulum, per cujus menfuram exhibenda eft Fluxionis quantitas fluens. Nempe fi R fit radix quadratica quantitatis affirmativæ, adeoque poffibilis: erit $R\left|\frac{R+T}{S}\right.$ menfura Rationis quam habet R+T ad S pro Modulo R. Sin R fit radix quadratica quantitatis negativæ, adeoque impoffibilis: affumi debet pro R radix quadratica ejufdem quantitatis affirmative fumptæ, eritque $R\left|\frac{R+T}{S}\right.$ menfura Anguli ad Modulum R, cujus Radius, Tangens & Secans funt inter fe ut R, T & S. Hanc

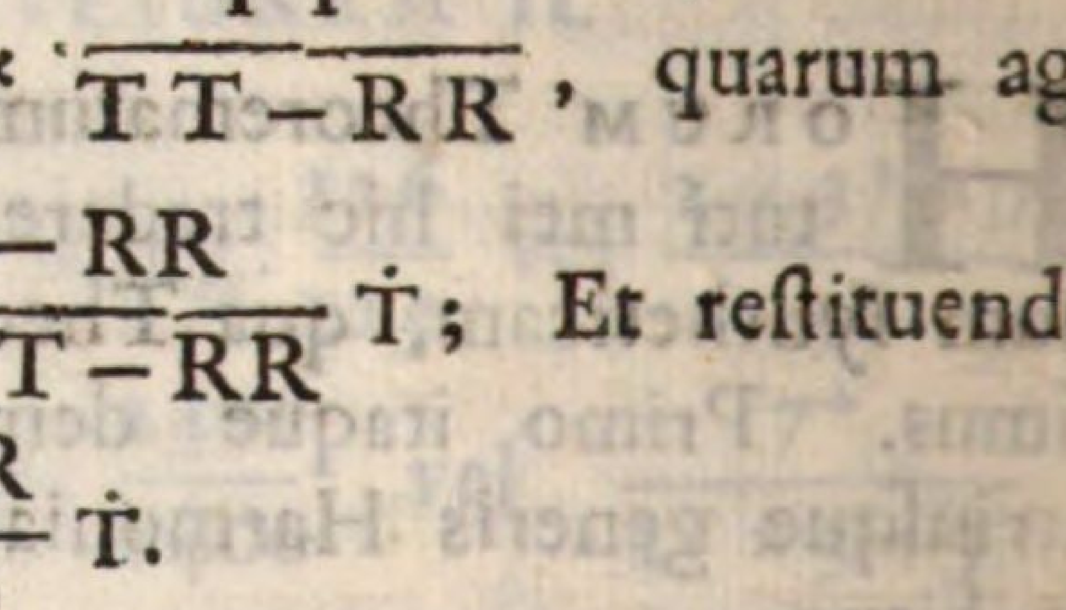
 vero

vero menſuram Angularem Auctor alibi ſic denotavit $R\left(\frac{R.T}{S}\right.$, ut appareat aliqua diſtinctio pro re nata.

In caſu priore ubi R eſt poſſibilis, oſtendam Fluxionem Menſuræ $R\left|\frac{R+T}{S}\right.$ eſſe $\frac{-RR}{SS}\dot{T}$. In caſu poſteriore ubi R eſt impoſſibilis ejuſque quadratum RR valorem habet negativum: ponatur $+KK = -RR$ & valor termini K erit poſſibilis. Quo poſito oſtendam Fluxionem Menſuræ $K\left(\frac{K.T}{S}\right.$ eſſe $\frac{+KK}{SS}\dot{T}$. Quæ ſi ita ſint, patebit Fluxiones Menſurarum utriuſque generis eſſe æquales. Et proinde quantitatem Fluentem exhibendam eſſe per alterutram menſuram, quam valor Moduli R indicaverit eſſe poſſibilem.

Inſpicienti Theoremata patebit eos eſſe valores quantitatum R, T, S, ut ſit ubique $SS = TT - RR$: & cum ratio quam habet $R+T$ ad S ſit exceſſus rationis $R+T$ ad R ſupra rationem S ad R vel $\sqrt{TT-RR}$ ad R: erit $R\left|\frac{R+T}{S}\right. = R\left|\frac{R+T}{R}\right. - R\left|\frac{\sqrt{TT-RR}}{R}\right.$. Harum menſurarum fluxiones ob datum R (per Cor. 1. Prop. I Logometriæ) ſunt $R \times \frac{\dot{T}}{R+T}$ & $-R \times \frac{T\dot{T}}{TT-RR}$, quarum aggregatum eſt $R \times \frac{T\dot{T} - R\dot{T} - T\dot{T}}{TT-RR} = \frac{-RR}{TT-RR}\dot{T}$; Et reſtituendo SS, fluxio menſuræ $R\left|\frac{R+T}{S}\right.$ eſt $\frac{-RR}{SS}\dot{T}$.

Erat $SS = TT - RR$: adeoque in hypotheſi caſus poſterioris, erit $SS = TT + KK$. Unde ſi conſtituatur triangulum rectangulum *BCE* faciendo crus alterutrum $CB = K$, alterum vero $CE = T$, centroque *B* & intervallo *BC* deſcribatur arcus *CSs*, rectæ *BE* atque hac proxime majori *Be*, occurrens in *S* & *s*: Angulus *CBE* erit is, cujus Radius, Tangens & Secans ſunt K, T, S. Hujus anguli menſura ad Modulum *BC* vel K eſt arcus *CS* per Corol. 1. Notæ I; adeoque menſuræ



Fluxio

Fluxio est ejusdem arcus incrementum Ss. Demisso autem EP normali ad Be, propter similia triangula SBs, EBP; itemque EPe, BCE, erit Ss ad EP ut BS ad BE, atque etiam EP ad Ee ut BC ad BE, & ex æquo Ss ad Ee vel $\dot{T}$ ut BCq ad BEq sive ut KK ad SS. Adeoque incrementum Ss vel Fluxio Mensuræ $K(\frac{K.T}{S}$ est $\frac{+KK}{SS}\dot{T}$. *Q.E.O.*

Notandum præterea, terminum T esse semper possibilem & quadratum ejus affirmativum: quod facile patebit conferenti valorem ejus cum Forma propria. Unde cum sit $SS=TT-RR$, si RR sit negativum: erit SS semper affirmativum, adeoque S semper possibilis ubi occurrit Anguli mensura. Quod si RR sit affirmativum: erit SS affirmativum & S possibilis, si sit T major quam R; eritque SS negativum & S impossibilis, si sit T minor quam R. Hoc ubi acciderit, valores terminorum T & S in pag. 64 adhiberi possunt. Nam cum alter valorum termini T sit semper major quam R, alter vero semper minor: (est enim valor termini R medius proportionalis inter binos valores termini T:) erit semper alter valorum termini S possibilis, alter vero impossibilis. Quæ quanquam ita sint, tamen bini valores termini S non sunt prorsus necessarii. Utervis enim sufficit vel in eo casu ubi S est radix quadratica quantitatis negativæ. Nam converso hujus quantitatis signo in positivum, ejus radix etiamnum adhiberi potest pro termino S. Est enim Ratio $\frac{R+T}{S\sqrt{-1}} = \frac{R+T}{S} \times \frac{1}{\sqrt{-1}}$ adeoque mensura $R\left|\frac{R+T}{S\sqrt{-1}}\right. = R\left|\frac{R+T}{S}\right. + R\left|\frac{1}{\sqrt{-1}}\right.$. Itaque ob datam quantitatem $R\left|\frac{1}{\sqrt{-1}}\right.$, cujus fluxio nulla est, eadem erit Fluxio mensuræ $R\left|\frac{R+T}{S\sqrt{-1}}\right.$ vel $R\left|\frac{R+T}{S}\right.$: & vicissim, ejusdem Fluxionis quantitas fluens erit vel $R\left|\frac{R+T}{S\sqrt{-1}}\right.$ vel $R\left|\frac{R+T}{S}\right.$.

Nota V.

De Methodo examinandi hæc Theoremata.

IN Nota præcedente, Fluxio Mensuræ $R\left|\frac{R+T}{S}\right.$ erat $\frac{-RR}{SS}\dot{T}$, & Fluxio Mensuræ $K\left(\frac{K.T}{S}\right.$ erat $\frac{+KK}{SS}\dot{T}$. Per utramvis Regulam satis commode computabis Fluxionem cujuscunque Theorematis. Ex qua patebit utrum theorema recte constructum fuerit nec ne. Exempli gratia, quæritur de Theoremate $\frac{2}{n}dP - \frac{2}{n}dR\left|\frac{R+T}{S}\right.$ pertinente ad Formam III, $d\dot{z}z^{\theta n-1}\sqrt{e+fz^n}$ ubi $\theta = 0$. In hac forma $P=\sqrt{e+fz^n}$, $R=\sqrt{e}$, $T=\sqrt{e+fz^n}$, $S=\sqrt{fz^n}$, unde $\dot{P}=\frac{\frac{1}{2}nf\dot{z}z^{n-1}}{\sqrt{e+fz^n}}=\dot{T}$, adeoque partis prioris $\frac{2}{n}dP$ fluxio est $d\dot{z}z^{-1}\times\frac{fz^n}{\sqrt{e+fz^n}}$. Est autem $\frac{-RR}{SS}=\frac{-e}{fz^n}$, atque inde fluxio mensuræ $R\left|\frac{R+T}{S}\right.$ (quæ per Regulam priorem erat $\frac{-RR}{SS}\dot{T}$) erit $\frac{-e}{fz^n}\times\frac{\frac{1}{2}nf\dot{z}z^{n-1}}{\sqrt{e+fz^n}}$. Itaque partis posterioris $\frac{-2}{n}dR\left|\frac{R+T}{S}\right.$ fluxio est $\frac{-2}{n}d\times\frac{-e}{fz^n}\times\frac{\frac{1}{2}nf\dot{z}z^{n-1}}{\sqrt{e+fz^n}}$ vel $d\dot{z}z^{-1}\times\frac{e}{\sqrt{e+fz^n}}$ & fluxionum summa fit $d\dot{z}z^{-1}\times\frac{e+fz^n}{\sqrt{e+fz^n}}$ $=d\dot{z}z^{-1}\sqrt{e+fz^n}$. Theorema igitur recte se habet. Similis esset operatio per Regulam posteriorem eademque conclusio, si modo Auctor posuisset $K=\sqrt{-e}$ & Theorema illud sic designasset $\frac{2}{n}dP-\frac{2}{n}dK\left(\frac{K.T}{S}\right.$. Nam Theoremata fluentes exhibent revera Logometricas, quæ hac unica mutatione convertuntur in Trigonometricas.

Ubicunque Rationum Mensuræ occurrant, quarum termini non designantur per terminos triangulares R, T, S, sed per quoslibetcunque absolutos: hujusmodi mensurarum fluxiones dantur immediate

diate per Cor. 1. Prop. I. *Logometriæ*. Sic in Forma I $\frac{d\dot{z}z^{\theta\eta-1}}{e+fz^{\eta}}$, si fuerit $\theta=0$ & quæratur fluxio Theorematis $\frac{-d}{\eta e}\Big|\frac{e+fz^{\eta}}{fz^{\eta}}$: hæc mensura æqualis est mensuris $\frac{-d}{\eta e}\Big|\frac{e+fz^{\eta}}{e}+\frac{d}{\eta e}\Big|\frac{fz^{\eta}}{e}$ simul sumptis, & earum fluxiones $\frac{-d}{\eta e}\times\frac{\eta f\dot{z}z^{\eta-1}}{e+fz^{\eta}}$ & $\frac{d}{\eta e}\times\frac{\eta f\dot{z}z^{\eta-1}}{fz^{\eta}}$ simul sumptæ faciunt $d\dot{z}z^{-1}$ in $\frac{-fz^{\eta}}{e\times\overline{e+fz^{\eta}}}+\frac{1}{e}$ vel $d\dot{z}z^{-1}\times\frac{-fz^{\eta}+e+fz^{\eta}}{e\times\overline{e+fz^{\eta}}}$ vel $\frac{d\dot{z}z^{-1}}{e+fz^{\eta}}$; Theorema igitur recte se habet.

Nota VI.

De Methodo Continuandi hæc Theoremata.

POSTQUAM unam alteramve e simplicioribus cujusque Formæ Fluentibus ad examen revocasti, innumeras exinde alias invenire possis per *Theoremata pro Continuatione Formarum & Fluentium ad infinitum*. Vide p. 66 in qua Theorema I sic se habet. Positis $Z=e+fz^{\eta}$; $\dot{A}=d\dot{z}z^{\theta\eta-1}Z^{\omega-1}$; $\dot{B}=d\dot{z}z^{\theta\eta+\eta-1}Z^{\omega-1}$; $\dot{F}=d\dot{z}z^{\theta\eta-1}Z^{\omega}$: Æquatio I erit $\frac{1}{\eta}dz^{\theta\eta}Z^{\omega}=\theta eA+\overline{\theta+\omega},fB$; ubi si detur A dabitur B & vice versa. Æquatio II erit $F=eA+fB$; ubi e tribus fluentibus A, B, F, si dentur duæ quævis dabitur tertia. Æquatio III erit $A=\frac{\frac{-1}{\eta}dz^{\theta\eta}Z^{\omega}+\overline{\theta+\omega},F}{\omega e}$; ubi si detur F dabitur A & vicissim. Æquatio IV erit $B=\frac{\frac{1}{\eta}dz^{\theta\eta}Z^{\omega}-\theta F}{\omega f}$; ubi si detur F dabitur B & vice versa. Atque operationibus successive repetitis, si in his Fluxionibus tam Index $\theta\eta-1$ perpetua additione vel subductione quantitatis η, quam Index ω perpetua additione vel subductione unitatis, utcunque augeatur vel minuatur: dabuntur Fluentes fluxionum omnium sic prodeuntium ad infinitum. Res exemplis illustrabitur.

Exemplum 1. Detur fluens $\frac{2}{\eta f} dR \left| \frac{R+T}{S} \right.$ fluxionis $\frac{\dot{z}z^{\frac{1}{2}\eta - 1}}{\sqrt{e+fz^\eta}}$, & quæratur fluens fluxionis $\frac{d\dot{z}z^{\frac{1}{2}\eta-1}}{\sqrt{e+fz^\eta}}$: ubi reliquis permanentibus quantitas η adjicitur indici $\frac{1}{2}\eta - 1$. In Theoremate I ponantur $\dot{A} = d\dot{z}z^{\theta\eta - 1} Z^{\omega - 1} = \frac{d\dot{z}z^{\frac{1}{2}\eta - 1}}{\sqrt{e+fz^\eta}}$; $\dot{B} = d\dot{z}z^{\theta\eta + \eta - 1} Z^{\omega - 1} = \frac{d\dot{z}z^{\frac{1}{2}\eta - 1}}{\sqrt{e+fz^\eta}}$, & erunt $\theta = \frac{1}{2}$, $\omega = \frac{1}{2}$: quibus scriptis in æquatione I^2, $\frac{1}{\eta} dz^{\theta\eta} Z^\omega = \theta e A + \overline{\theta + \omega}, fB$; evadet $\frac{1}{\eta} dz^{\frac{1}{2}\eta} \sqrt{e+fz^\eta} = \frac{1}{2} eA + fB$. Scribe fluentem datam pro A & habebis fluentem quæsitam $B = \frac{z^\eta}{\eta f} d\sqrt{\frac{e+fz^\eta}{z^\eta}} - \frac{e}{\eta ff} dR \left| \frac{R+T}{S} \right.$. Simili opera dabitur fluens fluxionis proxime superioris & sic ad infinitum.

Exemplum 2. Ab eadem fluente $\frac{2}{\eta f} dR \left| \frac{R+T}{S} \right.$ fluxionis $\frac{d\dot{z}z^{\frac{1}{2}\eta - 1}}{\sqrt{e+fz^\eta}}$ continuanda sit progressio in partem contrariam, quærendo fluentem fluxionis $\frac{d\dot{z}z^{\frac{-1}{2}\eta - 1}}{\sqrt{e+fz^\eta}}$: ubi quantitas η detrahitur ab indice $\frac{1}{2}\eta - 1$. In eodem Theoremate ponantur $\dot{A} = d\dot{z}z^{\theta\eta - 1} Z^{\omega - 1} = \frac{d\dot{z}z^{\frac{-1}{2}\eta - 1}}{\sqrt{e+fz^\eta}}$; $\dot{B} = d\dot{z}z^{\theta\eta + \eta - 1} Z^{\omega - 1} = \frac{d\dot{z}z^{\frac{1}{2}\eta - 1}}{\sqrt{e+fz^\eta}}$, & erunt $\theta = -\frac{1}{2}$, $\omega = \frac{1}{2}$: quibus scriptis in æquatione $\frac{1}{\eta} dz^{\theta\eta} Z^\omega = \theta e A + \overline{\theta + \omega}, fB$; evadet $\frac{1}{\eta} dz^{\frac{-1}{2}\eta} \sqrt{e+fz^\eta} = -\frac{1}{2} eA$. Est igitur quæsita fluens $A = \frac{-2}{\eta e} d\sqrt{\frac{e+fz^\eta}{z^\eta}}$. Simili opera dabitur fluens fluxionis proxime inferioris & sic ad infinitum.

Exemplum 3. Porro ex eadem fluente $\frac{2}{\eta f} dR \left| \frac{R+T}{S} \right.$ fluxionis $\frac{d\dot{z}z^{\frac{1}{2}\eta - 1}}{\sqrt{e+fz^\eta}}$ invenienda sit fluens fluxionis $d\dot{z}z^{\frac{1}{2}\eta - 1} \sqrt{e+fz^\eta}$: ubi reliquis permanentibus unitas adjicitur indici binomii $e+fz^\eta$. In Theoremate I ponantur $\dot{A} = d\dot{z}z^{\theta\eta - 1} Z^{\omega - 1} = \frac{d\dot{z}z^{\frac{1}{2}\eta - 1}}{\sqrt{e+fz^\eta}}$;

$\dot{F} = d\dot{z}z^{\theta\eta-1}Z^{\omega} = d\dot{z}z^{\frac{1}{2}\eta-1}\sqrt{e+fz^{\eta}}$; & erunt $\theta = \frac{1}{2}$, $\omega = \frac{1}{2}$. In æquatione tertia ad hanc reducta $\frac{1}{\eta}dz^{\theta\eta}Z^{\omega} + \omega e A = \overline{\theta+\omega}$, F ſcribantur iſti valores & evadet $\frac{1}{\eta}dz^{\frac{1}{2}\eta}\sqrt{e+fz^{\eta}} + \frac{e}{\eta f}dR\left|\frac{R+T}{S}\right. = F$, quæ erat invenienda. Et eadem opera dabitur fluens fluxionis $d\dot{z}z^{\frac{1}{2}\eta-1} \times \overline{e+fz^{\eta}}\big|^{\frac{3}{2}}$ & ſic ad infinitum.

Denique ſi fluens iſthæc primo propoſita ſcribatur pro F & quæratur A, continuabitur fluentium progreſſio in partem contrariam, perpetua ſubductione unitatis ab indice binomii in earum fluxionibus.

Explicato Theoremate primo reliqua exinde facile intelligentur. Eſt enim ſimilitudo omnium & cognatio quædam inter ſe. Notandum vero, fluentium progreſſionem infinitam, ab eadem fluente data verſus partes contrarias pergentem, non ſemper datam iri per regulas præcedentes. Evaneſcet enim quæſita fluens A vel B vel F ex æquatione Theorematis, ſi coefficiens ejus fuerit nihilo æqualis. Sic in Forma I, ſi detur fluens fluxionis $\frac{d\dot{z}z^{\eta-1}}{e+fz^{\eta}}$ & quæratur fluens fluxionis $\frac{d\dot{z}z^{-1}}{e+fz^{\eta}}$: relatis fluxionibus ad Theorema I, erit $\theta = 0$ & $\omega = 0$ adeoque evaneſcent A & B ex æquatione prima. In hoc caſu reducenda eſt fluxio prior ad formam poſterioris, ducendo numeratorem & denominatorem in $z^{-\eta}$. Sic enim $\frac{d\dot{z}z^{-1}}{e+fz^{\eta}}$ fit æqualis $\frac{d\dot{z}z^{-\eta-1}}{f+ez^{-\eta}}$ & ejuſdem formæ ac $\frac{d\dot{z}z^{\eta-1}}{e+fz^{\eta}}$. Quare ſcribendo $-\eta, f, e$ pro η, e, f erit $\frac{d\dot{z}z^{\eta-1}}{e+fz^{\eta}} = \frac{d\dot{z}z^{-\eta-1}}{f+ez^{-\eta}}$ & prioris fluens $\frac{d}{\eta f}\left|\frac{e+fz^{\eta}}{e}\right.$ evadet $\frac{-d}{\eta e}\left|\frac{f+ez^{-\eta}}{f}\right.$ vel $\frac{-d}{\eta e}\left|\frac{e+fz^{\eta}}{fz^{\eta}}\right.$, quæ proinde eſt fluens fluxionis $\frac{d\dot{z}z^{-1}}{e+fz^{\eta}}$.

NOTA

NOTA VII.

Demonſtratio Theorematum pro Continuatione Fluentium ad infinitum.

THEOREMATIS I æquatio prima $\frac{1}{n} dz^{\theta n} Z^{\omega} = \theta e A + \overline{\theta + \omega}, f B$ recte ſe habebit ſi partium fluxiones fuerint utrobique æquales. Fluxio partis prioris eſt $\theta d\dot{z}z^{\theta n - 1} \times \overline{e + fz^{n}}|^{\omega} + \omega f d\dot{z}z^{\theta n + n - 1} \times \overline{e + fz^{n}}|^{\omega - 1}$ ſive $\theta \times d\dot{z}z^{\theta n - 1} \times \overline{e + fz^{n}}|^{\omega - 1} \times \overline{e + fz^{n}} + \omega f d\dot{z}z^{\theta n + n - 1} \times \overline{e + fz^{n}}|^{\omega - 1}$ hoc eſt (per hypotheſin) $\theta e \dot{A} + \theta f \dot{B} + \omega f \dot{B}$, quæ eſt fluxio partis poſterioris.

Porro æquatio ſecunda $F = e A + f B$ fluxiones exhibet utrobique æquales. Nam per hypotheſin $\dot{F} = d\dot{z}z^{\theta n - 1} \times \overline{e + fz^{n}}|^{\omega} = d\dot{z}z^{\theta n - 1} \times \overline{e + fz^{n}}|^{\omega - 1} \times \overline{e + fz^{n}} = e \dot{A} + f \dot{B}$.

Denique ex his æquationibus datis dabuntur reliquæ per notas analyſeos regulas. Eodem modo demonſtrabis theoremata reliqua.

METHO-

METHODUS

COMPONENDI

DEMONSTRATIONES

LOGOMETRICAS ET TRIGONOMETRICAS

EXEMPLIS

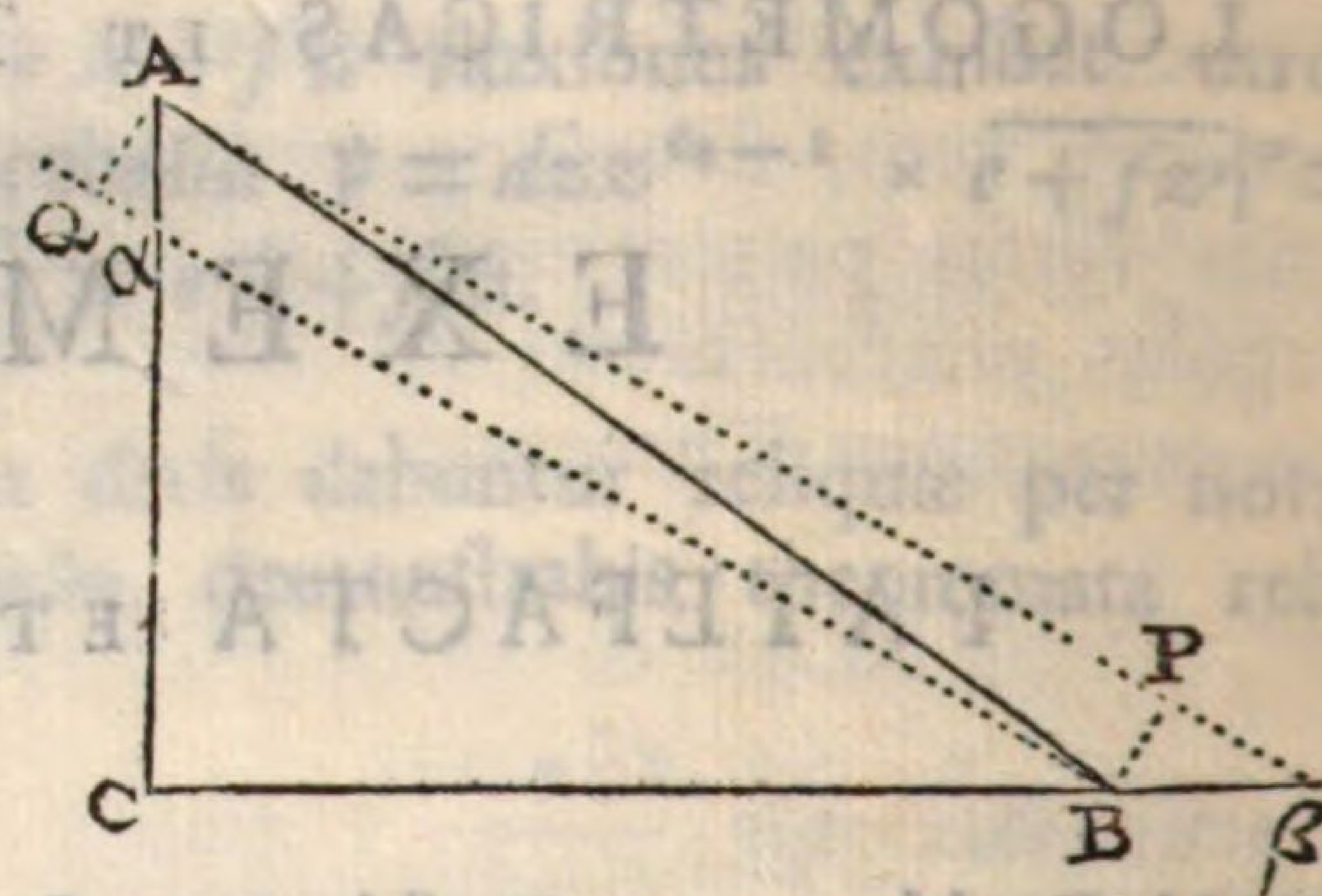
PATEFACTA ET EXPLANATA.

IN Problematum Reſolutione & Conſtructione, Methodi ſuæ præſtantiam Auctor noſter plurimis exemplis declaravit. Sed in Demonſtrationum Compoſitione exemplum dedit nullum; in eo nimirum conſtructionum caſu, qui ſæpiſſime occurrit, in quo dimetiendæ rationis termini triangulum rectangulum conſtituunt. Viſum eſt itaque de hac re Exempla quædam proponere. Hæc autem ſelegi ex Auctoris Conſtructionibus, cum de Curvarum Longitudine, Areis, Solidis inde genitis & ſolidorum Superficiebus, tum de Problematis phyſicis.

LEMMA.

Si triangulum rectangulum, per variationes quascunque minimas, mutetur in triangulum novum rectangulum utcunque positum: idem erit incrementum mensuræ Anguli utriusvis acuti ad Secantem pro modulo, ac mensuræ Rationis inter summam Secantis & Tangentis ejusdem anguli atque Radium ad Radium pro modulo.

MUTARI intelligatur triangulum *ABC*, rectangulum ad *C*, in triangulum Novum rectangulum utcunque positum; huic simile constituatur triangulum *AβC* vel *αBC*, & ad rectam *Aβ* vel *Bα* productam normalis sit *BP* vel *AQ*. Harum utravis est incrementum mensuræ Anguli *CAB* ad modulum *AB* per Prop. in Nota I. Porro cum Ratio proposita inter *AB*+*BC* & *CA* mutatur in rationem novam rationi inter *αB*+*BC* & *Cα* vel inter *Aβ*+*βC* & *CA* æqualem, & compositam ex rationibus *Aβ*+*βC* ad *AB*+*BC* & *AB*+*BC* ad *CA*: incrementum mensuræ rationis posterioris ad modulum *CA*, id est mensura prioris erit etiam *BP* vel *AQ* per Cor. 2. Prop. I. *Logometriæ*; quippe quæ est ad modulum *CA* ut *Bβ*+*βP* ad *AB*+*BC* vel (propter similia triangula *BβP*, *ABC*,) ut *BP* ad *CA*. *Q.E.D.*



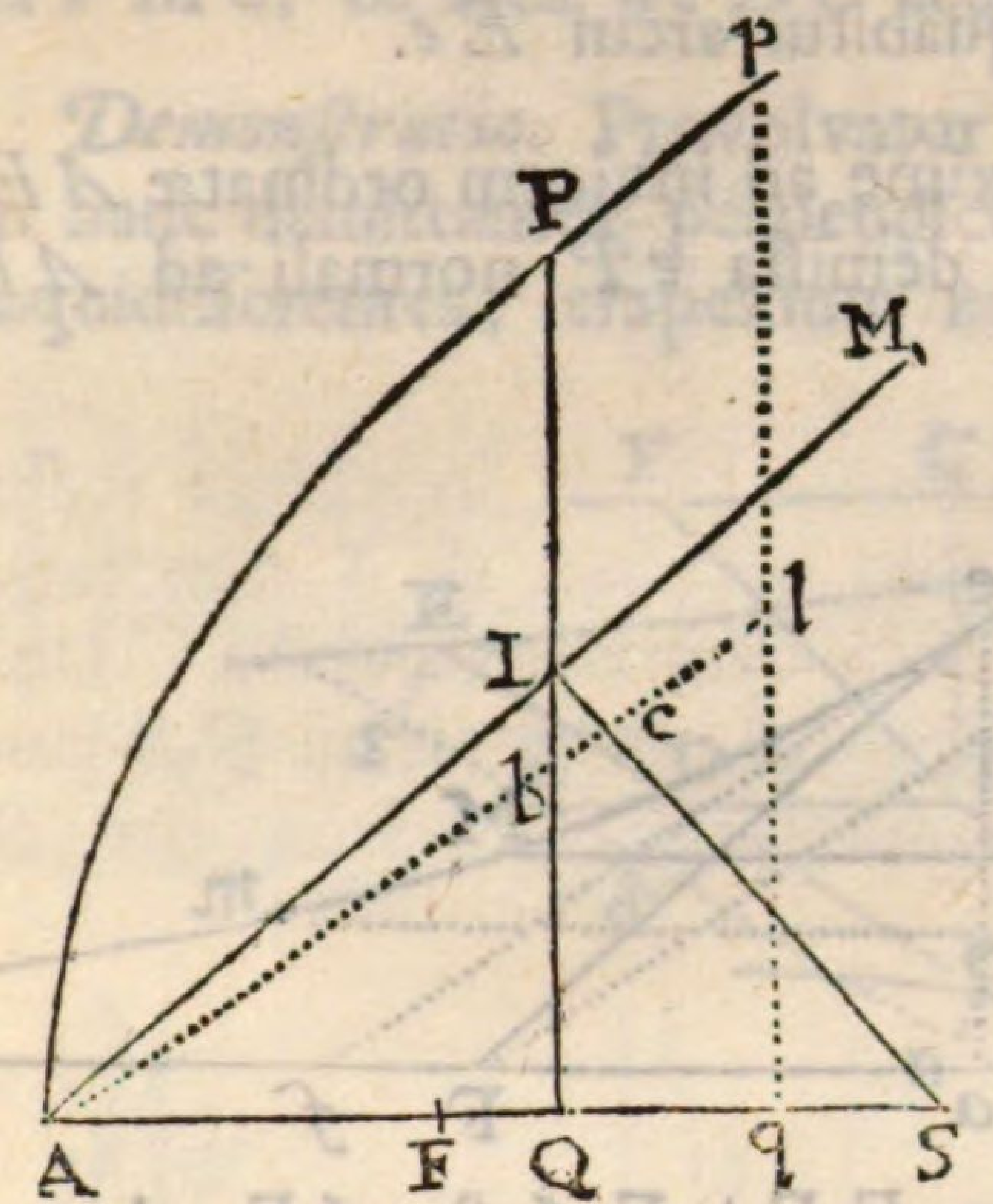
EXEMPLUM I.

De Longitudine arcus Parabolici.

Constructio. PARABOLÆ *Apollonianæ* *AP* sit *A* vertex, *F* focus, *AQ* axis, *PQ* ordinatim applicata ad axem. Ducatur *AL* quæ bifariam secet *PQ* in *L*, & productæ adjiciatur *LM* quæ sit mensura rationis inter *LA*+*AQ* & *QL* ad modulum *AF*: & recta *AM* æqualis erit arcui parabolico *AP*.

Demon-

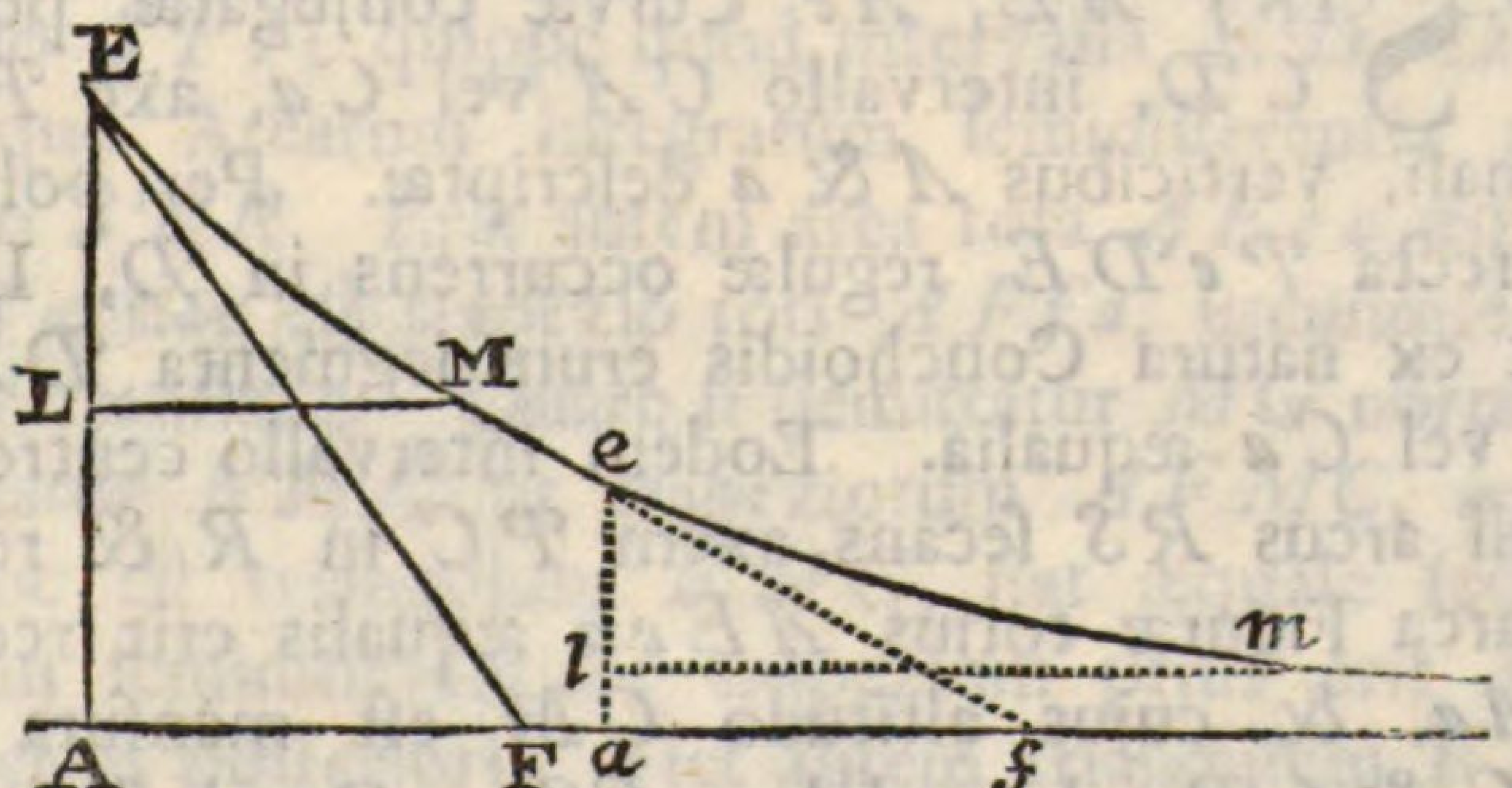
Demonstratio. Esto pq ordinata proxime major quam PQ; ducatur Al quæ bifariam dividat pq in l seceteque LQ in b: & ad AL normalis sit LS ipsi Al occurrens in c, axi vero in S. Erit ergo cl incrementum rectæ AL. Porro cL (quæ est incrementum mensuræ anguli QLA ad modulum LA) incrementum erit mensuræ rationis inter $LA+AQ$ & QL ad modulum QL per Lemma: adeoque propter proportionales cL & cb, QL & QS, erit cb incrementum mensuræ ejusdem rationis ad modulum QS per Cor. 3. Prop. I. *Logometriæ*, vel ex natura parabolæ ad modulum AF. Quare totius AM incrementum est bl sive Pp propter parallelogrammum $Pplb$ ex natura tangentis per p ductæ. Sed a nihilo simul incipiunt & curva AP & recta AL atque etiam mensura rationis inter $LA+AQ$ & QL, (quippe quæ recidente puncto P in punctum A fit ratio æqualitatis;) adeoque constat constructio. *Q.E.D.*



EXEMPLUM II.

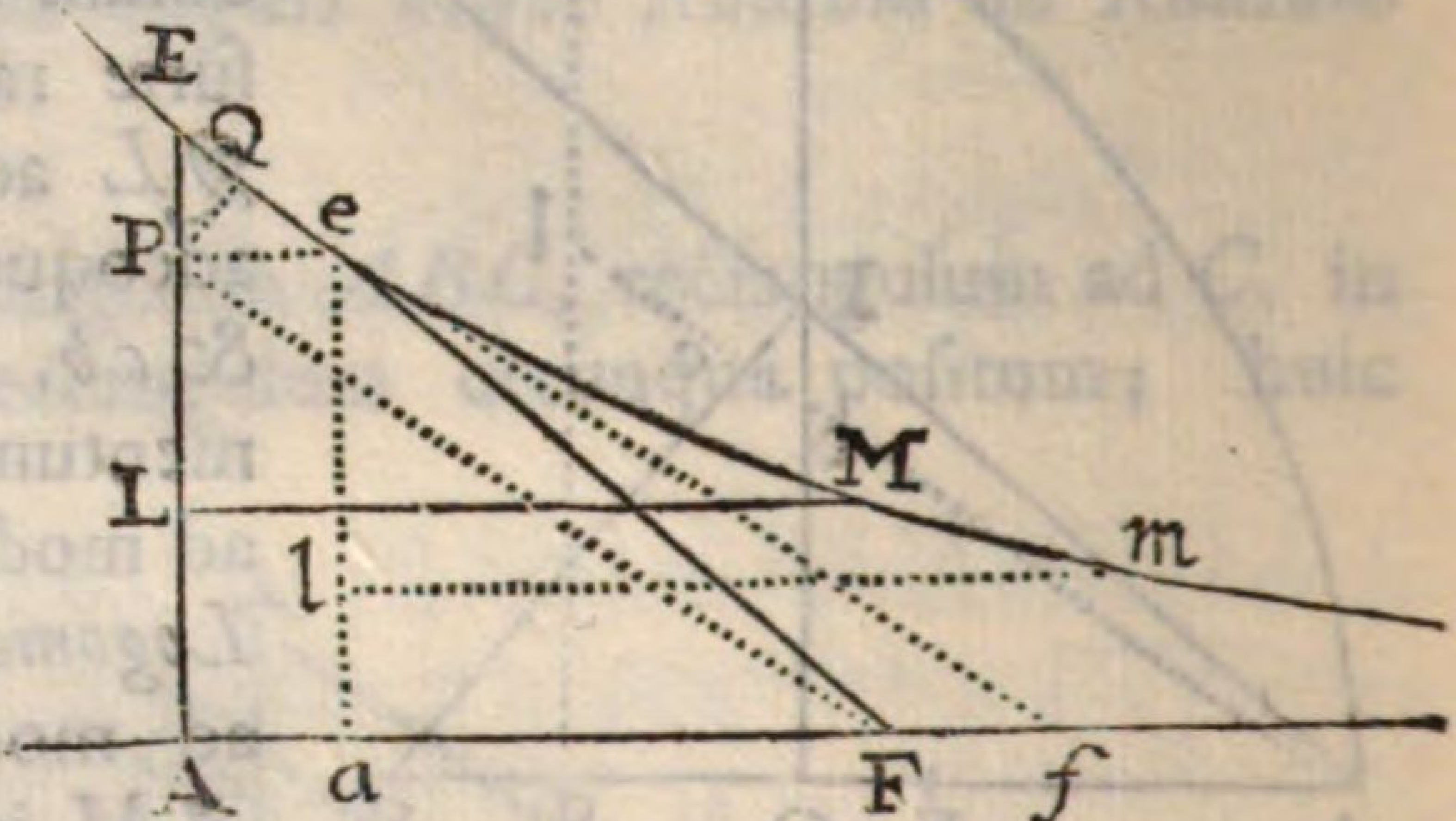
De Longitudine Lineæ Logisticæ.

Constructio. OBLATA sit Logistica $EMem$, cujus asymptotos $AFaf$: & quæratur longitudo arcus Ee. Demissis ad asymptoton perpendiculis ELA, ela & ductis tangentibus EF, ef; capiatur AL æqualis excessui quo tangens EF su-



perat subtangentem AF, & similiter al æqualis excessui quo tangens ef superat subtangentem af: & actis LM, lm asymptoto parallelis, si tangentium differentiæ $EF - ef$ adjiciatur parallelarum differentia $lm - LM$, aggregatum æquabitur arcui Ee.

Demonstratio. Accedant quam proxime ad invicem ordinatæ AE, ae reliquæque rectæ adscriptæ: & demissa eP normali ad AE ductaque PF, hæc erit parallela & æqualis tangenti ef, propter AF, af æquales ex natura Curvæ. Quare demissa PQ normali ad EF vel Ee, lineola EQ æqualis erit $EF - ef$. Porro QP (quæ est incrementum mensuræ anguli AEF ad modulum QF vel EF) incrementum erit mensuræ rationis inter $EF + FA$ & AE ad modulum AE per Lemma: adeoque propter proportionales QP & Qe, AE & AF, erit Qe incrementum mensuræ ejusdem rationis inter $EF + FA$ & AE vel inter AE & $EF - FA$ vel inter AE & AL ad modulum AF; & proinde (per Prop. V. *Logometriæ*) incrementum erit rectæ LM & æqualis $lm - LM$. Recedant jam ab invicem ordinatæ AE, ae, & colligendo similes ubique incrementorum summas constabit constructio. *Q.E.D.*



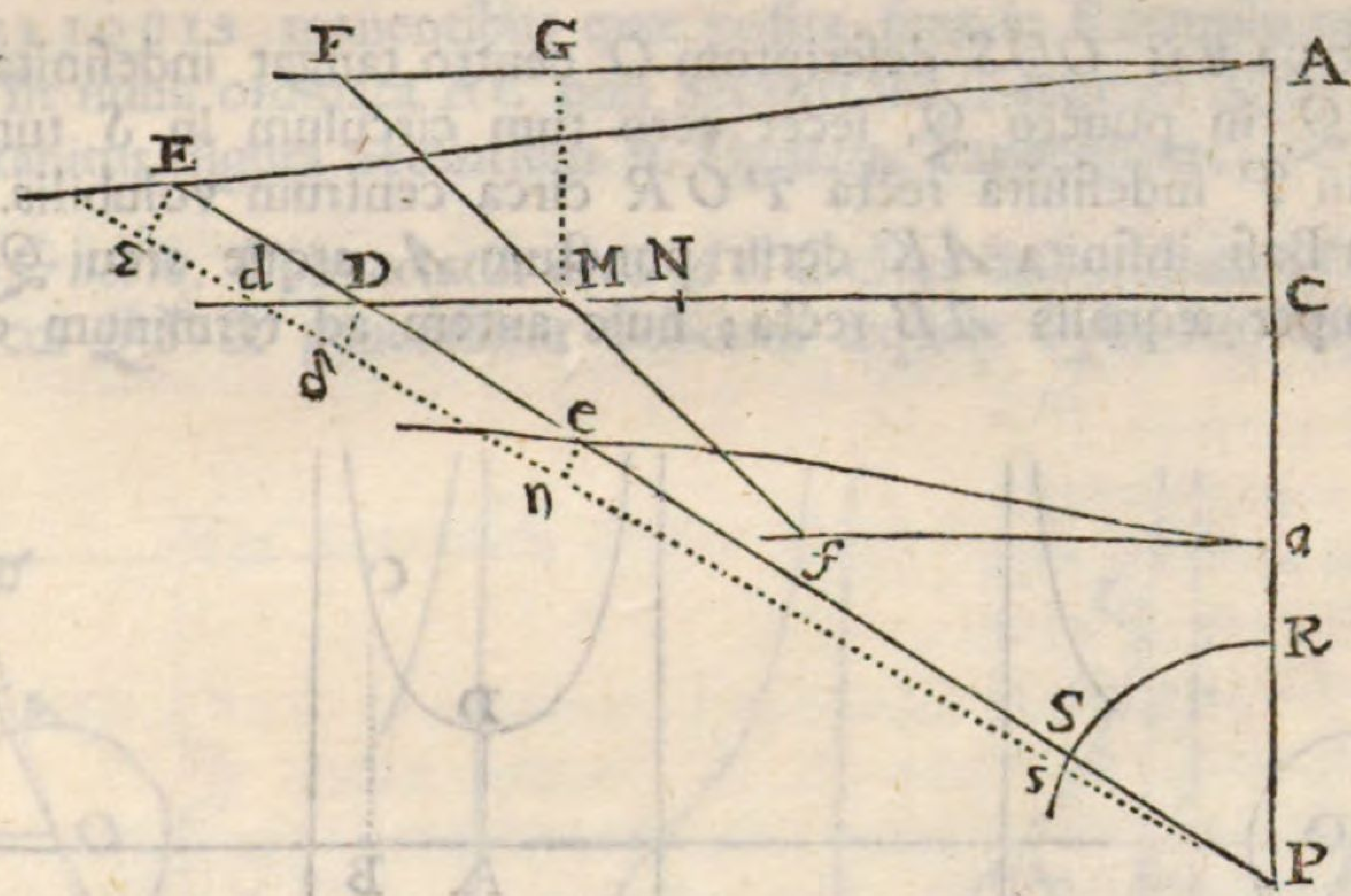
EXEMPLUM III.

De Area Conchoidis.

Constructio. SINT AE, Ae Curvæ conjugatæ polo P, regula CD, intervallo CA vel Ca, axe $PaCA$ ad regulam normali, verticibus A & a descriptæ. Per polum P ducatur ad libitum recta $PeDE$ regulæ occurrens in D, Lineæ vero in E & e: & ex natura Conchoidis erunt segmenta DE, De intervallo CA vel Ca æqualia. Eodem intervallo centroque P describatur circuli arcus RS secans axem PC in R & rectam PD in S: atque area Figuræ totius $AEea$ æqualis erit rectangulo cujus basis est Aa, & cujus altitudo CM est mensura rationis inter $PD + DC$ & CP ad modulum CP. Quod si desideretur quadratura partium $AEDC$, $aeDC$; ductis ad axem normalibus AF, af, in regula CD sumenda est CN quæ sit anguli CPD men-

mensura ad eundem Modulum CP: & acta per punctum M recta FMf quæ parallela sit rectæ jungenti puncta P, N, quæque occurrat normalibus in F & f; erit area $AEDC$ æqualis Trapezio $AFMC$, & area $aeDC$ æqualis Trapezio $afMC$.

Demonstratio. Provolvatur PSD in situm proximum Psd, & in hanc demittantur perpendiculares $E\varepsilon$. $D\delta$, $e\eta$; & cum hæ sint æquidifferentes, trapezium minimum $Ee\eta\varepsilon$, hoc est incrementum



Areæ $AEea$, erit æquale rectangulo $D\delta \times Ee$ vel $D\delta \times Aa$. Idem vero est incrementum Rectanguli $CM \times Aa$; quippe quod $D\delta$ est incrementum altitudinis CM id est mensuræ rationis inter $PD + DC$ & CP ad Modulum CP, per Lemma.

Incrementa arearum $AEDC$, $aeDC$ sunt æqualia trapeziis $ED\delta\varepsilon$, $eD\delta\eta$ respective, atque horum semidifferentia æqualis est sectori minimo SPs; quippe quod intervalla ED, De, SP sunt æqualia. Itaque Arearum integrarum semidifferentia æqualis est Sectori integro SPR. Erat autem area tota $AEea$ æqualis rectangulo $CM \times Aa$ hoc est trapezio toti $AFfa$, utcunque ducta fuerit FMf per M transiens. Quare si demittatur MG normalis ad AF, atque ita ducatur FMf, ut trapeziorum $AFMC$, $afMC$ semidifferentia, hoc est triangulum FMG, fiat æquale sectori SPR, id est arearum semidifferentiæ: trapezium prius erit æquale areæ priori & posterius posteriori. Fiet autem triangulum FMG æquale sectori SPR, si ponatur GF æqualis arcui RS, id est mensuræ anguli RPS ad Modulum RP vel GM: Atqui CN est mensura

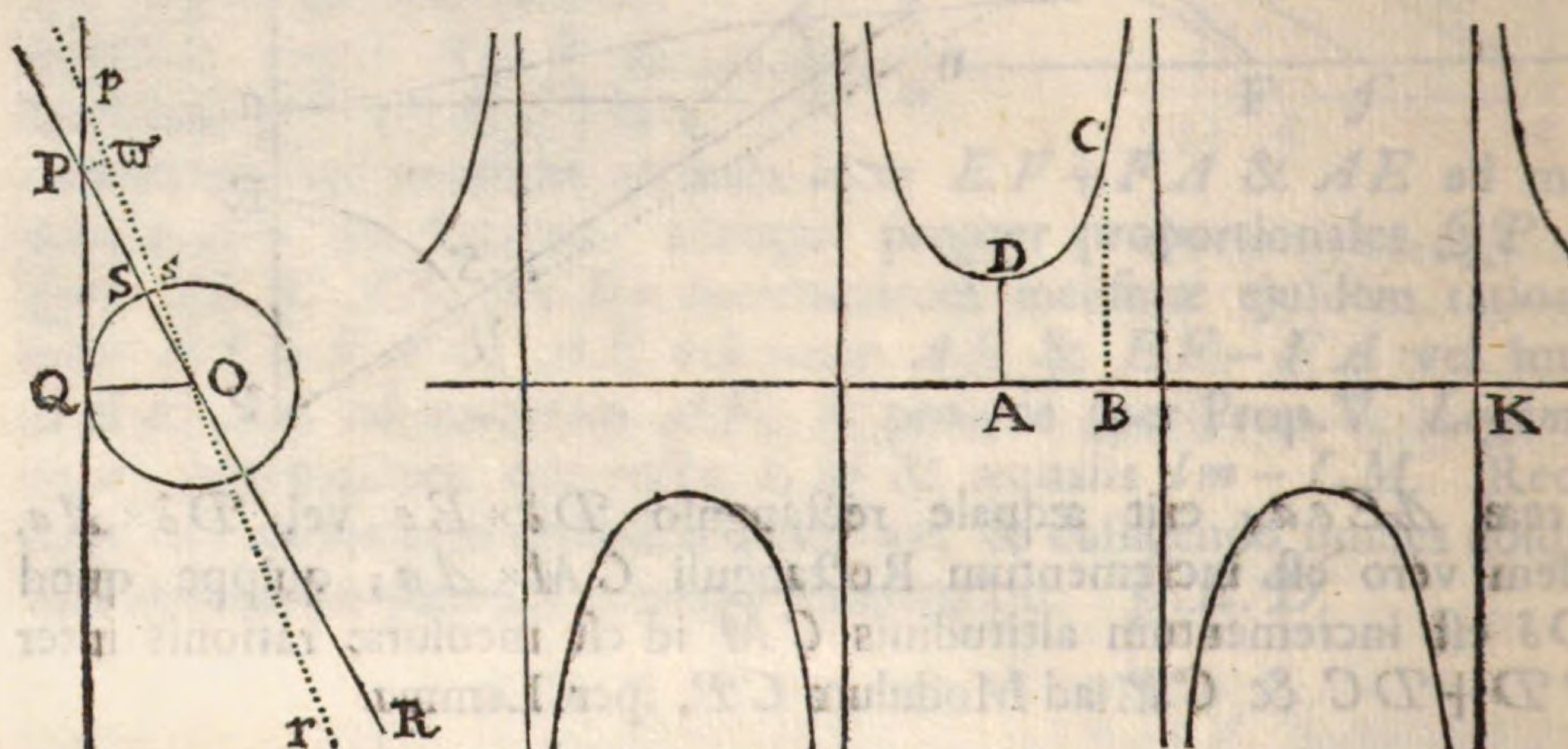
 ejusdem

ejusdem anguli ad Modulum CP, adeoque (per Corol. 2. Prop. in Nota I.) proportionales erunt GF & GM, CN & CP. Et proinde recta FMf parallela est rectæ jungenti puncta P, N. *Q. E. D.*

EXEMPLUM IV.

De Area Figuræ Secantium.

CIRCULUM OQS descriptum O centro tangat indefinita recta PQ in puncto Q, secet vero tum circulum in S tum tangentem in P indefinita recta POR circa centrum volubilis. Deinde si in Basi infinita AK detur punctum A, atque arcui QS ponatur semper æqualis AB recta; huic autem ad terminum ejus B



insistat ordinata rectangula BC Secanti OP æqualis: generabit hæc ordinata ductu ejus in basem AK Figuram illam Secantium quam quadrare volumus.

Constructio. Ducta ordinata AD; Area $ABCD$, quæ Basem habet AB æqualem arcui QS, æqualis erit mensuræ rationis inter summam Secantis & Tangentis ejusdem arcus atque Radium ad Quadratum radii pro modulo.

Demonstratio. Provoluta quam minimum recta OSP in situm rectæ Osp & in hanc demissa normali $P\varpi$: Incrementum mensuræ rationis istius inter $OP+PQ$ & OQ ad modulum OQ erit

$P\varpi$

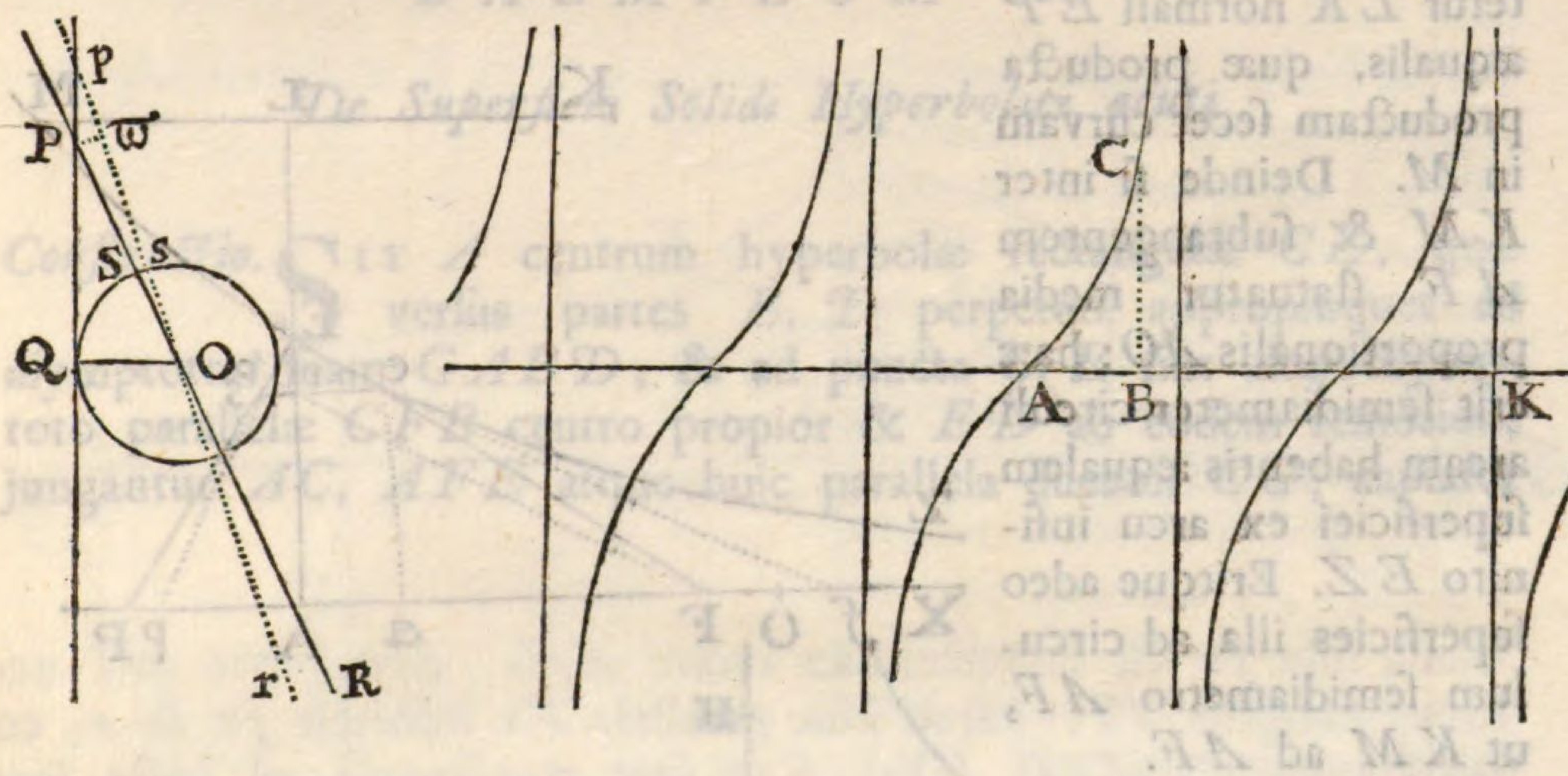
$P\varpi$ per Lemma. Adeoque incrementum mensuræ rationis ejusdem ad modulum OQ *quad.* est $OQ \times P\varpi$ vel $OS \times P\varpi$ vel $OP \times Ss$ vel $BC \times Ss$, quod est incrementum areæ $ABCD$. *Q.E.D.*

EXEMPLUM V.

De Solido Figuræ Tangentium.

RELIQUIS manentibus quæ posita sunt in Exemplo novissimo, sit nunc ordinata BC non Secanti sed Tangenti QP æqualis, & mutabitur Figura Secantium in Figuram Tangentium.

Constructio. Convertatur area ABC circa basem suam AB æqualem arcui QS & generabitur Solidum æquale Cylindro, cujus Basis



est Figuræ circulus genitor $OQSs$ & cujus Altitudo est excessus Tangentis ejusdem arcus supra arcum ipsum.

Demonstratio. Concipe ductam esse ϖt perpendicularem ad Pp, & quoniam Cylindri altitudo est $QP - QS$, altitudinis incrementum erit $Pp - Ss$ vel pt. Nam cum sit $P\varpi$ ad Pt ut OP ad OQ, & $P\varpi$ ad Ss in eadem ratione: erit Ss æqualis ipsi Pt. Est autem pt ad Pt ut pt^2 ad ϖt^2 ut pQ *quad.* vel BCq ad OQq sive ut solidi Tangentium basis circularis ad basem cylindri. Unde pt ducta in basem cylindri æqualis est Pt vel Ss ductæ in basem Solidi, hoc est incrementa utriusque Solidi sunt semper æqualia. *Q.E.D.*

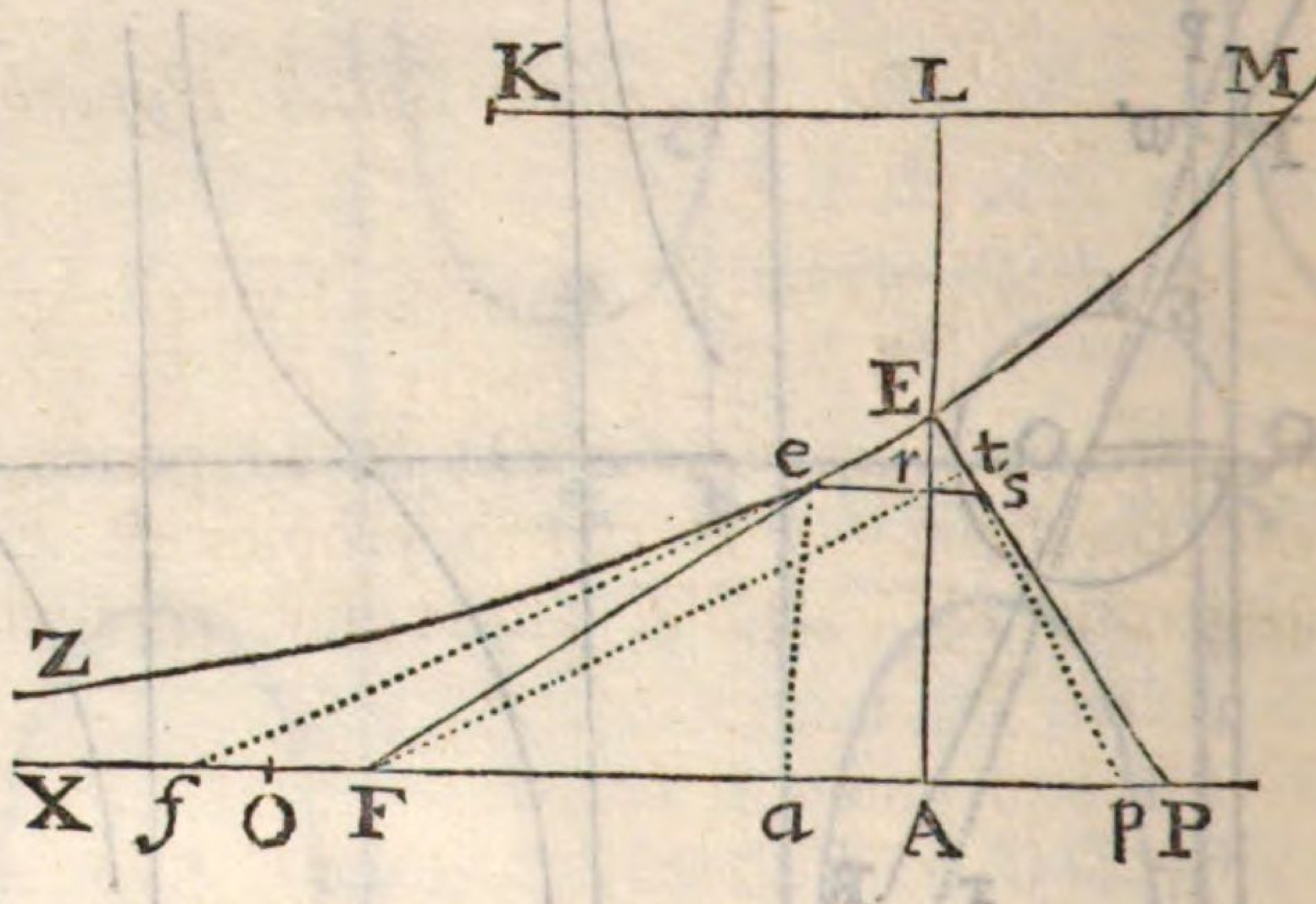
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EXEMPLUM VI.

De Superficie Solidi Logistici.

REVOLVATUR Logisticæ arcus infinitus EZ circum asymptoton suam AX, ad quam perpetuo appropinquet versus partes Z, X: & quæratur AO Radius Circuli cujus Area sit æqualis Superficiei ex isto motu genitæ.

Constructio. Fiat EP normalis ad curvam in puncto E & attingat asymptoton in P; producatur ordinata rectangula AE ultra punctum E ad L, ut sit AL æqualis aggregato linearum AP, PE; huic ad quas partes jacet curva EZ perpendicularis excitetur LK normali EP æqualis, quæ producta productam secet curvam in M. Deinde si inter KM & subtangentem AF statuatur media proportionalis AO: hæc erit semidiameter circuli aream habentis æqualem superficiei ex arcu infinito EZ. Eritque adeo superficies illa ad circulum semidiametro AF, ut KM ad AF.



Demonstratio. Ducatur tum ordinata ae proxime minor quam AE, tum recta ers asymptoto parallela & occurrens rectis AE, EP in r & s; agatur Frt normali EP occurrens in t; & completo parallelogrammo $Fref$, erit ef tangens nova propter subtangentes AF, af æquales ex natura Curvæ. Porro sit sp normalis ad fe vel Fr & attingat asymptoton in p. His positis, rectæ KL vel normalis EP decrementum erit $Es+sP-sp$ vel $Es+ts$. Nam propter similia triangula sPp, FEr, erit $sP-sp$ ad $FE-Fr$ vel rt, ut sP vel EP ad FE ut ts ad rt. Porro tE (quæ est decrementum mensuræ anguli AFE ad modulum FE) erit decrementum mensuræ rationis inter $FE+EA$ & AF vel inter

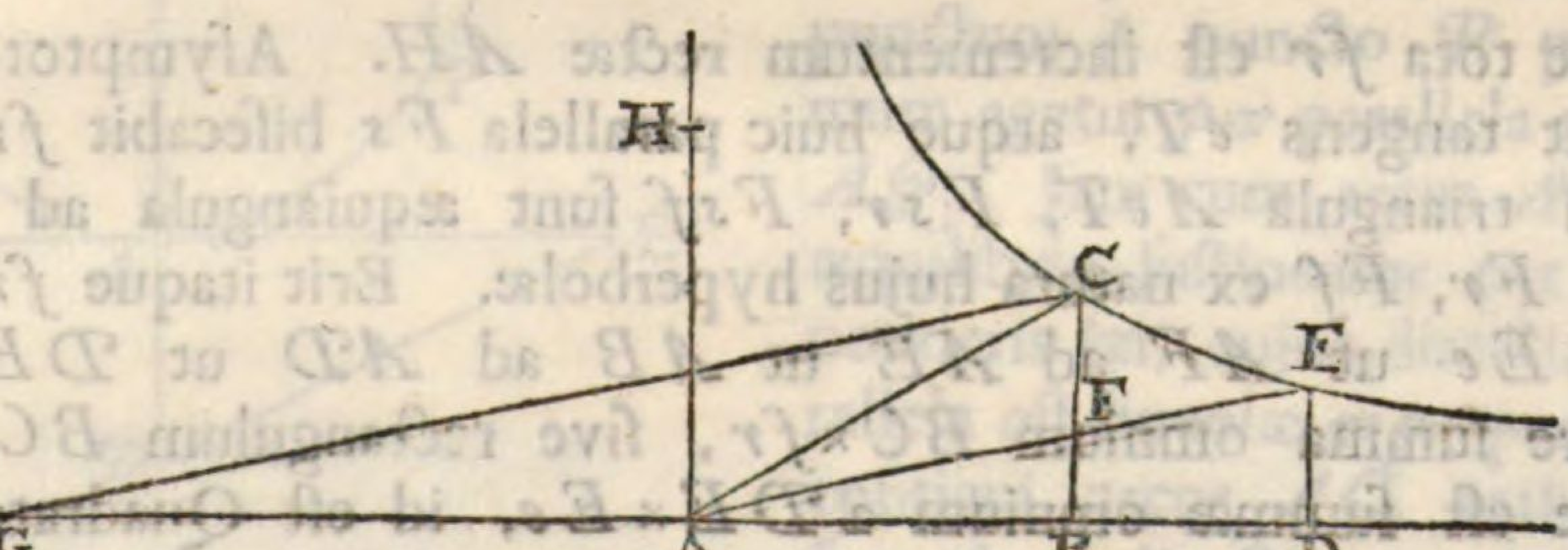
EP

$EP+PA$ & AE vel inter AL & AE ad modulum AF per Lemma; adeoque erit decrementum rectæ LM per Prop. V. *Logometriæ*. Quare $2Es$ est decrementum totius KM; atque inde propter proportionales Es & Ee, AE & AF, summa omnium $2Es \times AF$, sive rectangulum $KM \times AF$ æquale est summæ omnium $2AE \times Ee$ hoc est AO *quad.* Nam summa omnium $2AE \times Ee$ est ad Superficiem ex elementis similibus similiter conflatam, in data ratione elementorum: id est ut circuli diameter ad peripheriam, vel ut semidiameter ad semiperipheriam, vel denique ut Quadratum semidiametri ad Circulum ipsum. Quare si consequentes æquantur, etiam antecedentes æquales erunt. *Q. E. D.*

EXEMPLUM VII.

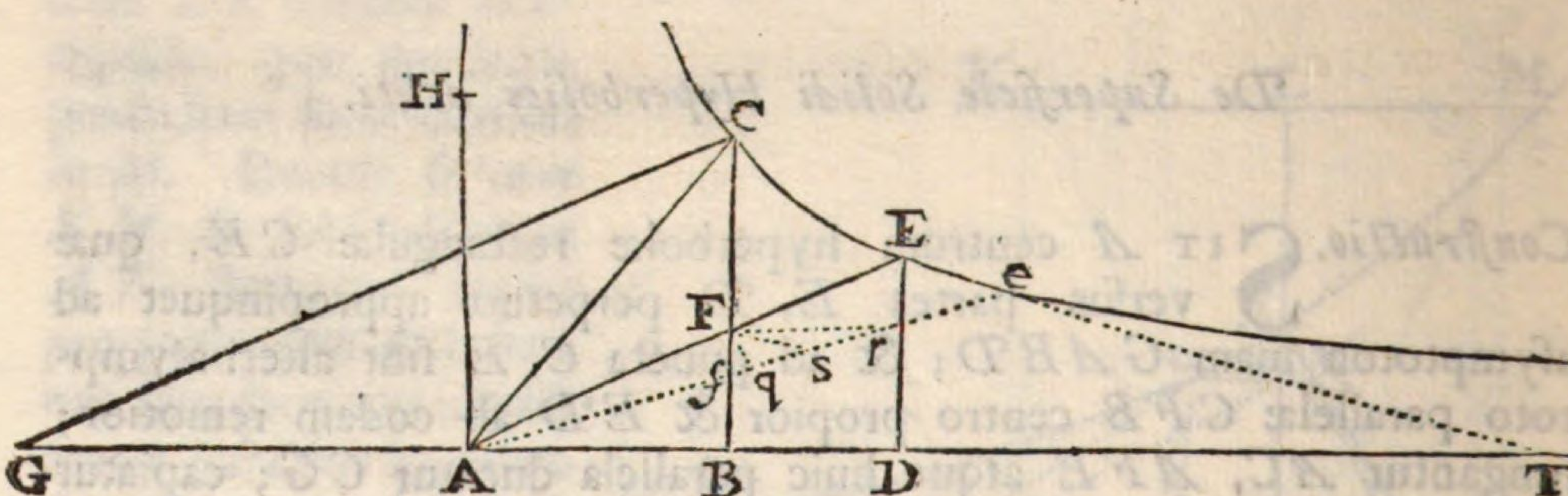
De Superficie Solidi Hyperbolici acuti.

Constructio. SIT A centrum hyperbolæ rectangulæ CE, quæ versus partes E, D perpetuo appropinquet ad asymptoton suam $GABD$; & ad puncta C, E sint alteri asymptoto parallelæ CFB centro propior & ED ab eodem remotior; jungantur AC, AFE atque huic parallela ducatur CG; capiatur



AH æqualis excessui rectæ AC supra AF & simul mensuræ rationis inter $CG+GB$ & $CA+AB$ ad modulum AB: & Superficies quæ generatur ex arcu CE circum BD asymptoton convoluto, erit ad Circulum semidiametro BC descriptum, ut AH ad BC.

Demonstratio. Augeatur arcus CE particula quam minima Ee, & erigantur normales Fq ad FA, Fr ad FB, quas secet recta Ae in q & r ipsamque FB in f. Erit ergo fq decrementum rectæ AF, vel incrementum excessus rectæ AC supra AF. Porro qF (quæ est incrementum mensuræ anguli BFA ad modulum FA) erit incrementum mensuræ rationis inter $FA+AB$ & BF ad modulum BF per Lemma: adeoque propter proportionales qF & rq, BF & AB, erit rq incrementum mensuræ ejusdem rationis inter $FA+AB$ & BF vel inter $CG+GB$ & BC, vel (subducendo rationem datam $CA+AB$ ad BC ut fluentes statuantur æquales) inter $CG+GB$ & $CA+AB$ ad modulum AB.



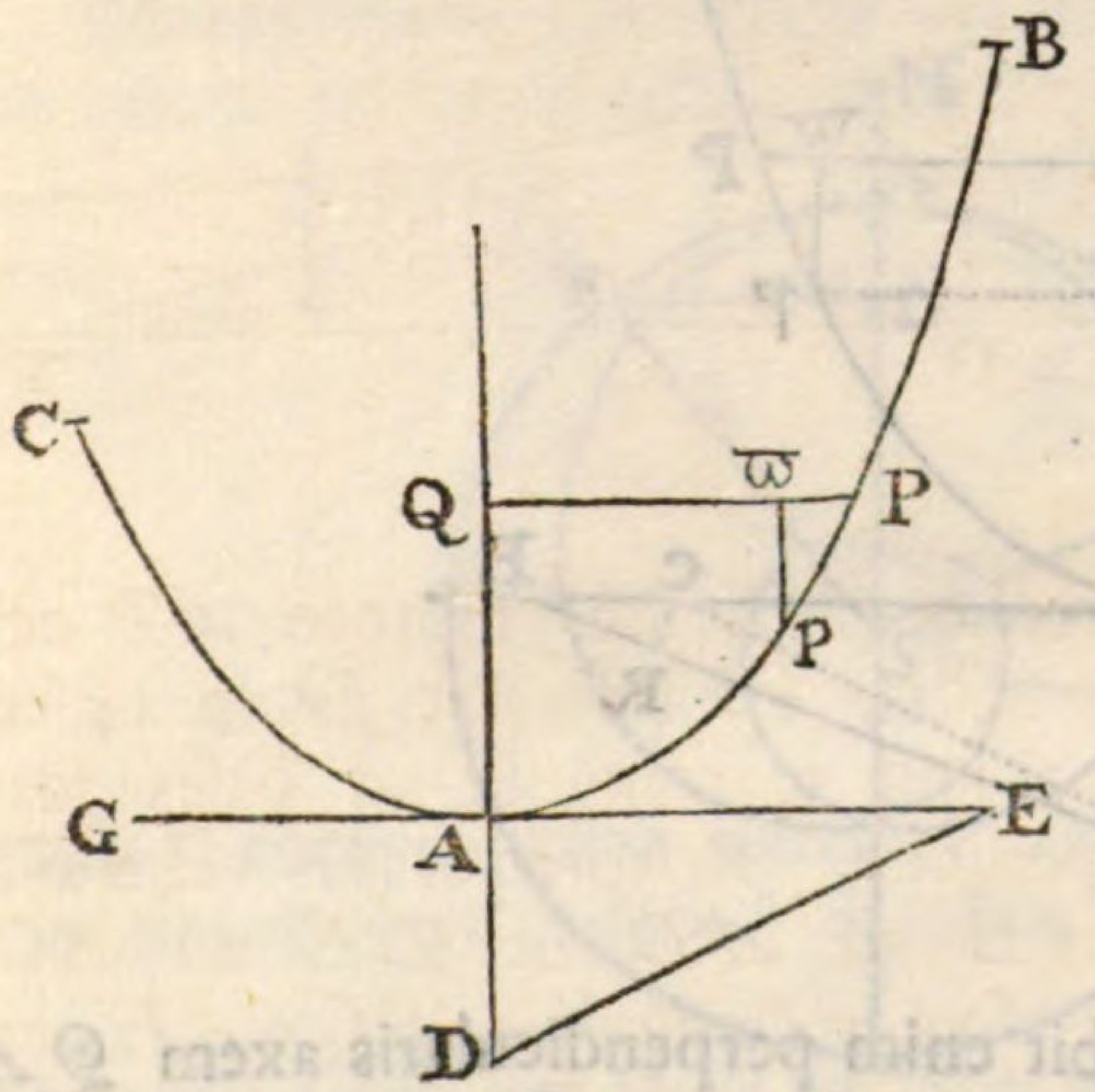
Unde tota fr est incrementum rectæ AH. Asymptoto AT occurrat tangens eT, atque huic parallela Fs bisecabit fr in s; eo quod triangula AeT, Fsr, Fsf sunt æquiangula ad bases suas AT, Fr, Ff ex natura hujus hyperbolæ. Erit itaque fr vel $2Fs$ ad $2Ee$ ut AF ad AE ut AB ad AD ut DE ad BC. Quare summa omnium $BC \times fr$, sive rectangulum $BC \times AH$ æquale est summæ omnium $2DE \times Ee$, id est Quadrato ex semidiametro circuli propositæ superficiei æqualis. Unde Superficies illa vel (quod perinde est) circulus eidem æqualis, erit ad Circulum semidiametro BC, ut $BC \times AH$ ad BC *quad.* vel, ut AH ad BC. *Q.E.D.*

EXEM-

EXEMPLUM VIII.

Invenire quotlibetcunque puncta Catenariæ.

Esto *BAC* catena tenuiſſima, ſeu potius linea mathematica per totam longitudinem ſuam vi quavis minima flexibilis, quæ neque extendi poſſit neque contrahi. Hæc a terminis *B* & *C* dependens vi proprii ponderis, per omnes ſuas particulas æquales æqualiter diffuſi, componatur in figuram curvam *BPpAC*: oportet hujus Figuræ quotlibetcunque puncta invenire.

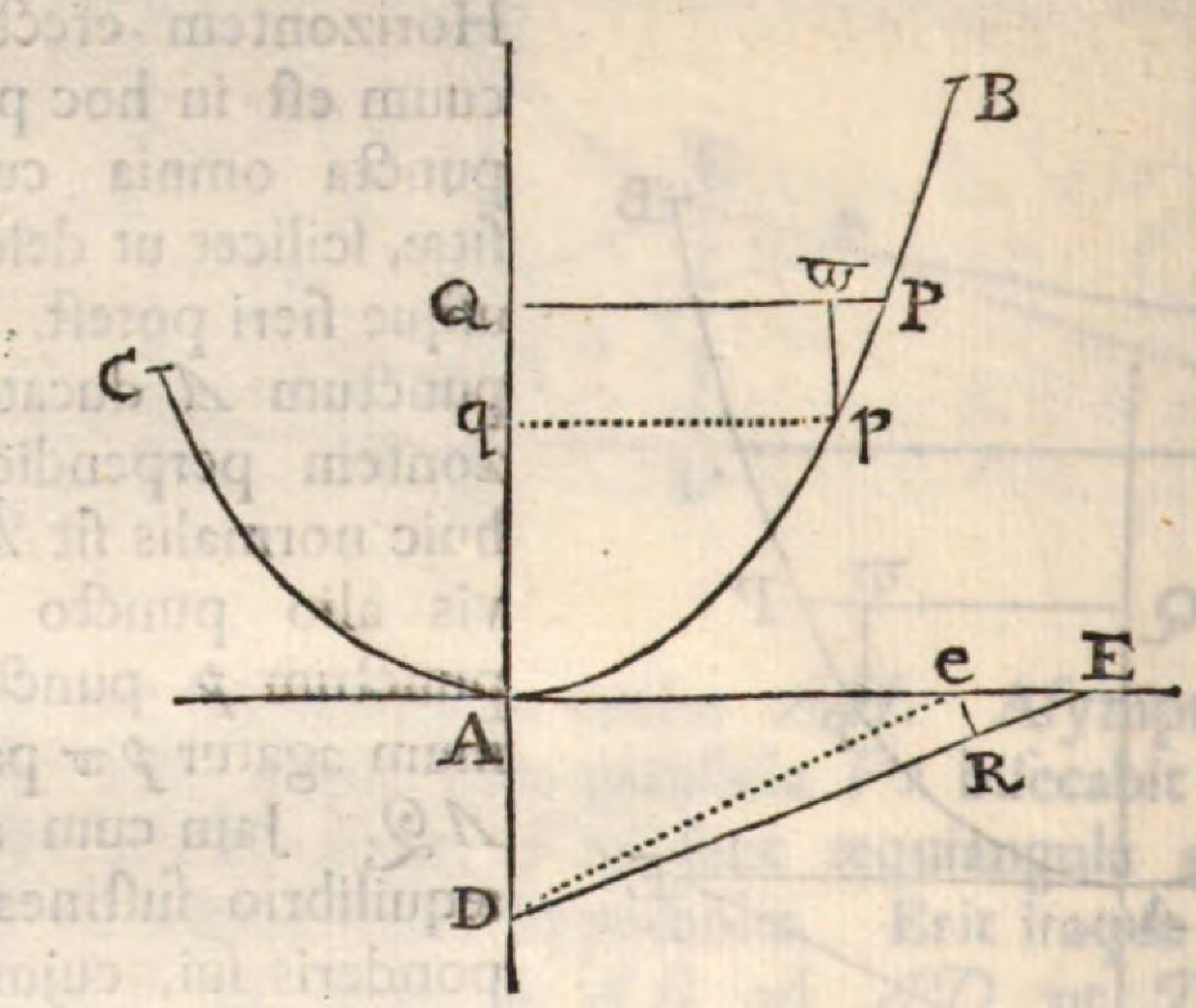


Si per terminos *B* & *C* tranſire intelligatur planum, quod ſit ad Horizontem erectum: perſpicuum eſt in hoc plano ſita eſſe puncta omnia curvæ propoſitæ, ſcilicet ut deſcendant quouſque fieri poteſt. Per infimum punctum *A* ducatur ad horizontem perpendicularis *AQ*, huic normalis ſit *PϖQ* a quovis alio puncto *P*, & per punctum *p* puncto *P* proximum agatur *pϖ* parallela rectæ *AQ*. Jam cum arcus *AP* in æquilibrio ſuſtineatur per vim ponderis ſui, cujus directio parallela eſt lineolæ *ϖp*; per vim contigui arcus *AC*, trahentis ſecundum directionem tangentis ad *A* lineolæ *Pϖ* parallelam; perque vim contigui arcus *PB*, trahentis in directione lineolæ *pP*: patet ex Mechanicis eſſe has vires ad invicem ut *ϖp*, *Pϖ*, *pP*. In recta horizontali *AeE* capiatur *AE* æqualis longitudini curvæ *AP*; & cum hujus pondus abſolutum, horizontem verſus, ſemper ſit ut longitudo ejus *AE*, ſit pondus illud ad datam fili tenſionem in *A*, ut *AE* ad datam rectam *AD*, in axe *QA* deorſum producto aſſumptam; actaque *DE*, ſimilia erunt

trian-

triangula DAE, $P\varpi p$. Capiatur porro Ae æqualis arcui Ap, actaque De, abſcindatur ex DE longitudo DR æqualis De & jungantur e, R; poſtremo ducatur pq axi AqQ perpendicularis: & triangula ERe, $p\varpi P$ erunt ſimilia & æqualia; ſimilia quidem propter eorum ſimilitudinem eidem triangulo EAD, & æqualia ob æquales lineolas Ee, Pp. Quare rectæ DE, DQ quæ coincidentibus punctis A, E, Q æquales extitere, & quæ æqualibus deinde acceſſionibus elementorum ER & Qq vel ϖp ſimiliter crevere, etiam per omnia æquales erunt: id quod pariter dicendum eſt de ordinata QP & eidem contemporanea menſura rationis inter $DE+EA$ & AD ad modulum AD, quarum incrementa $P\varpi$ & eR perpetuo æquantur.

Unde ſi in tangente AE, per infimum punctum A ducta, capiatur AE æqualis longitudini arcus alicujus dati AP, cujus altitudo ſit AQ, dabitur Parameter AD, jungendo QE & eandem



biſecando perpendiculariter; ſecabit enim perpendicularis axem QA deorſum productam in D. Data vero ſemel AD, deinde dabitur AE longitudo cujuſvis arcus AP, cujus datur altitudo AQ; centro D & intervallo DQ deſcribendo circulum qui ſecet AE in E. Atque hæ ſunt relationes mutuæ parametri AD, arcus AP & altitudinis ejus AQ.

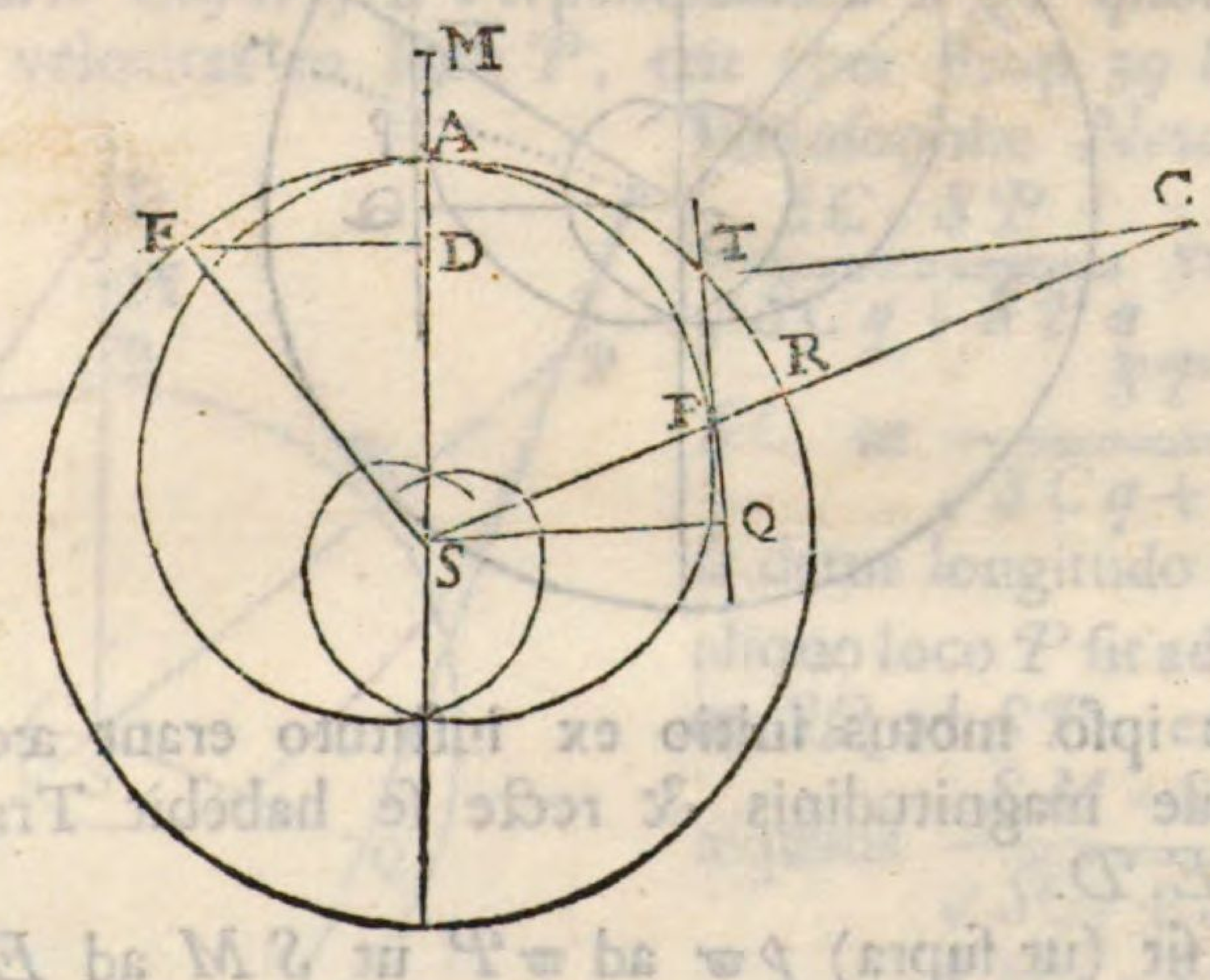
Amplitudo autem PQ erit menſura rationis inter $DE+EA$ & AD vel inter AD & $DE-EA$, vel dividendo inter $AP+AQ$ & $AP-AQ$ ad modulum AD. Dato itaque vel invento ut ſupra modulo AD, ſi aſſumantur in axe AQ quotlibetcunque puncta Q, invenientur totidem curvæ correſpondentia puncta P. $Q.E.I.$

EXEM-

EXEMPLUM IX.

Proponatur Trajectoriarum species enumerare, in quibus corpora moveri possunt, quæ a viribus tum centripetis tum centrifugis in ratione distantiarum triplicata decrescentibus seorsum agitantur; quæque de loco dato, data cum velocitate, secundum datam rectam egrediuntur. Consule Auctoris Constructiones in HARMONIÆ MENSURARUM p. 30. & 98.

Demonstratio Casus 1. PERPENDICULUM SQ, quod ubique est reciproce ut corporis velocitas in loco P, erit (per Prop. 39 & 40. Lib. 1. Philosophiæ *Newtonianæ*) ut $\frac{SC \times SP}{\sqrt{SCq - SPq}}$ vel, cum detur SC, ut $\frac{SP}{\sqrt{SCq - SPq}}$. Quare si detur longitudo SM, quæ in aliquo loco P sit ad $\sqrt{SCq - SPq}$ ut SQ ad SP: erit ubique SQ æqualis $\frac{SM \times SP}{\sqrt{SCq - SPq}}$ & SQq

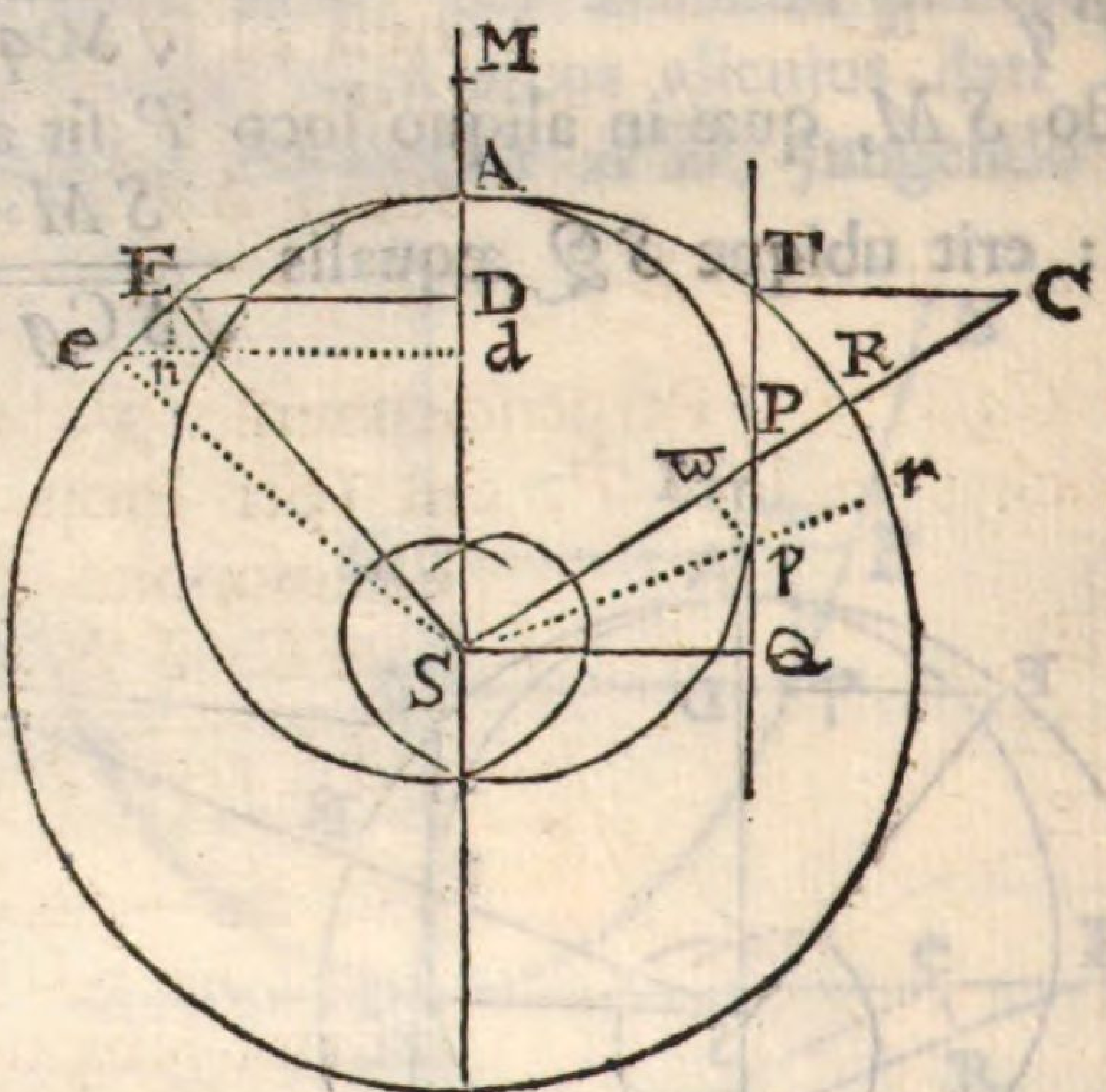


æquale $\frac{SMq \times SPq}{SCq - SPq}$ & QPq æquale $SPq - \frac{SMq \times SPq}{SCq - SPq}$ & QTq æquale $SCq - \frac{SMq \times SCq}{SCq - SPq}$ & $SQq + QTq$ æquale $SCq - SMq$. Dabitur itaque perpetuo $\sqrt{SQq + QTq}$ vel SR, eritque SM æqualis $\sqrt{SCq - SRq}$, & SQ jam erit ad QP ut SM ad ED vel $\sqrt{SRq - SPq}$. Hæ sunt proprietates Trajectoriæ

 jectoriæ

jectoriæ describendæ, quæ jam conferenda est cum illa quam in hoc loco Auctor descripsit.

Repetatur eadem constructio pro distantia Sd proxime minore quam SD & sint puncta d, e, r, p analoga punctis D, E, R, P; rectisque $S\varpi P$, dne normales sint $p\varpi$, En. His positis erit eE ad Rr in ratione composita ex rationibus eE ad ϖP, ϖP ad $p\varpi$, $p\varpi$ ad Rr. Sed eE est ad ϖP vel Dd vel En ut SE ad ED; & ϖP ad $p\varpi$, vel QP ad SQ ut ED ad SM (ut supra); & $p\varpi$ ad Rr, vel SP ad SR ut SD ad SE. Quare conjunctis his rationibus, erit eE ad Rr ut SD ad SM. Est autem eE incrementum mensuræ rationis inter $SE+ED$ & SD ad modulum SD per Lemma: adeoque Rr est incrementum mensuræ rationis ejusdem ad modulum SM. Unde arcus AR & men-

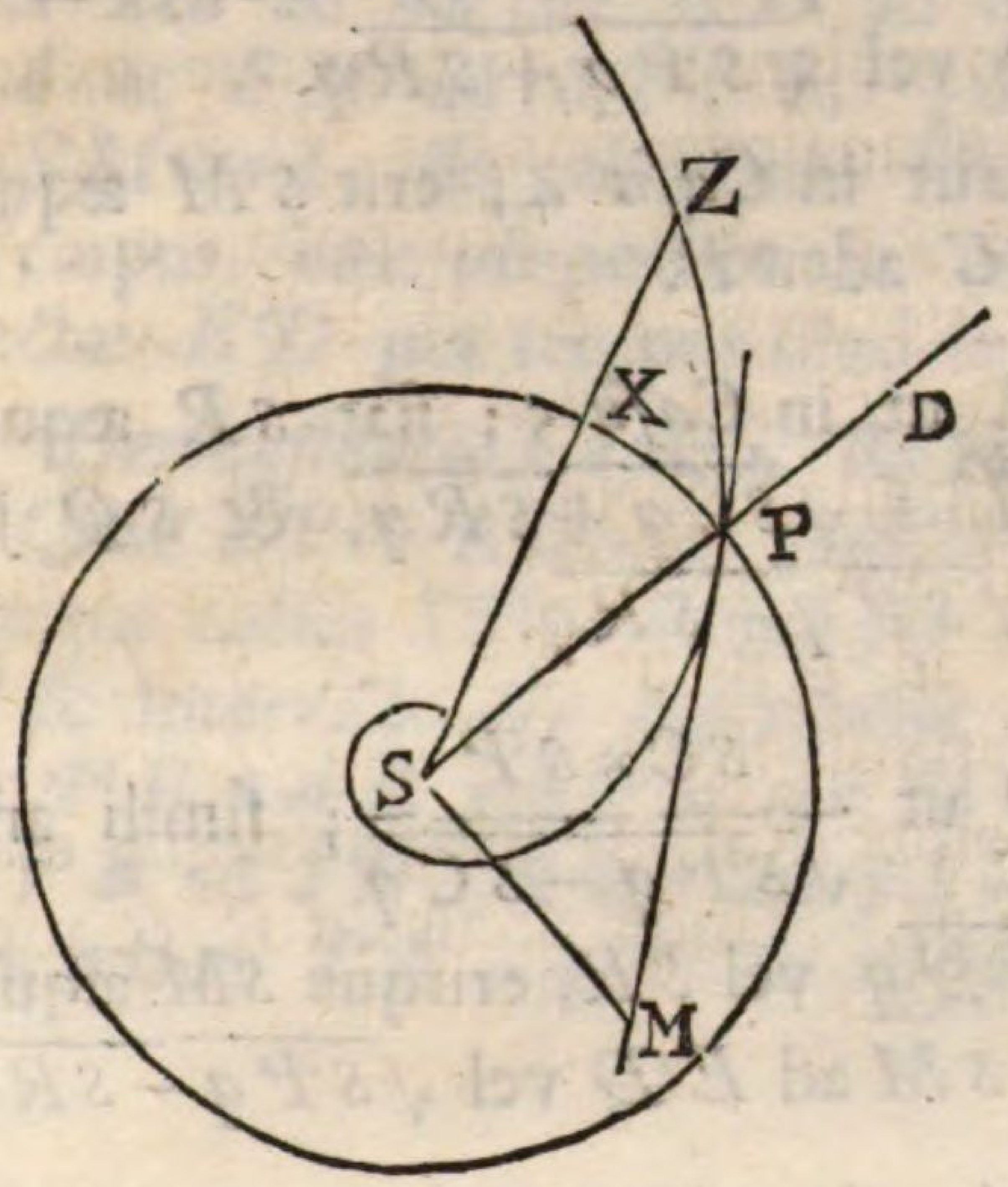


sura illa, quæ ipso motus initio ex instituto erant æqualia, erunt ejusdem ubique magnitudinis & recte se habebit Trajectoriæ descriptio. $Q.E.D.$

Porro cum sit (ut supra) $p\varpi$ ad ϖP ut SM ad ED, & ϖP vel En ad en ut ED ad SD vel SP: erit ex æquo $p\varpi$ ad en ut SM ad SP, & $\frac{1}{2}SP \times p\varpi$ vel areola SPp æqualis $\frac{1}{2}SM \times en$; & colligendo similes ubique summas, erit area ASP æqualis $\frac{1}{2}SM \times ED$; & Tempus, huic areæ proportionale, erit ut ED. Denique punctum A est Apsis summa; quia coeuntibus A & P, erit (ut supra) SQ ad SP ut SM ad $\sqrt{SCq - SRq}$, hoc est in ratione æqualitatis. Unde angulus SPQ rectus erit; & simili argumento, si Trajectoria altius ascenderet, foret SQ major quam SP, quod est absurdum.

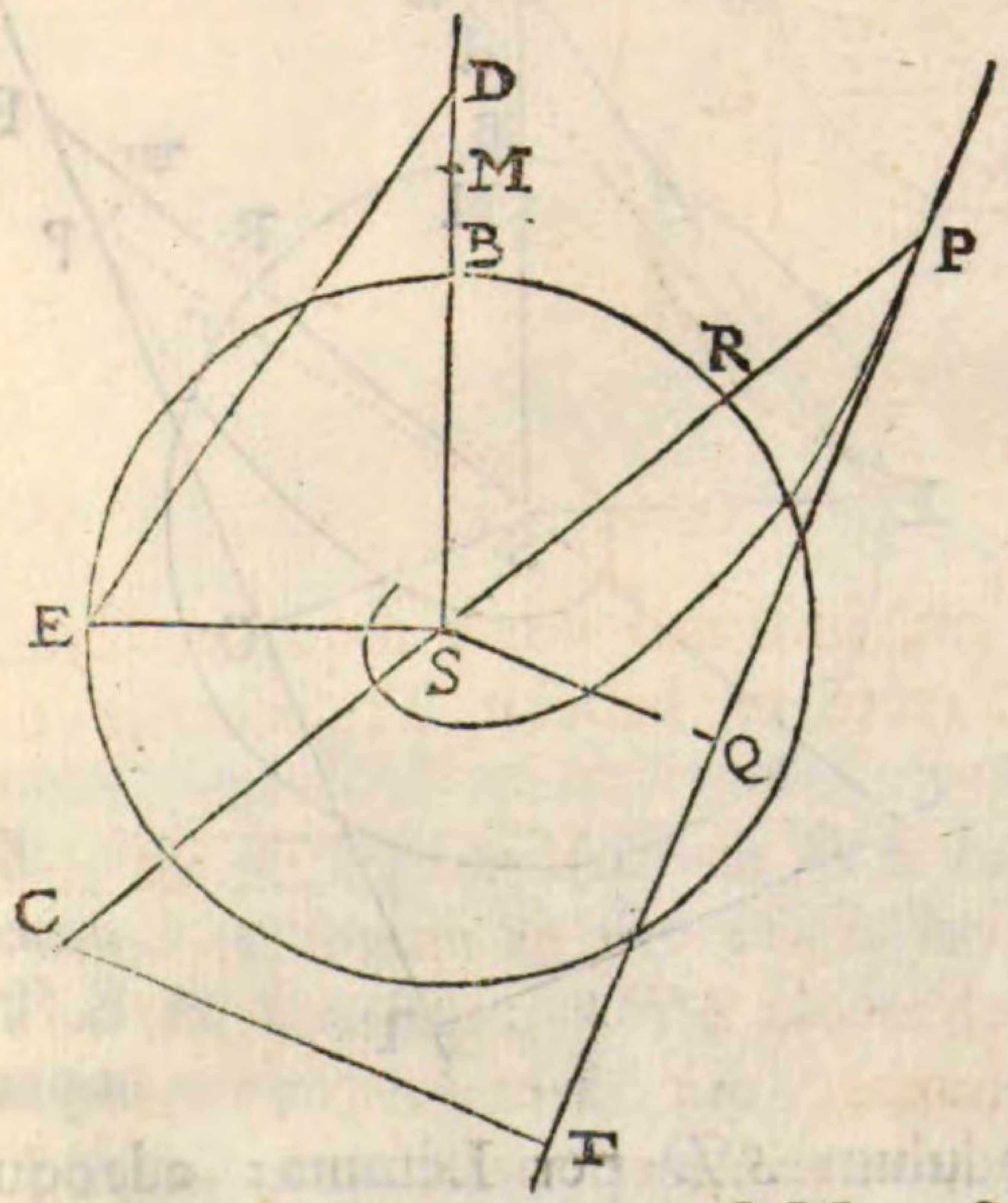
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Demonstratio Casus 2. Perpendiculum a centro S in tangentem PM demissum, quod ubique est reciproce ut velocitas in loco P, erit (per Prop. 39 & 40. Lib. I. Philosophiæ *Newtonianæ*) ut SP: adeoque dabitur ubique angulus SPM & Trajectoria erit Spiralis Æquiangula; cujus descriptio recte se habet per Prop. VI. *Logometriæ*. *Q.E.D.*



Porro Areæ totæ ab SZ & SP ad centrum usque protensæ (cum sint figuræ similes) erunt ad se invicem ut SZq ad SPq: atque adeo earum differentia SPZ, Tempusque huic areæ proportionale, erit ut $SZq - SPq$.

Demonstratio Casus 3. Perpendiculum SQ, quod ubique est reciproce ut velocitas in loco P, erit (per Prop. 39 & 40. Lib. I. Philosophiæ *Newtonianæ*) ut $\frac{SC \times SP}{\sqrt{SCq + SPq}}$ vel, cum detur SC, ut $\frac{SP}{\sqrt{SCq + SPq}}$. Quare si detur longitudo SM, quæ in aliquo loco P sit ad $\sqrt{SCq + SPq}$ ut SQ ad SP: erit SQ ubique æqualis $\frac{SM \times SP}{\sqrt{SCq + SPq}}$ & SQq æquale $\frac{SMq \times SPq}{SCq + SPq}$ & QPq æquale $SPq - \frac{SMq \times SPq}{SCq + SPq}$



& TQq æquale $SCq - \frac{SMq \times SCq}{SCq + SPq}$ & $TQq - SQq$ æquale $SCq - SMq$. Dabitur itaque perpetuo $TQq - SQq$. Ex quo tres oriuntur Trajectoriarum species; prout TQ vel major, vel æqualis, vel minor fuerit quam SQ.

Nam

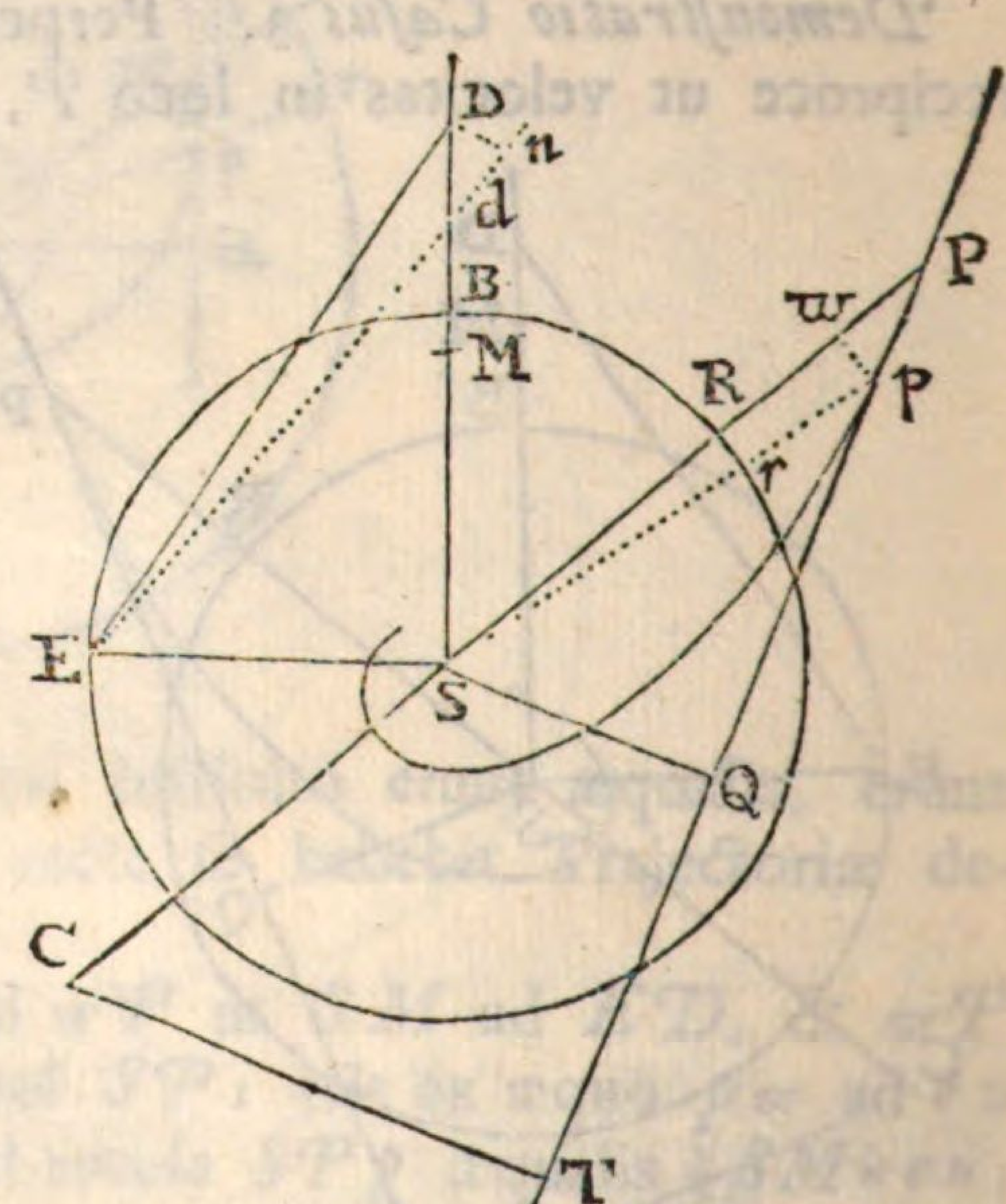
Nam si TQ fuerit major quam SQ, ut in hoc *Casu*; fiat SR æqualis $\sqrt{TQq - SQq}$, eritque SM æqualis $\sqrt{SCq - SRq}$, & SQ jam erit ad QP ut SM ad ED vel $\sqrt{SPq + SRq}$.

Si TQ fuerit æqualis ipsi SQ, ut in *Casu* 4; erit SM æqualis SC, & SQ jam erit ad QP ut SC ad SP.

Si TQ fuerit minor quam SQ, ut in *Casu* 5; fiat SR æqualis $\sqrt{SQq - TQq}$, eritque SM æqualis $\sqrt{SCq + SRq}$, & SQ jam erit ad QP ut SM ad ED vel $\sqrt{SPq - SRq}$.

Denique in Prob. X, cum sit SQ ut $\frac{SC \times SP}{\sqrt{SPq - SCq}}$; simili argumento dabitur perpetuo $\sqrt{SQq + TQq}$ vel SR, eritque SM æqualis $\sqrt{SRq - SCq}$, & SQ ad QP ut SM ad ED vel $\sqrt{SPq - SRq}$.

Cum his inventis jam conferendæ sunt Trajectoriæ, quas in his Casibus Auctor descripsit.

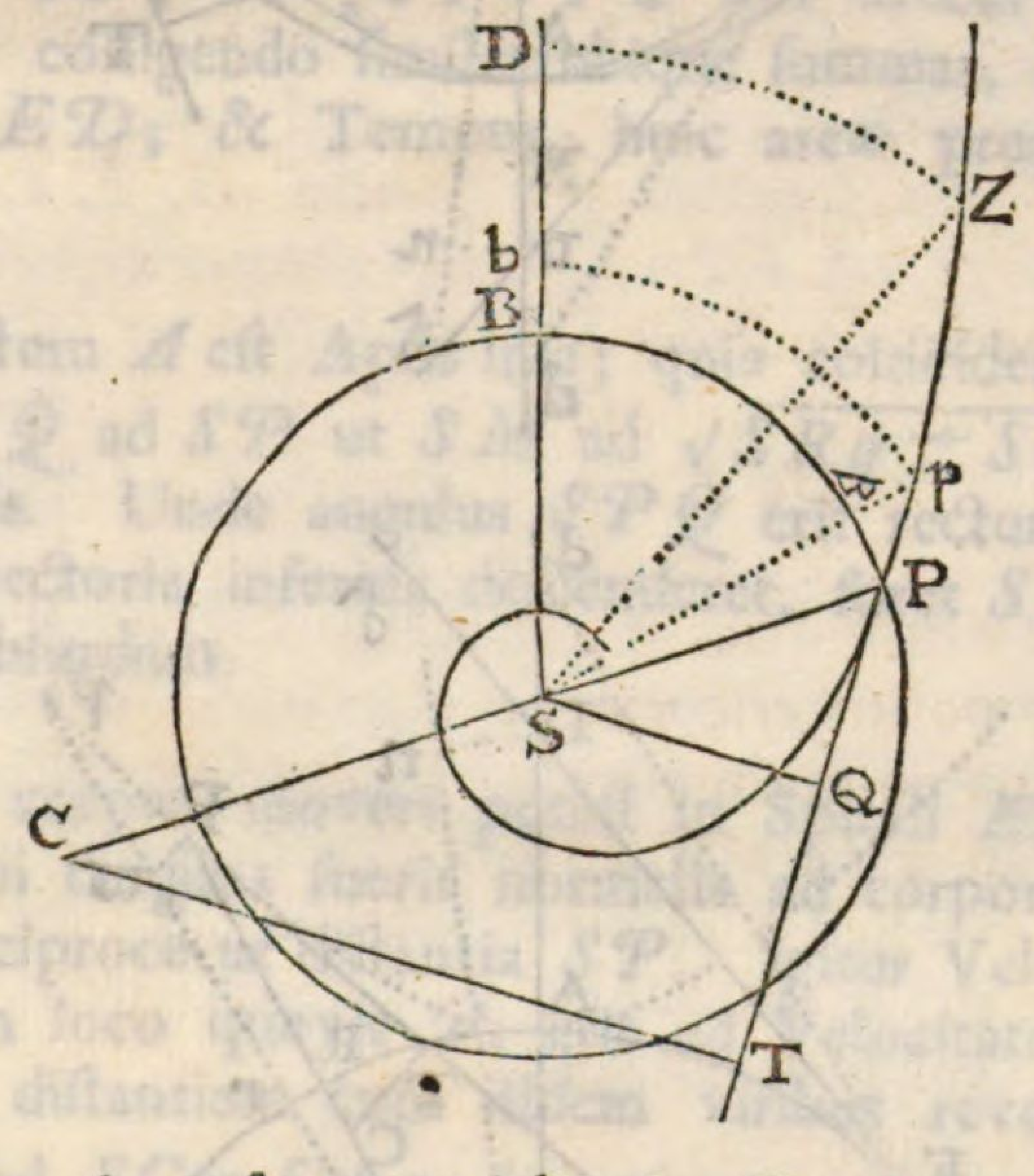


Repetatur eadem constructio pro distantia Sd proxime minore quam SD & sint puncta d, r, p analoga punctis D, R, P; rectisque $S\varpi P$, Edn normales sint $p\varpi$, Dn. His positis erit nD ad Rr in ratione composita ex rationibus nD ad ϖP, ϖP ad $p\varpi$, $p\varpi$ ad Rr. Sed nD est ad ϖP vel Dd ut SE ad ED; & ϖP ad $p\varpi$, vel QP ad SQ ut ED ad SM (ut supra); & $p\varpi$ ad Rr, vel SP ad SR ut SD ad SE. Quare conjunctis his rationibus, erit nD ad Rr ut SD ad SM. Est autem nD incrementum mensuræ rationis inter $DE + ES$ & SD ad modulum SD per Lemma: adeoque Rr est incrementum mensuræ rationis ejusdem ad modulum SM. Unde arcus BR & mensura illa, quæ ipso motus initio ex instituto erant æqualia, erunt ejusdem ubique magnitudinis & recte se habebit Trajectoriæ descriptio. *Q. E. D.*

Porro

Porro cum sit (ut supra) $p\varpi$ ad ϖP ut SM ad ED, & ϖP vel Dd ad nd ut ED ad SD vel SP: erit ex æquo $p\varpi$ ad nd ut SM ad SP; & $\frac{1}{2}SP \times p\varpi$ sive areola SPp æqualis $\frac{1}{2}SM \times nd$; & colligendo similes ubique summas, area quævis & Tempus, huic proportionale, erit ut incrementum vel decrementum rectæ ED per tempus illud factum.

Demonstratio Casus 4. Reliquis manentibus, erat SQ ad QP ut SC ad SP. Ascendat jam corpus P in locum proximum p, feraturque radius SP in situm Sp secans arcum PB in ϖ; dein centro S & intervallo Sp describatur arcus pb occurrens ipsi SD in b: & $PB - \varpi B$ vel $P\varpi$ erit ad ϖp ut SQ ad QP, sive ut SC vel * PB ad SP; & ϖp ad $pb - \varpi B$ ut SP vel $S\varpi$ ad ϖB; quare ex æquo $PB - \varpi B$ est ad $pb - \varpi B$ ut PB ad ϖB, hoc est,

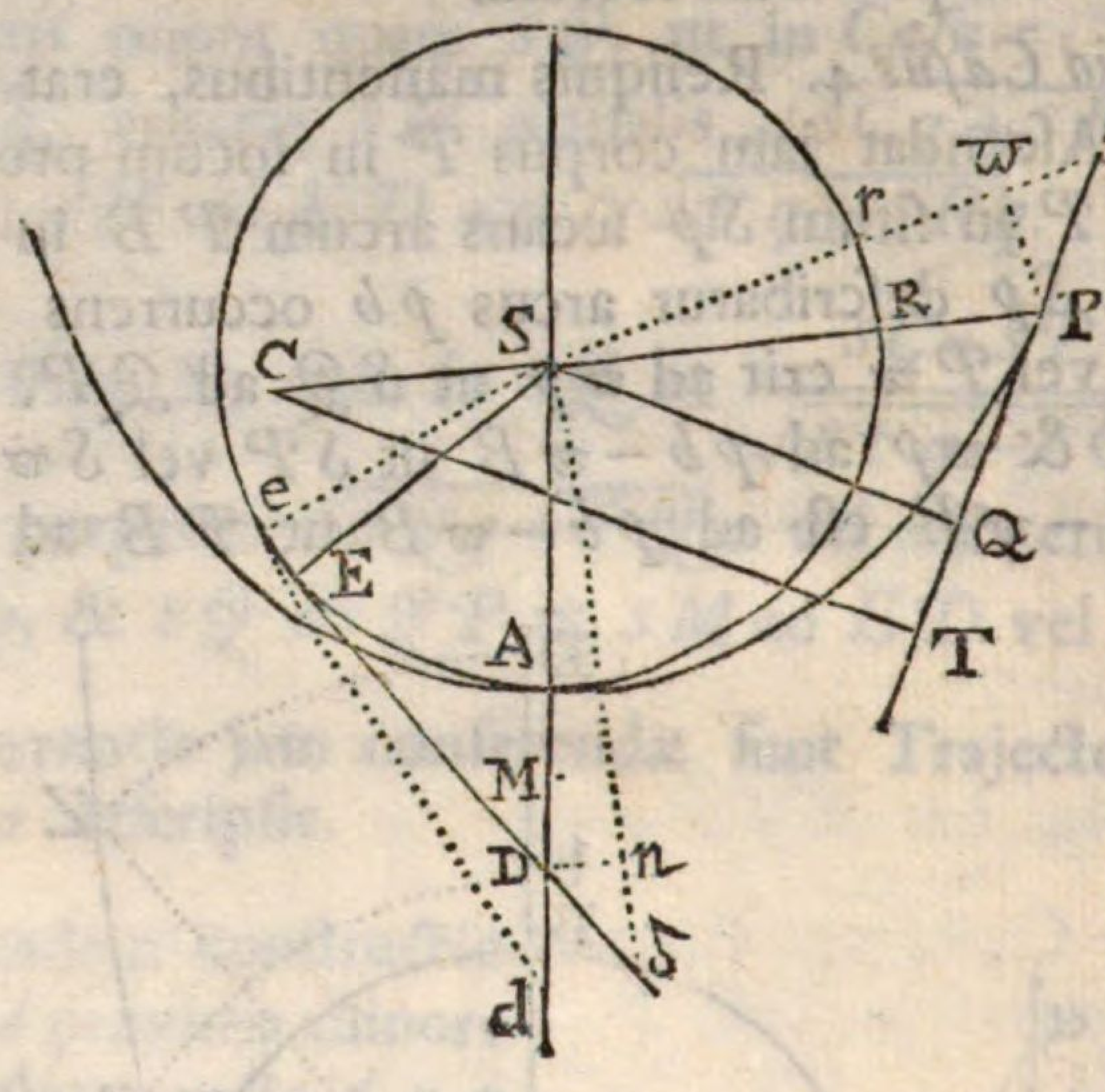


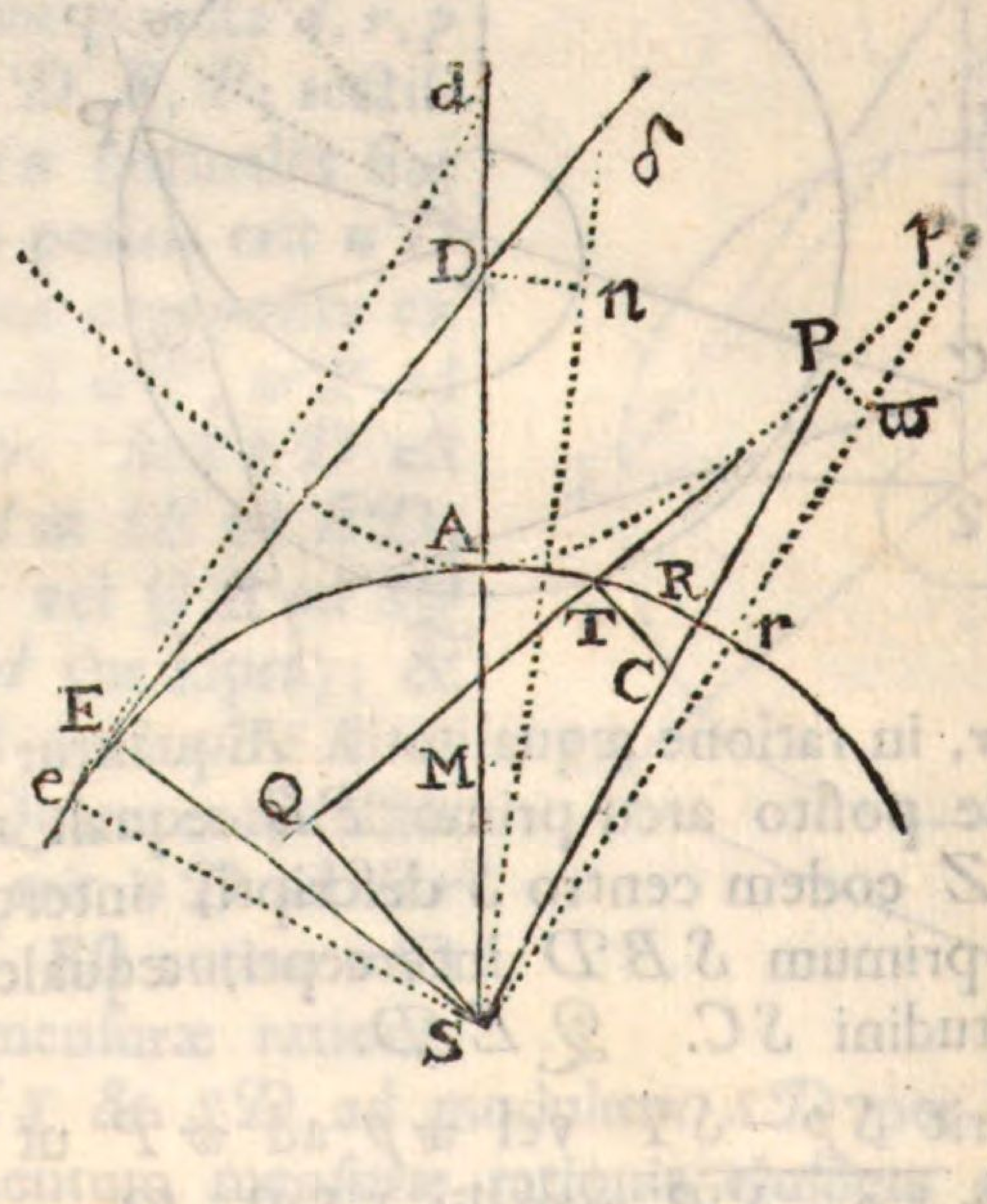
coeuntibus P & ϖ, in ratione æqualitatis. Æquantur itaque arcus PB & pb, & proinde posito arcu primo PB æquali rectæ SC; arcus omnes reliqui DZ eodem centro S descripti, interque Trajectoriam PZ & Radium primum SBD intercepti, æquales erunt inter se atque etiam longitudini SC. $Q.E.D.$

Porro cum fuerit $\overline{Sp - SP}$ vel ϖp ad ϖP ut SP ad SC (ut supra); erit $\frac{1}{2}SC \times \overline{Sp - SP}$ æqualis $\frac{1}{2}SP \times P\varpi$ vel areolæ SPp: & colligendo similes ubique summas, $\frac{1}{2}SC \times \overline{SZ - SP}$ erit æqualis areæ toti SPZ & Tempus, huic areæ proportionale, erit ut $SZ - SP$.

* Per Constructionem.

Demonstratio Casus 5. & Prob. X. Reliquis adhuc manentibus, repetantur Auctoris constructiones pro distantia *Sd* proxime majore quam *SD* & sint puncta *d*, *e*, *r*, *p* analoga punctis *D*, *E*, *R*, *P*; trian-





gulo *Sed* fiat æquale triangulum *SEδ*, in angulo nempe *SED* inscribendo *Sδ* æqualem ipsi *Sd*; rectisque *Snδ*, *Sϖp* normales sint *Dn*, *Pϖ*. His positis, erit *Dn* ad *Rr* in ratione compo-

sita

ſita ex rationibus $\mathcal{D}n$ ad ϖp, ϖp ad $\mathcal{P}\varpi$, $\mathcal{P}\varpi$ ad Rr. Sed $\mathcal{D}n$ eſt ad ϖp vel $\mathcal{D}d$ vel $n\delta$ ut SE ad $E\mathcal{D}$; & ϖp ad $\mathcal{P}\varpi$, vel $\mathcal{Q}\mathcal{P}$ ad $S\mathcal{Q}$ ut $E\mathcal{D}$ ad SM (ut ſupra); & $\mathcal{P}\varpi$ ad Rr, vel $S\mathcal{P}$ ad SR ut $S\mathcal{D}$ ad SE. Quare conjunctis his rationibus, erit $\mathcal{D}n$ ad Rr ut $S\mathcal{D}$ ad SM. Eſt autem $\mathcal{D}n$ incrementum menſuræ anguli $\mathcal{D}SE$ ad modulum $S\mathcal{D}$, adeoque * Rr eſt incrementum menſuræ anguli ejuſdem ad modulum SM. Unde arcus AR & menſura illa, quæ ipſo motus initio ex inſtituto erant æqualia, erunt ejuſdem ubique magnitudinis & recte ſe habebunt Auctoris conſtructiones. $\mathcal{Q}.E.\mathcal{D}.$

Porro cum ſit (ut ſupra) $\mathcal{P}\varpi$ ad ϖp ut SM ad $E\mathcal{D}$, & ϖp vel $n\delta$ ad $\mathcal{D}\delta$ ut $E\mathcal{D}$ ad $S\mathcal{D}$ vel $S\mathcal{P}$: erit ex æquo $\mathcal{P}\varpi$ ad $\mathcal{D}\delta$ ut SM ad $S\mathcal{P}$; & $\frac{1}{2}S\mathcal{P}\times\mathcal{P}\varpi$ vel areola $S\mathcal{P}\varpi$ æqualis $\frac{1}{2}SM\times\mathcal{D}\delta$; Et colligendo ſimiles ubique ſummas, erit area $AS\mathcal{P}$ æqualis $\frac{1}{2}SM\times E\mathcal{D}$; & Tempus, huic areæ proportionale, erit ut $E\mathcal{D}$.

Denique punctum A eſt Apſis ima; quia coincidentibus A & $\mathcal{P}$, erit (ut ſupra) $S\mathcal{Q}$ ad $S\mathcal{P}$ ut SM ad $\sqrt{SRq \pm SCq}$, hoc eſt in ratione æqualitatis. Unde angulus $S\mathcal{P}\mathcal{Q}$ erit rectus; & ſimili argumento, ſi Trajectoria inferius deſcenderet, foret $S\mathcal{Q}$ major quam $S\mathcal{P}$, quod eſt abſurdum.

Velocitas qua corpus moveri poteſt in Spirali Æquiangula, hoc eſt in Circulo ubi tangens fuerit normalis ad corporis a centro diſtantiam, erat reciproce ut diſtantia $S\mathcal{P}$. Igitur Velocitas corporis in Trajectoriarum loco quovis $\mathcal{P}$, erit ad Velocitatem qua in Circulo ad eandem diſtantiam cum iiſdem viribus revolvi poſſet, ut $\sqrt{SCq - S\mathcal{P}q}$ ad SC in Caſ. 1; in ratione æqualitatis in Caſ. 2; ut $\sqrt{SCq + S\mathcal{P}q}$ ad SC in Caſ. 3, 4 & 5; ut $\sqrt{S\mathcal{P}q - SCq}$ ad SC in Prob. X. $\mathcal{Q}.E.\mathcal{D}.$

Denique ut majori obſequio lectorem demerear, Demonſtrationem Trajectoriarum in Scholio Prob. X. deſcriptarum, ex Auctoris adverſariis depromptam, hic in lucem proferam.

* Per Corol. 2. Prop. in Nota I.

LEMMA

LEMMA.

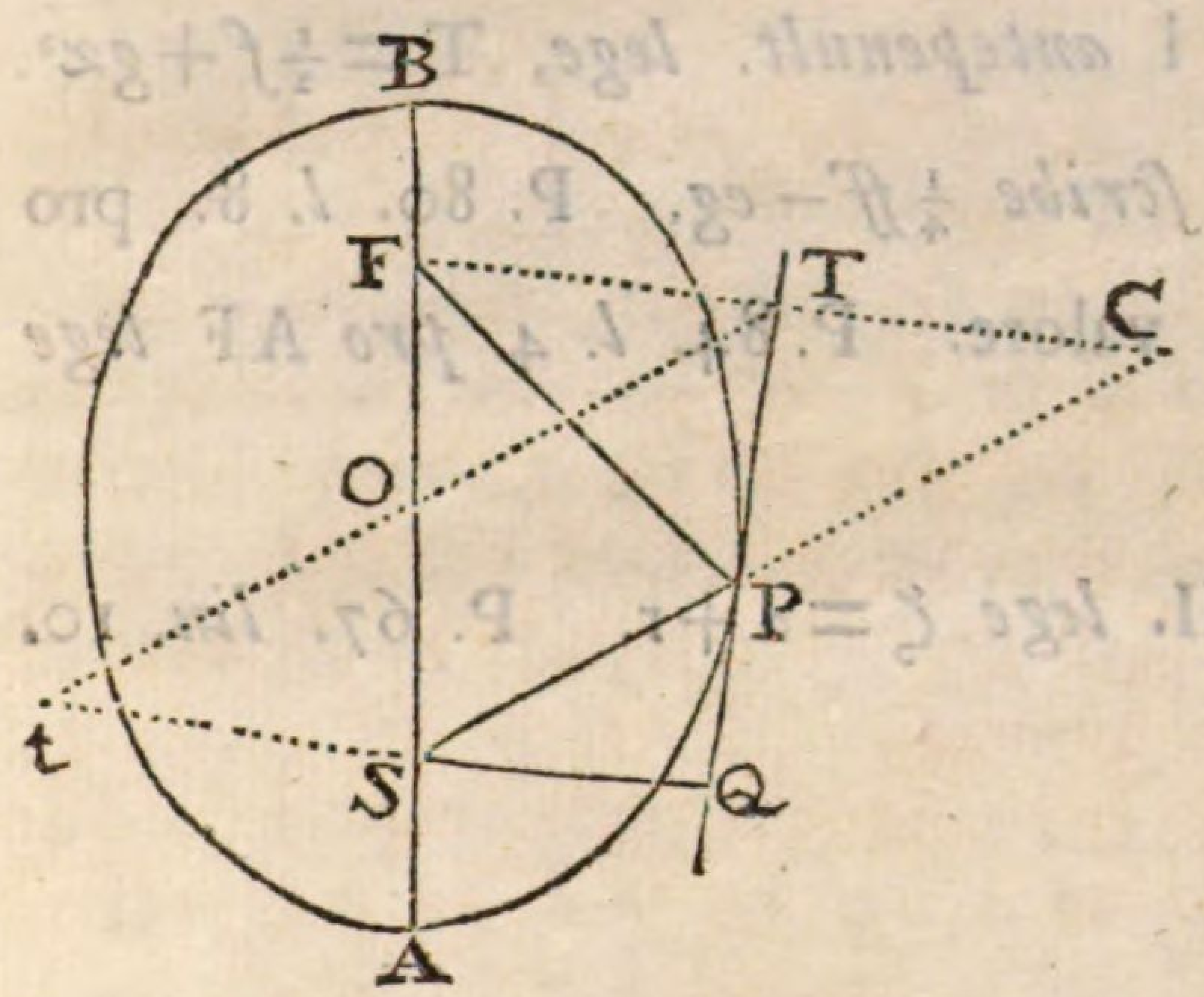
Si Ellipsin vel Hyperbolam quamvis datam APB *tangat indefinita recta* QPT *ubivis in* P; *& ab umbilicorum* S, F *alterutro* S *ad tangentem illam demittatur perpendicularis* SQ; *& ducantur* SP, FP: *dico perpendicularem* SQ *fore ut* $\sqrt{\frac{SP}{FP}}$.

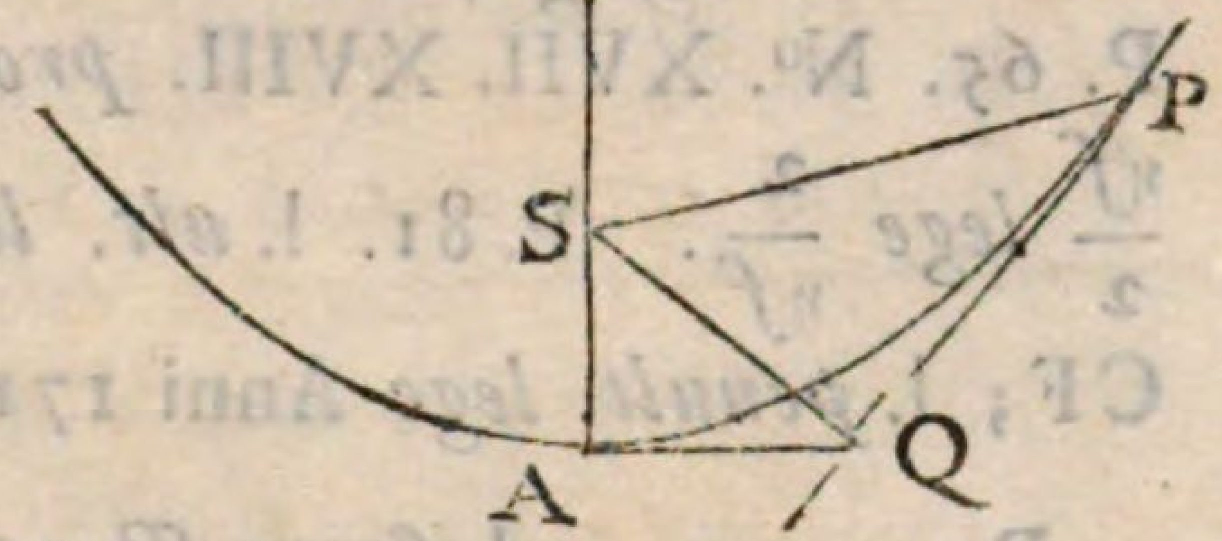
AD eandem tangentem *QT* demittatur altera perpendicularis *FT*, quæ producta occurrat ipsi *SP* (productæ) in *C*: & recta *TO* quæ bisecat *FS* in centro *O*, propter æquales angulos *TPF*, *TPC*, bisecabit etiam *FC* in *T*; & proinde parallela erit ipsi *SC* & æqualis ejus dimidio; vel dimidio summæ aut differentiæ rectarum *SP*, *PF*; vel semiaxi principali *OA* vel *OB*. Productâ itaque *TO* donec occurrat ipsi *QS* (productæ) in *t*; circulus centro *O* descriptus transibit per puncta *B*, *T*, *t*, *A*, *Q*: adeoque rectangulum *tSQ* vel $FT \times SQ$ erit ubique æquale rectangulo dato *ASB*. Unde cum sit $FT \times SQ$ ad *SQq* ut *FT* ad *SQ* vel ut *FP* ad *SP*; erit *SQq* ut $\frac{SP}{FP}$ & *SQ* ut $\sqrt{\frac{SP}{FP}}$. *Q.E.D.*

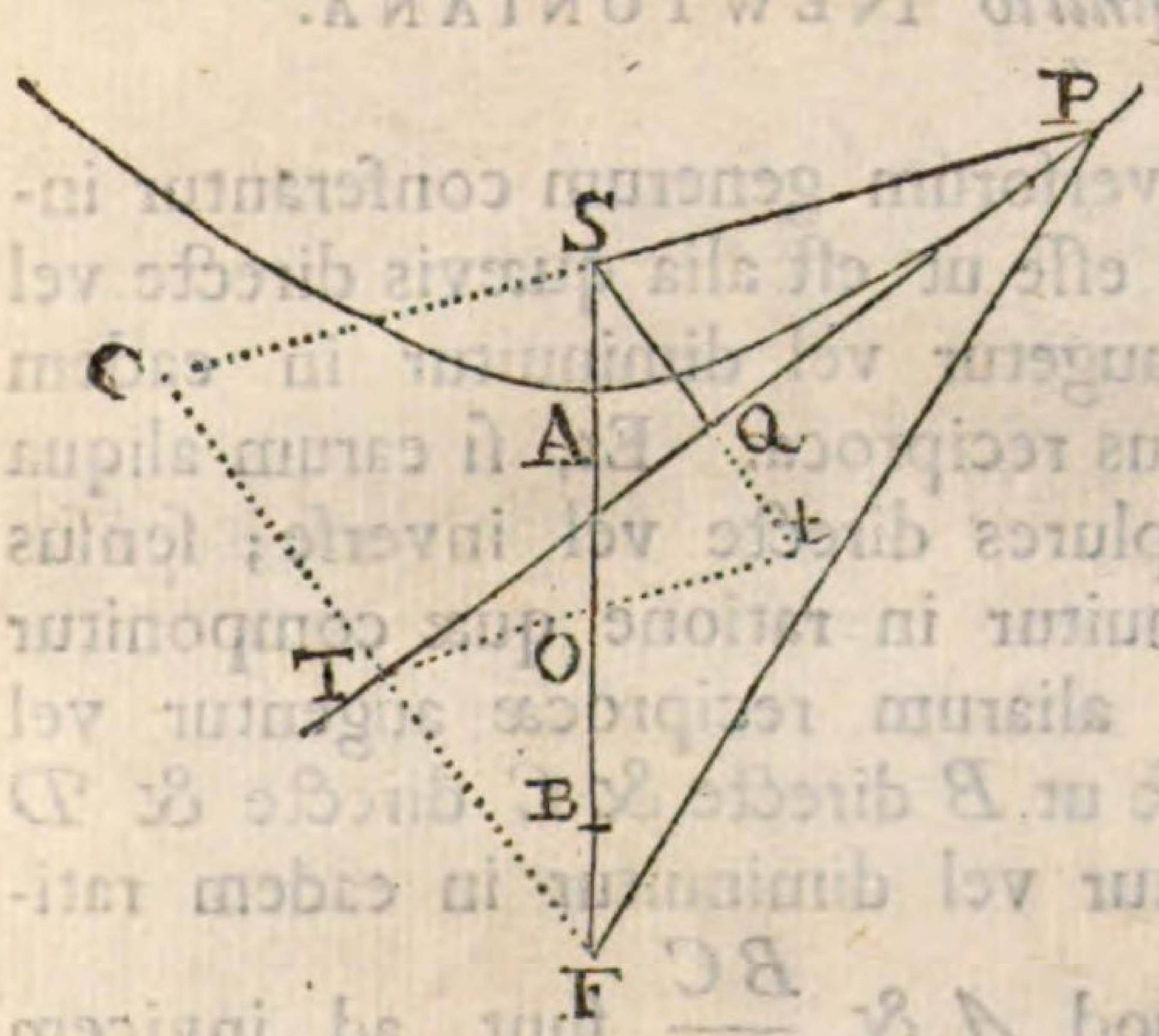
Corol. Manentibus *S* & *A* punctis; si *F* & *B* recesserint ad distantiam infinitam, mutabuntur hæ figuræ in Parabolam: & cum detur *FP*; erit jam *SQ* in subduplicata ratione rectæ *SP*; vel *SQq* ut *SP* & ubique æquale $SA \times SP$, prout fit in vertice *A*. Unde cum sint *SA*, *SQ*, *SP* proportionales; &, ex natura Parabolæ, anguli *ASQ*, *QSP* æquales: si connectantur *A*, *Q*; triangula *ASQ*, *QSP* erunt æquiangula.

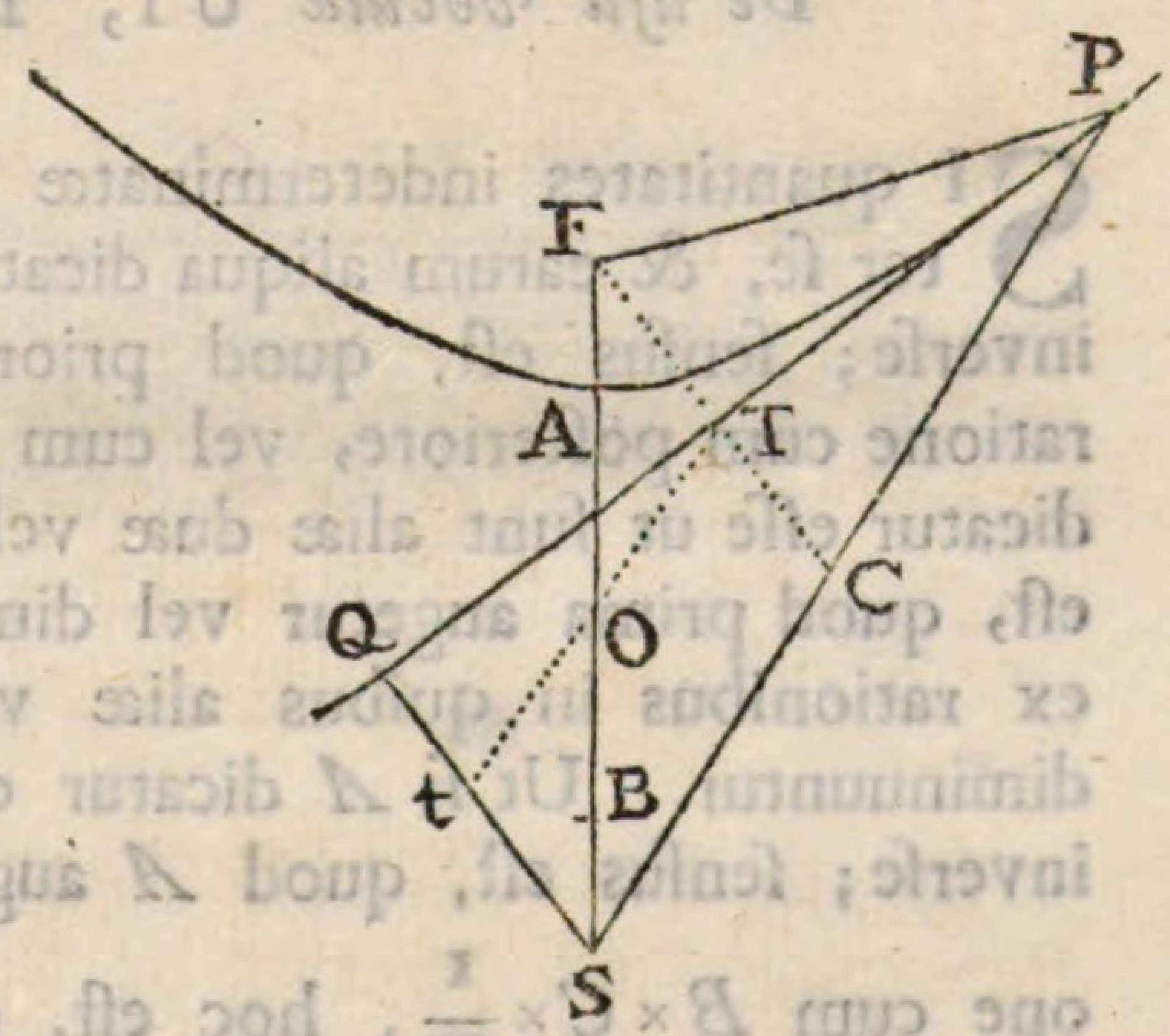
Demonstratio Scholii Prob. X. p. 102. Perpendicularis *SQ*, quæ ubique est reciproce ut Velocitas corporis in loco *P*, erit (per Prop. 39 & 40. Lib. I. Philosophiæ *Newtonianæ*) ut $\sqrt{\frac{SP}{SC-SP}}$ in Cas. 1; in subduplicata ratione distantiæ *SP* in Cas. 2; ut $\sqrt{\frac{SP}{SC+SP}}$ in Cas. 3; ut $\sqrt{\frac{SP}{SP-SC}}$ in Cas. 4. Constabit igitur ex Lemmate, Trajectoriarum descriptiones recte se habere. *Q.E.D.*

Si V ſit quantitas vis abſolutæ centralis; adeoque vis acceleratrix jam ut $\frac{V}{SPq}$: conſtabit per eaſdem propoſitiones Velocitatem in his omnibus fore ubique ut $\sqrt{\frac{V \times FP}{AB \times SP}}$. Hæc autem quantitas in Circulo ſit $\sqrt{\frac{V}{AB}}$; in Parabola ſit $\sqrt{\frac{V}{SP}}$.









Corol. Corpus in Ellipſi quavis revolvens, motu ſuo ſurſum verſo, aſcendet ad diſtantiam a centro virium æqualem axi principali figuræ.

F I N I S.

CORRIGENDA.

PAG. 11. *lin.* 27. *lege*, quadrare. P. 14. *l.* 6. *lege* Newtonianæ; *lin.* 32. *lege*, Libet. P. 47. *Adde*, In hac Forma adhibere licet vel R. T. S. vel *r. t. s*; nam $e\,r\left|\frac{r+t}{s}\right. = -f\,R\left|\frac{R+T}{S}\right.$. P. 57. l. *penult.* *lege*, $s = \sqrt{\frac{eh-fg}{h.\overline{g+hz^n}}}$. P. 58. l. *antepenult.* *lege*, $T = \frac{1}{2}f + gz^n$. P. 65. N°. XVII. XVIII. *pro q ſcribe* $\frac{1}{4}ff - eg$. P. 80. *l.* 8. pro $\frac{nf}{2}$ *lege* $\frac{2}{nf}$. P. 81. l. *ult.* *lege*, valore. P. 84. *l.* 4. *pro* AF *lege* CF; l. *penult.* *lege* Anni 1712.

PAG. 27. *ad finem Prop.* III. *lege* $\zeta = \alpha + r$. P. 67. *lin.* 10. *lege* $\frac{81}{2}$ ②.

De uſu voculæ UT, *Definitio* NEWTONIANA.

SI quantitates indeterminatæ diverſorum generum conferantur inter ſe, & earum aliqua dicatur eſſe ut eſt alia quævis directe vel inverſe; ſenſus eſt, quod prior augetur vel diminuitur in eadem ratione cum poſteriore, vel cum ejus reciproca. Et, ſi earum aliqua dicatur eſſe ut ſunt aliæ duæ vel plures directe vel inverſe; ſenſus eſt, quod prima augetur vel diminuitur in ratione quæ componitur ex rationibus in quibus aliæ vel aliarum reciprocæ augentur vel diminuuntur. Ut ſi *A* dicatur eſſe ut *B* directe & *C* directe & *D* inverſe; ſenſus eſt, quod *A* augetur vel diminuitur in eadem ratione cum $B \times C \times \frac{1}{D}$, hoc eſt, quod *A* & $\frac{BC}{D}$ ſunt ad invicem in ratione data.

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